invisible visible" (to borrow from the title of Devlin's book) and in so doing set the stage for students and teachers alike to do something about the problem, to make the world more fair or more kind.

After studying the issue of domestic violence and interpreting graphs and charts, patterns and trends, my students develop a pamphlet on the issue and hit the streets of Toronto for a morning, talking to people and collecting donations for local women's shelters (we don't graph the number of pennies, nickels, dimes and quarters that we collect ...) After studying global warming and carbon dioxide equivalents, we all look at how a diet of food that comes from within 100 kilometres of Toronto has so much less environmental impact than if we eat our grapes from California and our mangos from Central America. The point is that mathematics empowers students to make informed choices about issues central to their lives in a way that may transform the world for the better.

Keeping relevancy, in the most honest sense of the word, at the heart of real-world problems must be our goal. And if we're not going to betray the idea of education, the notion that problems will encourage students to transform our society for the better must also take priority. Students love talking about fairness, and given the chance and a good reason to do so, will move mountains to be kind to others. Let's give them a *real* reason to do so.

Communication: simulation, reality, and mathematical word problems

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A comment after reading 'Word problems as simulations of real-world situations: a proposed framework', Palm, 26(1):

In contemporary theory, terms like 'real world', 'simulations' and 'language' are anything but transparent and unambiguous. Our networked electronic media have restructured the balance of our senses and our sense of the relationships amongst self, others and reality. We no longer live in a world in which frameworks, grids and checklists can capture the complex relationship between human-made simulations and an assumed external reality.

Baudrillard (1981/2001) is a key theorist addressing issues of simulation and reality in our electronically-mediated world. Baudrillard presents the idea that simulations now *precede*, and in fact *supplant* reality, existing entirely without any corresponding or matching referent, and interacting primarily with other simulations. Baudrillard writes that

> [i]t is no longer a question of imitation, nor of reduplication [...] It is rather a question of substituting signs of the real for the real itself [...] A hyperreal [...] sheltered [...] from any distinction between the real and the imaginary, leaving room only for the orbital recurrence of models and the simulated generation of difference. (Baudrillard, 1981/2001, p. 170)

Simulations ('reality' TV shows, computer games, faked political crises, theme parks) precede and create events which may be indistinguishable from simulated events, and which interact with other simulations.

Baudrillard's characterization of simulations bears a resemblance to characterizations of genres in language, literature and film. Examples of any particular genre are made in imitation, not of life but of other exemplars of the genre - so exemplars of a genre interact primarily with one another, rather than with any external 'reality'. (Think of the train of imitative genre references generated by a series of films like *Frankenstein, Bride of Frankenstein, Son of Frankenstein, Curse of Frankenstein*.) Examples of a genre refer in only the most cursory way to 'real' objects and processes, and the intentions embedded in the history of the genre are carried forward with its use, regardless of the conscious intentions of the person using the genre as a communicative medium. Similarly, word problems refer only glancingly to the realities of the workaday world, referring primarily to other word problems (Gerofsky, 2004).

Baudrillard's simulations and simulacra go beyond genre to create cultural worlds where there is no boundary between real and imaginary. I will attempt to address the place of word problems in relation to Baudrillard's insights on simulations, and to relate this to Palm's framework for judging the degree to which word problems simulate "real-world situations".

Simulations and reality

Baudrillard writes about the historical relationship of representation (or image) with the real, and offers the following "successive phases of the image":

1. It is the reflection of a basic reality.
2. It masks and perverts a basic reality.
3. It masks the *absence* of a basic reality.
4. It bears no relation to any reality whatever: it is its own pure simulacrum. (Baudrillard 1981/2001, p. 173)

A simple example of this progression relates to cultural meanings of money. If a gold coin is an example of a kind of reality, since the gold has intrinsic value, then a paper note that can be exchanged at any time for a lump of gold is a *reflection of a basic reality*; a counterfeit version of such a note would *mask and pervert a basic reality*, while leaving that reality intact. A system in which the 'gold standard' is removed but the paper money remains *masks the absence of a basic reality*, and a system in which electronic pulses travelling globally by satellite change numbers in electronically tallied accounts may *bear no relation to any reality whatever* insofar as gold is concerned.

What would constitute an image of Baudrillard's first type, a *reflection of a basic reality* within a mathematics education context? Perhaps an accurate map or diagram of an actual, physically existing place or object would qualify

(for example, the plan of a house or a schematic diagram of a machine), or an accurate table of measured data (the number of cars passing a particular corner at rush hour, or the results of an opinion survey actually carried out).

An example of Baudrillard's second type of representation, which *masks and perverts a basic reality*, would be a diagram, chart or table of data which *purports* to represent a real, known situation but knowingly lies about this data. Such deliberately counterfeit representations are rarely seen in mathematics education; we are more likely to find them in politics and the news media, where a power advantage may be gained by misleading people.

Word problems belong to the third type of representation, which *masks the absence of a basic reality*. Word problems cannot be considered transparent, simple simulations of "real-world situations". When they appear to refer to actual situations, word problems mask the fact that they represent situations impossible in real life, and possible only in the conceptual 'world' of mathematical relationships. Word problems are constitutionally, generically unable to be faithful emulations of real-life tasks.

If, with Palm, we demand that word problems represent the everyday experience of the real world, we will be sorely disappointed in their lack of effectiveness. The three word problems that Palm has used as examples - the dissection of a Swiss roll, the 269 people waiting for a lift that only carries 14, and the 540 Little League members who are planning bus transportation - are all far from representative of the contingencies and purposes of everyday situations, as Palm himself has pointed out. Tellingly, Palm has not provided any examples of word problems which "could be considered simulations with high representativeness" (Palm, 2006, p. 46).

In a recent conference presentation, Kavousian [1] gave a good example of the problems that arise when a literal, real-life interpretation is made of the referential intentions of a mathematical word problem. Kavousian gave this word problem to her college students on a test:

> How many ways can ten people sit around a circular table?

In her role as a mathematics instructor, Kavousian meant the roundness of the table to signal that the permutations were invariant under rotation, so that the number of permutations would be 9! rather than 10! However, one of her students insisted that the answer ought to be 10! The student interpreted the table as a real one, in a real room, with windows on one side, bookshelves on another and so on. In a real room, there would certainly be 10! ways that 10 people could sit at a round table, since it would be different to sit near the window, the bookshelves or the blackboard. Kavousian acknowledged the real-life sensibleness of the student's interpretation, even though it short-circuited the mathematical question she had meant to ask.

Kavousian referred to the mathematical intentions of word problems as a "secret language" that students are expected to know, without having had any explicit instruction or attention paid to it. This "secret language" is a hidden set of meanings and references that live within the world of formal mathematical relationships. A novice cannot be expected to know these references while in the process of acquiring them; it is only in retrospect, once one is familiar with the mathematical structures in question, that the referential universe of the word problem and its answer can make any sense. If one assumes that the word problem refers to real objects and situations, the word problem masks the absence of such situations as table seatings invariant under rotation. Like the example of paper money without the gold standard, which operates on our collective faith in the illusion alone, word problems pretend to refer to a reality which does not actually exist, which survives only by our collective acceptance of the illusions built into the genre.

As for Baudrillard's fourth type of image, one which *bears no relation to any reality whatever*, I believe that this level of simulacrum goes beyond the capacities of word problems on the printed page. Examples of "pure simulacra", perhaps as yet unrealized in mathematics education specifically, might include computer gaming, interactive and networked text, image, sound, video, even tactile and CAD interfaces in which there is fluid movement between various virtual and actual representations. In such a system of simulation, the image would precede and possibly pre-empt reality; actual physical artefacts would be no more than instantiations of online patterns and designs (see Sterling, 2005; Gerofsky [2] for a further discussion of such potential systems of simulations).

Alternative models to think with

In Palm's article, he hints at several alternate ways to consider the problematic relationship between word problems and 'real life'. Although Palm mentions these in passing, and often introduces them only to reject them, the following seem most interesting and fruitful to me:

High fidelity: Palm refers to word problems as simulating real life with low or higher fidelity (Palm, 2006, p. 46). In its more common usage, 'high fidelity' referred to hi-fi stereo systems in the 1960s. Contrary to the most extravagant advertising claims of the time, these systems were not really expected to fool anyone into thinking that there was an orchestra or pop singer in their living room - they were not meant to be taken as transparent representations of an existing reality. In fact, it was at this time that innovative highly-engineered studio-produced record albums used multi- tracking to produce synthetic performances that could never take place in a live performance - an early example of Baudrillard's simulacra.

Because the term 'high fidelity' has these associations of a higher, but never possibly complete, faithfulness to some aspects of an originating reality, while at the same time invoking new possibilities of the synthetic and artificial (which represent things that never existed), I think it holds promise in the exploration of word problems and other representational modes.

Semi-reality: Palm cites Alrø and Skovsmose (2002) writing about different kinds of "worlds" related to different learning tasks. The concept of "semi-reality" is an interesting way to describe the referential world of word problems, "a world that is fully described by the text of the task and in which all measurements are exact" (Palm, 2006, p. 43). It is

rather mysterious but also fascinating to try to conceive of the tidy world of the semi-real. How would a semi-real world compare to a virtually real world, for example? (see, for instance, Sismondo, 1997). Are there other learning tasks aside from word problems that deal in semi-reality? (Alrø and Skovsmose (2002, p. 47) give some other examples within the realm of mathematics education.) How do learners and teachers engage differently with semi-real tasks?

Problem solving in role: Palm writes, "In many word problems, of which Example 3 is one, it is not known in what role the students are solving the task." (Palm, 2006, p. 45) I am intrigued by the idea of mathematical problem solving *in role*, especially as it relates to Heathcote and Bolton's work on drama in education (Wagner, 1989; Bolton, 2003). Heathcote, Bolton and others have developed ways of teaching across academic disciplines using whole group improvised drama, with teacher and students in role. I have experimented in using this kind of drama in education for teaching mathematics, and have found it particularly effective as a way to incorporate emotion and a sense of purposefulness in problem-solving. I would like to hear more about the ways that Palm would involve students working on a task in role, and whether he would include fictional roles as part of the learning and teaching repertoire.

Reality and imagination: Palm quotes Van den Heuvel-Panhuizen (2005) writing about context and 'reality' in word problems from the perspective of the Dutch theory of *Realistic Mathematics Education* (RME):

> The task context is suitable for mathematization - the students are able to imagine the situation or event so that they can make use of their own experiences and knowledge [...] The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves. (quoted in Palm, 2006, p. 43)

Palm rejects Van den Heuvel-Panhuizen's characterization of "realistic" (*i.e.*, vividly imaginable) contexts as insufficient to "facilitate an experience of mathematics as useful in real life beyond school" (Palm, 2006, p. 43). However, from the perspective of Baudrillard's theorization, Van den Heuvel-Panhuizen's sense of reality as the imaginable is the most appropriate one for our age in which the boundaries between the virtual and the real are dissolving. I am much more concerned with finding vividly imaginable, emotionally engaging modes and far less concerned than Palm with the simulation of out-of-school 'real life' situations in mathematics education.

Note

[1] Kavousian, S. (2006) Presentation in the panel discussion 'Obstacles to understanding', chaired by Dubiel, M., conducted at the Pacific Institute of Mathematical Studies conference *Changing the culture*, Vancouver, BC. Contact e-address: skavousi@langara.bc.ca

[2] Gerofsky, S. (in press) 'Moving fluidly among worlds: multisensory math software', poster presentation, *Proceedings of the 30th Annual Conference of the International Group for the Psychology of Mathematics Education*, PME30, Prague, Czech Republic

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What is required now is that educators of all kinds make themselves vulnerable to the awareness of awareness, and to mathematization, rather than to the historical content of mathematics.

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