An Experience with Some Able Women Who Avoid Mathematics

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> And on the eighth day, God created mathematics He took stainless steel, and he rolled it out thin, and he made it into a fence forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules, theorems, axioms and pointed reminders. 'Invert and multiply ' 'The square on the hypotenuse is three decibels louder than one hand clapping ' 'Always do what's in the parentheses first ' And when he was finished, he said 'On one side of this fence will reside those who are good at math. And on the other will remain those who are bad at math, and woe unto them, for they shall weep and gnash their teeth.'

> Math does make me think of a stainless steel wall – hard, cold, smooth, offering no handhold, all it does is glint back at me Edge up to it, put your nose against it, it doesn't give anything back, you can't put a dent in it, it doesn't take your shape, it doesn't have any smell, all it does is make your nose cold I like the shine of it – it does look smart, intelligent in an icy way. But I resent its cold impenetrability, its supercilious glare

Many students of mathematics believe that the subject is only rules to be memorized, skills to be practiced, and methods to be followed precisely I would like to propose that, for some adults, this view is not consistent with their more relativistic view of knowledge in general; this discrepancy in world view and view of mathematics causes discomfort with mathematics; and closing the gap between these disparate world views can make these adults feel more comfortable in approaching mathematics.

Lenses to view mathematics

I have spoken to many people, especially to mature women who avoid mathematics or feel apprehensive about it Many discuss experiences with timed tests or flash cards that led to embarrassment, an embarrassment that can be felt even years later when recalling the events. Many believed that if they were good at mathematics, then they should be very quick and competitive with it, and the failure to be quick and competitive led to feelings of inadequacy – especially if they were able people who thought reflectively. Many simply felt powerless in the face of mathematics because, as one woman put it, "the wicked mathematician has all the answers in the back of the book" [see Potter, 1978 for more of her statement].

Their experiences and feelings indicate a conception of mathematical knowledge that is termed "dualistic " I use "dualistic" as it is used by William G. Perry, Jr. [1970, 1981] in his developmental scheme describing how adults view knowledge (See Note 1). He defines dualism as:

> Division of meaning into two realms – Good versus Bad, Right versus Wrong, We versus They All that is not Success is Failure, and the like. Right Answers exist *somewhere* for every problem, and authorities know them. Right Answers are to be memorized by hard work Knowledge is quantitative Agency is experienced as "out there" in Authority, test scores, the Right Job [Perry, 1981, p. 79]

Several women with whom I have worked in a small group setting express this dualistic perception of mathematical knowledge vividly, and also the discomfort that comes with it. One expression of this view opens this paper and two others follow:

> I think of math problems or situations as having right and wrong answers (very black and white), but having a variety of ways to reach the answer. Unfortunately, my math teachers never stressed the fact that there could be more than one way to approach a problem. For this reason, and there are other reasons, I do not see math as a "creative activity " It is most definitely not linked to language, or music, or the other humanities. *Sonya* (See note 2)

> It is encouraging to me, when reading (about mathematics) to see the acceptance or existence of unanswered questions and "puzzlement " I feel that some of the "pressure" is taken off me to produce THE ANSWER via THE METHOD *Sophie*

Must mathematics be viewed dualistically? Is mathematics only a collection of correct answers and proper methods? It is clear that this is not the view of those who write in *For the Learning of Mathematics*. The mathematics discussed in these pages evolves through a dynamic process that is exciting to those who discuss and experience it For example, Henderson [1981] believes "that mathematics has meaning that can be experienced and imagined." Brown [1981] states:

> I believe that it is a serious error to conceptualize of (sic) mathematics as anything other than a human enterprise which among other things helps to clarify who we are and what we value.

In contrast to the experiences of these FLM writers, the women with whom I worked have experienced mathematics in a dualistic mode They see it as a discipline that is rigid, removed, aloof, and without human ties, rather than one that is being discovered and developed It is a collection of answers rather than a dynamic process that is alive and changing. The Authorities, the mathematicians, are mistrusted and suspect.

The women quoted above do not view other areas of their lives and experiences in this same dualistic way. General data from each of them have been rated at the positions Perry calls "relativism," in which one accepts all knowledge and all values as relative and contextual:

> Comparison, involving systems of logic, assumptions, and inferences, all relative to context, will show some interpretations to be "better," others "worse," many worthless Yet even after extensive analysis there will remain areas of great concern in which reasonable people will reasonably disagree It is in this sense that relativism is inescapable and forms the epistemological context of all further developments. [Perry, 1981, p 88]

The acceptance of relativism as the way the world is, is a major – in fact, drastic – change. It is a change from stressing quantity to stressing quality, from relying on Authority, who has the answers and passes them on, to interacting with authority as an expert who can help in the intellectual search It is a shift from an external source for action, for power, and for responsibility, to an internal one. A relativist becomes caught up in the excitement of ideas, the interrelationships of ideas, and an urge to play around with ideas. Asking questions and listening to the ideas of others is easier, but much learning is an active process in which the learner remains open to ideas, is self-processing, and even initiates the exploration Care is taken to be precise in thought, to reason systematically, and to keep the context in focus. The security of relativism comes in this careful exploration of alternatives in many different areas of life

The insecurity of relativism centers around the variety of alternatives and the realization that choices need to be made in some of these areas. Movement beyond relativism in Perry's scheme comes with the awareness that the individual herself/himself is the only one who can make these commitments At issue now is responsibility, the individual's responsibility for personal commitments as a means of orienting himself/herself in a relativistic world. Perry calls this "Commitment in Relativism"

The discrepancy between the way these women viewed mathematical knowledge and the way they viewed knowledge in general both puzzled and disturbed me How could women who were so able and intellectually mature as these women happened to be, seriously view mathematics in such a dualistic way? More importantly, could these women be helped to see mathematics from a perspective more closely aligned with their view of the world in general?

The study

To answer the latter question I designed a study [Buerk 1981] in which five women, with general data rated as relativistic in Perry's scheme but retaining dualistic beliefs about the nature of mathematical knowledge, shared as a small group in mathematical experiences designed to help them to see the discipline of mathematics from a new perspective I chose experiences and presented them in ways that would encourage growth through successive positions in the Perry scheme. A particular emphasis in the sessions was placed on "experiencing" a problem or question individually before discussing it as a group When discussion did ensue its focus was on the question rather than an answer The women were encouraged to ask questions about the meaning of the problem, to clarify any puzzling terms, and to share the mental images that the problem brought to mind. I believe that this "experiencing" step was important since it allowed each woman to make the problem meaningful for herself and to clarify it both visually and verbally Once each woman "saw" the problem, resolution became the focus.

In addition, I asked the women to reflect on statements about mathematics and challenged them to articulate their own perceptions of mathematics, of mathematicians, and of the nature of the work mathematicians do These reflections, and also their reactions and responses to the five sessions, were recorded in a journal which circulated among the participants during the 10 to 21 day intervals between the sessions. In addition, the women were interviewed before and after the sessions to determine their background and past experience with mathematics, and to allow them to express their feelings about mathematics

The experience of five women

The first mathematical experience of the group was the "hand-shaking" question which I first thought about in a mathematics education course taught by Marion Walter at the State University of New York at Buffalo. I presented it to the five women in the following way:

> If the six of us wanted to meet by shaking each others' hands, how would you envision the number of handshakes?

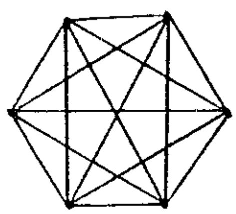
Please, before you, the reader, read further, stop and think about this question How would *you* envision the number of handshakes? Try to envision it in more than one way. Think about it for a moment. What questions follow for you as you think about "hand-shaking?"

I chose this experience because it is a question that can be approached in many ways, but which for most people does not bring to mind a formula or model from traditional classroom mathematics. The problem allows for a diversity of methods to reach THE ANSWER. It builds confidence since most people (with encouragement) do have a response It also has some ambiguity which will allow some people to interpret it in ways that may yield more than more answer. (Is Maria's shaking of Sophie's hand the same as Sophie's shaking of Maria's hand? Is a right-handed handshake the same as a left-handed handshake? Can I shake my own hand?)

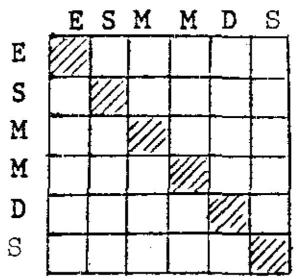
The hand-shaking question was posed to the women in the group, each of whom had a unique approach to it. Some approaches included:

The process of elimination Person 1 shakes 5 hands, person 2 has shaken hands with person 1 and has only 4 handshakes so: 5 + 4 + 3 + 2 + 1 + 0 = 15 handshakes This approach of Maria was very insightful, but she wished she had come up with a visual one.

The diagram approach. Here Sonya represented each person as a dot and each handshake as a line between two persons. A discussion arose about ways to count the lines.



The grid approach In this drawing Emmy represented the people along the outside and each box represented a handshake Shaded are handshakes with one's self. The squares above the diagonal duplicate those below the diagonal. This was an extremely helpful visualization, but Emmy didn't trust it



The square dance approach Mary's approach was to fantasize a square dance, choosing to count every handshake by considering person 1 shaking person 2's hand as different from person 2 shaking person 1's hand. Therefore, each of 6 people make 5 handshakes or 30 handshakes altogether. She then delighted the group with her ideas about styles of handshakes – firm, limp, joke, bone-crushing, and left-handed.

Once the methods were shared, it became apparent that each woman felt a strong sense of insecurity about her own approach Comments made included the following:

"I wasn't very mathematical about it."
"It didn't occur to me to make a diagram"
"I think that your way is 'better' than mine."

In general, group members did not exude confidence in their own methods.

Members raised many questions and gradually shifted the focus to some questions that might grow out of the initial problem. Suppose there were forty-five people. Which of our methods would work? Which would be convenient? Which would be efficient? How do you add the integers from 1 to 45 without getting bored? How could you make a formula to represent the problem? What form would a formula take? Is there more than one formula? Some comparison of methods, especially in terms of similarities and differences, ensued and served as a first approximation of "what mathematicians do"

The hand-shaking question involved these women in mathematics by first asking them to bring their own meaning to the question at hand Each, on her own, could visualize the problem in her own way The fact that Mary defined a handshake differently from the others led to a discussion of the need to agree on basic notions in a problem

Following a second session which was on symbolizing, the third one, "tree-planting," was presented to the women in the following way:

> Imagine a flat plane – a geometric plane – completely flat Does it have color? Weight? Can you see it? Plant 100 trees on your plane. Plant them in perfect rows and columns. You can have as many or as few as you like in any given row or column. Do you have a special kind of tree you are planting? These are very special trees of whatever variety you choose, because they are very tall and infinitely thin. They are perfectly straight. Are they all planted? Okay, please chop down one of the trees – any one at all. Replace it with yourself. As you stand on the stump you become infinitely thin as well. From wherever you put yourself, can you see out of your forest? Or, from your perspective and without bending to look around trees, can you see all of your trees? If not, how many can you see? You may look in as many directions as you like. [Adapted from a problem in Copes, 1980.]

Again, as reader of this paper, stop and fantasize your own forest Once you have it clearly visualized place your trees on graph paper. From the place you placed yourself, can you see out? Suppose that you planted an infinite number of trees. Could you see out of your forest then? [See Copes, 1980]

This experience was chosen to give participants the chance to create their own problems Each participant had at least three choices to make: the arrangement of the 100 trees, the positioning of the self, and the choice between two questions. If the person was on the outside edge of her forest, then the question of seeing out became trivial; the other question was "better" and more challenging. The answers to both questions were clearly dependent on the forest that each woman chose. The experience lends itself well to a "multiplistic" conception of mathematical knowledge "Multiplicism" is the name given to the positions in Perry's Scheme that lie between "dualism" and "relativism." It is described by Perry [1981] as:

> Diversity of opinion and values is recognized as legitimate in areas where right answers are not yet known Opinions remain atomistic without pattern or system. No judgments can be made among them so "everyone has a right to his own opinion; none can be called wrong"

In the first three sessions, the women were encouraged to adopt a multiplistic conception of mathematical knowledge. They were encouraged to see a problem from their own perspectives and to pursue their own directions Individual ideas were encouraged and reinforced Every attempt was made to build confidence, to talk in terms other than right and wrong, and to avoid traditional mathematical jargon This direction was chosen to promote growth away from a dualistic conception of mathematical knowledge It was also chosen to begin to compensate for the ways in which the dualistic perspective may have caused the feelings of apprehension about mathematics that these women have experienced: (1) by reinforcing an expectation of "needing to be quick," (2) by causing them to reject their intuition and with it their self-confidence in mathematics, and/or (3) by creating confusion about the role of conceptualization in an area viewed as dualistic

In the fourth and fifth sessions, more stress was placed on supportive evidence, on evaluating, and on looking for "better" ways These sessions were designed to encourage the relativistic view of mathematics

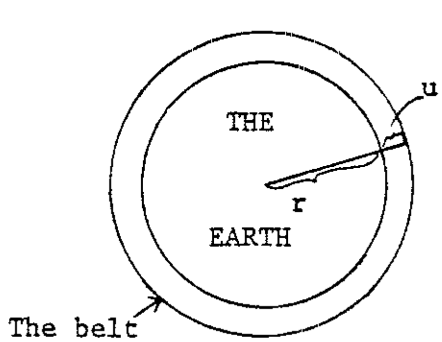
The experience of the fourth gathering proved the most powerful of the five. Please experience it for yourself as you read it and resolve it to your own satisfaction before you read on. If it is new to you be prepared to be surprised. It was presented to the five women as follows:

> I'm going to present you with a question about the world. Please focus on your first "gutlevel" answer Then let's talk together about the question to be sure that we really "see" it. The focus is to be on understanding the question Try not to focus on a method to "solve it." Think again about what your intuition tells you. Keep a note of your intuitions They need not be shared.
>
> The question is about the earth. Think of the equator. Put a flexible steel belt around the earth at the equator so that it follows exactly the contours of the earth Now add 40 feet to the length of that belt and arrange it so that the belt is above the equator for its entire length The belt still follows the contours of the earth at the equator and is raised above the equator by the same distance at every point. The model of a monorail track is a very nice one. (Thanks to Mary.)
>
> The question is, what will fit between the earth and the belt? That is, what is the distance between the earth and the belt? [Adapted from Stephen Brown, SUNY Buffalo Math Education course]

I chose this fourth experience to generate a conflict, since for most people, this problem provides a situation in which the intuition and the theory (logic, data) are inconsistent. Once a conflict arises between the intuition and the data, what are the ways that people choose to deal with the inconsistency? I have seen it lead some to a relativistic view of mathematical knowledge and others to the crisis that leads them to take the responsibility for making commitments within the framework of a relativistic world.

I posed the "belted earth" problem with instructions to focus first on an intuitive answer rather than on a method of solution. Materials available and visible included: an old basketball, a rough-surfaced dyed orange, a ball of pie dough trying to flatten, an encyclopedia with the earth's measurements, and a calculator. I directed discussion through the following stages:

1. What is your initial intuition? Time was spent encouraging each person to become comfortable with her own perception about the size of the space between the earth and the belt. Most thought that it was minuscule, although Maria first thought that her hand could fit in the space and Mary saw some space because she envisioned the belt as a monorail track.
2. What questions do you have about the problem? Care was taken to encourage questions and clarify what the problem was really saying. The belt was viewed as a loose bracelet, a tight thing stretched, a monorail track. Members wondered if it got wet from ocean waves; if it followed the ocean bottom or the ocean surface, and if the belt conformed to the ocean at high tide or low tide. They wondered how it could be kept uniformly distant from the surface of a constantly moving earth. Did the 40 feet mean that the belt was 40 feet away from the earth? Could a circle be the model? What effect would the changes in the contours of the earth have? Members decided that the changes in the contours would average out and that a circle was a good model. A discussion of π evolved This clarification process was extremely important in order to get each person in touch with the problem in some depth and to be sure that the members were addressing the same problem.
3. What theory applies? The problem was envisioned with the following diagram where:
   C = the circumference of the earth in feet.
   $C + 40$ = the circumference of the belt in feet



r = the radius of the earth in feet
u (for unknown) = the increase in the radius in feet
π = the ratio of the circumference to the diameter in any circle.
$C = 2 \pi r$ represents the circumference of the earth.
$C + 40 = 2 \pi (r + u)$ represents the circumference of the belt.
$C + 40 = 2 \pi r + 2 \pi u$, but $C = 2 \pi r$.
$C + 40 = C + 2 u$ (C can now be subtracted from both sides).
$40 = 2 \pi u$
$u = 40/2 \pi = 20/ \pi \backsim 20/3.14 \backsim 6.37$ ft
($\backsim$ indicates an approximate value)

Therefore the belt stands a little more than 6 feet above the earth.

4. Participants were aware of a conflict. They were asked, "How do you resolve a conflict between your intuition and theory (data, logic)?" Some of the comments following the finding of $u \backsim 6$ feet include:

Maria: That's too much. I don't believe that.
Sonya: I deal with the conflict by asking someone else.
Emmy: The circumference of the earth *has* to matter.
Maria: By using C, we've made it so the circumference could be 2 feet or 30,000 feet – as if it would all come out the same – and it can't!
Sonya: Is π in feet? Is 6.37 feet right? Why are we getting the wrong answer?
Sophie: Let's try it for the orange. If the diameter is 3 inches that would be one-quarter of a foot.

Throughout this discussion, Mary was busy with the calculator, pencil, and paper. She assumed a radius of 2 feet and went through the computation. $u = 6.37$ feet! She assumed the orange had a diameter of 3 inches and got $u = 6.37$ feet! She felt triumphant in reporting that the only way to change $u = 6.37$ feet is to change the 40 feet.

Sophie pursued her quest for more data. She didn't want to use the numbers for the diameter and circumference of the world but wanted to use numbers. She talked about the orange. Finally, she went to the blackboard to pursue the question using the basketball. She assumed its circumference was 2 feet. She went through the computations on the board and found that u (the distance the band stands above the basketball at every point on its "equator") was 6.37 feet.

The session ended with Mary feeling triumphant, not only because she had discovered a concept, but also because her intuition had been a monorail track above the earth. The others were all in conflict. Sonya was asking how π could remain constant. Maria, too, was questioning the use of π and not believing that 6 feet could be an answer. Sophie was excited by how well she could breeze through the computations on the board, but frustrated because she did not understand the answer. Emmy was confused. She wanted to believe her intuition, but could not find a way to reject the theory.

The session left members feeling more confident in their skills in using the algebra, and yet in conflict about its result.

That conflict, coming after they had gained some confidence and courage to try mathematical questions and to share their own ideas, was a powerful vehicle for changing their conception of mathematical knowledge.

To illustrate that power let's look at Maria's responses. Her fourth journal entry began with her experience of making the belted earth problem her own:

> Circumference problem. I first visualized the stainless steel band, shiny, about 5-6 " in width sloping across the earth, over grasses, down under marshes, under water. Wondered how long it would stay there before it began to look worn, like scrap metal. Seemed a perfect image for what mathematics does – take something organic and colorful and constrict it with something metallic and silver and rigidly out of place. (Why couldn't we have put a woven rush braid around the earth?)

It continued with a description of her conflict concluding with the following:

> So the problem must be in $C = 2 \pi r$ – that you can't use π as a fixed and actual "number." This is what it comes down to, but I'm not even convinced. The real problem lies with equating one $C + 40'$ with any *other* $C + 40'$ but I'm not sure how to solve it. *Still* believe *a* formula will apply, but can't see what I'm misapplying. *Still* trusting my first instinctive visualization. No exit.

In her final interview Maria indicated that she saw mathematics as a process and realized that speculation was necessary for the process to evolve. She concluded that:

> I feel like I have given birth to this new little creature, "math," and I have to take it home with me and where is it going to fit into my life now?

Note that Maria at this stage no longer sees mathematics as dualistic. She sees it as a process. She still wants to resolve her conflict in connection with the belted earth problem, but she also realizes that the responsibility is now hers and she must choose how, when, and if she will relate to mathematics. She states:

> Need to know if I "let math into my life," what would I *do* with it? (I have an image of opening my screen door, and a short furry 2-legged creature trotting in ...)

Maria did not learn many new mathematical skills in five one-hour sessions, but she came to see mathematics from a different point of view. With that new perspective on mathematics she has taken responsibility for her own relationship to mathematics. Should she choose to take a course, she will not be without apprehension, but it will be an apprehension that she can deal with more readily than the apprehension that comes with the feelings of powerlessness because "the wicked mathematician has all the answers in the back of the book."

Summary

Many people who have moved beyond a dualistic view of knowledge in general, have difficulty with mathematics because they continue to see it only dualistically My experience with the women in my study indicates that a more relativistic view of mathematics, coupled with a sense of personal responsibility for their own learning, made mathematics more approachable for them

I am not making a general claim Clearly my small and specialized sample is much too limited for that. I share this experience instead because of its depth and richness There is much we can learn from listening to the way our students view mathematics as a field of knowledge. This group of able women provide us with a particularly articulate voice.

I share this experience also because I believe that a look at apprehension about mathematics from the perspective of a developmental scheme like William Perry's provides an alternative not usually present in work with the math avoidant. It is unfortunate that the term "math anxiety" has become such a popular one for it carries such a negative connotation.

Let me conclude with an excerpt from Emmy's final journal entry:

> In this group I have very much enjoyed the philosophic insights the (statements and journal) entries gave. They sent me off into regions I love, and I apologize (particularly to Maria) that those regions were sometimes strange and irritating to others. I guess I got carried away I do feel however that this wandering has helped me to think through the values that define my own character as well as mathematics. This seems to me an important step in repairing the flawed "relationship" I have with math To think philosophically about math and to *play* with math problems simultaneously seem to me a lovely teaching method. For the first time I think I have had the experience of learning something about myself from doing math I feel somehow relieved that we all do not have the same symptoms or sources of "math anxiety": the fact that my problems are particular makes them seem easier to overcome. I almost feel foolish enough to give calculus another whirl without a pre-calc. review course! I am suddenly very curious, though, about how to find out the math books and teachers that will teach to my strengths (visual intuition) and not to my weaknesses (mechanics: arithmetic, algrebra, trig , etc). If mathematical thinking can take on so many styles, so can math teaching. I have not seen so many math teaching styles, though, at least not consciously

Notes

1. For a detailed description of Perry's scheme and its application to mathematics see the recent article by Larry Copes, "The Perry Development Scheme: a metaphor for learning and teaching mathematics", *For the Learning of Mathematics*. 3, 1: 38-44
2. The names that I have given to my subjects are the names of women prominent in the history of mathematics:

   Sonya Kovalevskaya (1850-1891) – Russian
   Sophie Germain (1776-1831) – French
   Emmy Noether (1882-1935) – German
   Maria Agnesi (1718-1799) – Italian
   Mary Somerville (1780-1872) – English

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a mathematical proof but who still feels uncertain about the generality of the corresponding statement – has not yet attained this fundamental synthesis He is still oscillating between a formal – but not powerful enough – understanding of a mathematical statement and the intuitive, empirical tendencies which push him to seek new, additional facts which, he feels, may increase his conviction, his belief in the general validity of the already proved statement

In fact what he needs is to confer the strength and the universality of a belief on the formal conviction derived from the proof without, by this, destroying the sense, the conceptual legitimacy of the proof itself.

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