The Development of Spatial Competencies through Alternating Analytic and Synthetic Activities [1][2]

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Introduction

By the dawn of the year 2000 the National Research Council [NRS, USA, 1990] has clearly indicated that programs for students, particularly in the case of geometry, should emphasize the student's ability to construct his or her own knowledge, the acquisition of visualization skills, the exploration of mathematical concepts which promote the development of models of reality, etc., and all this, in a problem solving context.

Geometry, as it is taught today at the pre-university levels, is too often oriented strictly towards linguistic and graphic representations of forms and concepts [Pallascio *et al.*, 1991]. Traversi [1979] points out that there is much more to be gained in terms of understanding if we attribute to the teaching of geometry an operational and experimental character where knowledge is transformed into know-how through tasks which are centered on the manipulation, construction, and comparison of geometric models. Parzysz [1988] and Bishop [1983] confirm that the drawing of real objects is very difficult for young students to understand due to the numerous conventions required (a square must often be represented by a parallelogram, hidden lines by dotted lines, etc.), but also due to the restricted nature of the learning situations. There is more to the acquisition of spatial competency than the problem of decoding a drawing.

The study that follows presents a different portrait of geometry teaching which takes into consideration the directions of the new programs and the difficulties mentioned above. Aiming at a better grasp of spatial elements, it suggests a dynamic approach which alternates between activities which are analytic in nature—where, for example, one observes shapes—and activities which are more dynamic—where, for example, the construction of scale models plays a key role. These activities are organized around a developmental matrix of spatial competencies according to various geometric modes and intellectual processes. Examples of the activities are presented and discussed following the description of a teaching experiment with some high school students.

Research questions

Two important facts have emerged from the general research on visualization. First, two types of mental images seem to exist: one pertains to the immediate intake of data (*perception*, which relates to analytic competencies), while the other deals with the purely mental reconstruction of objects (*representation*, which relates to synthetic competencies). It is generally agreed that the first type of mental image involves the perception of three-dimensional objects only. With respect to two-dimensional images of three-dimensional objects, Bishop [1983] suggests that the mechanisms involved are not the same, as such images constitute a type of representation which must be processed in some manner to be understood.

The second point of agreement is that the formal aspect of representation seems to be a function of learning—more precisely, access to the higher levels of logical organization remains a function of the learning situations in which a person is immersed. Even though there currently seems to be widespread acceptance of the hypothesis that such access is not strictly due to biological maturation but depends on stimulation of the mental operations that trigger it, there are nevertheless differing views on the nature of such operations. We have discussed the nature of these intellectual operations from a geometric point of view [Mongeau *et al*, 1991; Pallascio *et al*, 1992].

The objective of the experiments that we will highlight was to study the development of spatial competencies through the use of activities focussing alternatively on analytic and synthetic competencies. The activities involved placing young adolescents in situations in which they had to generate geometric forms, thus going beyond the limited two-dimensional symmetrical transformations generally taught in school programs. Our exploratory research, which could provide the basis for an effective teaching model, enabled us to answer a certain number of questions that we had previously posed: What should be the role of hands-on activities in the creation or generation of spatial representations? What role should topological and projective figures play in the activities used? Which operations could gradually lead to the development of deductive reasoning? Should the detailed analysis of spatial relationships be encouraged among younger students? Which operations are most effective in fostering the development of visualization competencies?

Structural perception of space

Space can be characterized in various ways: physically, socially, geometrically, etc. [Alsina *et al*, 1987]. Our research examines the perception of geometric space, which can be looked at from the point of view of either form or structure. While form perception consists in the quantitative internalization of a spatial model by means of analyzing and synthesizing its properties in terms of relationships, proportions, measurements, and coordinates, structural perception considers instead the qualitative internalization of a spatial model through the analysis and syn-

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thesis of its topological, projective, affine, or metric properties [Baracs, 1988]. The latter approach has been adopted in this study. "Spatial representation is an internalized action, not simply the imagining of some external information" [Piaget and Inhelder 9148, 539].[3]

The order in which geometric properties are integrated from a genetic point of view corresponds to the stages identified by Jean Piaget. He demonstrated that the order parallels the axiomatic structure of geometric properties and is, consequently, the inverse order to that of their historical development. "First of all, the topological relationships and, secondly, the projective and euclidean relationships presuppose a growing number of increasingly complex correlations among the actions themselves..." [ibid., 538].[4] Although Piaget does not actually distinguish between projective and affine geometry, his works contain the partially ordered use of (a) topological space, in which a relationship is drawn among points within a curve or a closed surface; (b) projective space, in which a relationship is established between an object and a particular viewpoint (i.e., above/below, near/far, right/left, over/under); and (c) euclidean (or metric) space, which places the subject in a spatial relationship with one or more objects, with reference to a system of coordinates that are distinct from both the subject and the objects.

Matrix of the development of geometric spatial competency

The instrument we have developed [Baracs and Pallascio, 1981; Baracs *et al.*, 1983; Pallascio *et al.*, 1985; Mongeau, 1989] is defined by means of a two-way table. One dimension of the table lists the five intellectual operations that correspond to spatial competencies (*spatial relationships* and *spatial visualization*), while the other shows the four geometric modes.

Competency	Operation	Geometric Mode			
		Topological	Projective	Affine	Metric
Spatial relationships (analytic)	Classifying	X			
	Structuring		X		
Spatial relationships (analytic) / Spatial visualization (synthetic)	Transposing			X	
Spatial visualization (synthetic)	Determining		X		
	Generating				X

Figure 1
Matrix of the development of geometric spatial competency

The five intellectual operations are *classifying, structuring, transposing, determining* and *generating*. *Classifying* consists in grouping together spatial structures according to certain common geometric properties or parameters. *Structuring* involves identifying the geometric properties of a spatial structure and the geometric combinations it contains. *Transposing* consists in establishing correlations and equivalences, and bridging the various representational modes (physical, linguistic, algebraic and geometric) and the geometric modes. *Determining* consists in defining the elements or parameters that are determined by the geometric limitations that apply to a spatial structure. The last operation, *generating*, involves producing or modifying a spatial structure so that it meets certain predetermined geometric criteria. (Some examples are presented in Appendix 1.)

The four modes are *topological, projective, affine* and *metric* (see Figure 2). The *topological* mode corresponds primarily to the study of those properties of adjacency and connectedness in spatial structures which are conserved following one or several continuous transformations, such as extending, contracting, folding, and torsion. The *projective* mode mainly refers to the study of those properties of incidence and flatness which are conserved following a central projection. The *affine* mode corresponds primarily to the study of those properties of parallelism and convexity which are conserved following a parallel projection. The *metric* mode corresponds mainly to the study of the properties of distance and angle.

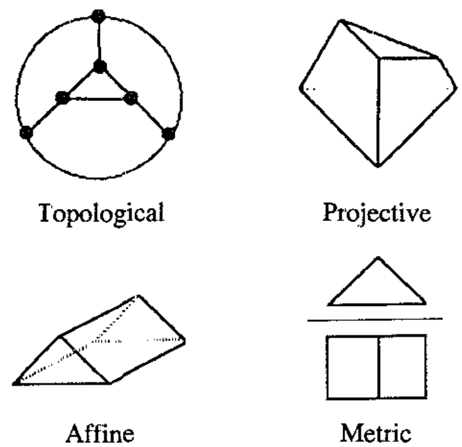


Figure 2
Representation of a triangular prism according to the various geometric modes

In the final analysis, the *spatial-relationship* competency [Bishop, 1988], which is on an *analytic* level, involves the mental activity of the recognition of forms and their properties, whereas the competency of *spatial visualization*, which is on a *synthetic* level, in effect involves the transformation of shapes.

After identifying preadolescence as a period which is conducive to the natural development of spatial competencies [Mongeau, 1989], we endeavored to gain a better understanding of the respective roles of the intellectual operations and the various types of spatial representation in a person's development of visualization. In order to do so, we worked with two 12-year-old students, a boy and a girl, and two 14-year-old girls. The sessions consisted of five hours of three-dimensional geometrical activities that were recorded and analyzed afterwards. The two 12-year-old students scored above the 90th percentile on a spatial-aptitude test, while the two 14-year-olds scored above the 85th percentile (in each case, in relation to students of the same age).

The alternate use of analytic and synthetic activities

Contemporary research carried out from a constructivist perspective has demonstrated that the student must be involved in active learning if he/she is to grasp concepts. More specifically, certain principles belonging to genetic psychology support this hypothesis: the dynamic principle, for example, according to which the manipulation of objects plays a very important role in the development of intelligence; or the construct principle, according to which students are virtually incapable of analyzing a situation or a concept without having constructed it beforehand; that teaching must encourage both concept formation—even from an empirical perspective—and concept analysis [Aebli, 1966]. We therefore advance the hypothesis that, within a constructivist teaching framework, alternation between analytic and synthetic activities is an effective approach, especially for the development of spatial competencies.

One of the aims of our teaching strategy was therefore to promote frequent switching between analytic and synthetic intellectual operations, while fostering dynamic exploration—guided at first, and then—in order to arrive at deductive reasoning and effective problem solving: the fundamental objectives of mathematical activities. Our teaching sequence consisted of the following:

Competency	Operation	Type of Activity
(a) Analytic/synthetic	Transposing	From linguistic input to a physical model
(b) Operation	Generating	Guided construction of a physical model
(c) Analytic	Structuring	Recognition of the characteristics of the model
(d) Analytic/synthetic	Transposing	From a physical model to a drawing
(e) Analytic	Classifying	Examination of the possibilities
(f) Synthetic	Determining	Reasoning on the basis of the model and drawing
(g) Synthetic	Generating	Unstructured problem-solving activities

Figure 3
Types of activities used in the teaching sequence

Our basic premise was that students would gain understanding by creating mental representations (i.e., via synthetic activity) and using them for the purposes of analysis and, conversely, by using increased competencies in analyzing geometric objects to construct figures or forms. Our observation of secondary-school students has provided significant findings in this regard.

Results and discussions

The generation of three-dimensional figures

In an activity on intersecting planes (see Figure 4), since the students had no concrete experience in this area they did not show—through comments or questions, for instance—any signs of having formed mental representations. The activity consisted first in using wire, constructing rings to represent infinite planes and making a drawing to show where the planes intersect (craft activity, an adding process). Secondly, it consisted of sectioning a polystyrene sphere, using as few cross-sections as possible, (analytic ◄► synthetic) to create a tetrahedron, the simplest type of polyhedron (sculptural activity, a subtracting process). We had to introduce means of concretization. For example, two sheets of paper were placed randomly, without any parallel edges, to incite the students to create a mental image (synthetic ◄► analytic) so that they could interpret and rationalize the activity of generating intersecting planes and discovering the points of intersection. Figure 4a represents the projection of any three planes whereas Figure 4b seemed an acceptable representation of "any three planes" for the two younger students.

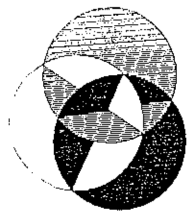


Figure 4a

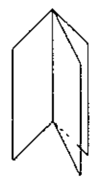


Figure 4b

Intersection of three planes

The actual construction of models enabled the students to anticipate the subsequent operations. For example, in one activity which involved parallel and central projections (see Figure 8), the model served as a point of reference and a means of verifying the various projections sought (synthetic ◄► analytic). The students completed their drawings while looking at the model (see Figures 5, 8 and 9) Since students are generally familiar with the type of model used in these activities, the use of models presents few obstacles to teaching. For those who follow the Van Hiele theoretical model, the global recognition of the generated shapes places the students most of the time at level 1 [Gutiérrez, 1992] For example, using "D-Stix" and rubber joints, an activity was to construct a tetrahedron ABDE symmetrical with respect to a given plane π (see Figure 5, where the triangle ABC is in the plane π), to determine how long the sticks should be, to draw the faces of the resulting object, and to complete the drawing on the basis of the model

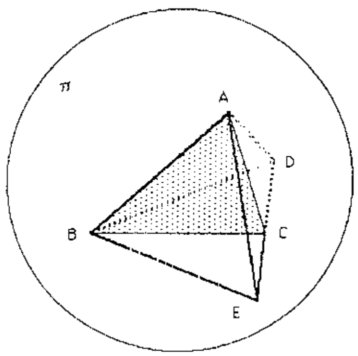


Figure 5
Isometric three-dimensional figures created by reflection

In other activities it was easier for the students to find the properties of the resulting objects as they were able to look directly (synthetic ◄━► analytic) at the models they had constructed (see Figures 5, 8 and 9). In some cases, the model created confusion, as in the case illustrated in Figure 5, where the construction components were confused with parts of the geometric figure generated. The two youngest students saw two tetrahedra, rather than just one, as they thought that the plane of reflection represented a cross-section rather than a plane of symmetry independent of the figure itself.

Discovery of the invariance of volume through successive transformations (see Figure 6) is not directly dependent on the model itself. Here the activity consisted in constructing a right quadrilateral prism using four wooden blocks, finding how the quadrilateral prism can be transformed into an oblique prism (analytic ◄━► synthetic), and vice-versa, by means of slide transformations and truncation (the solution is to make an orthogonal truncation along four parallel edges), observing the transformations, and formulating a hypothesis on the volume of each object. This is seen as a three-dimensional puzzle, since the primary aspect of the activity is discovery through trial and error. The activity does not allow for free creation as the object is rigid. The activity is therefore analytical in nature.

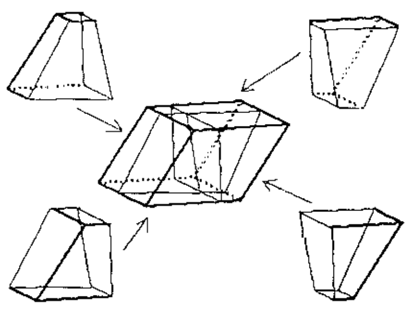


Figure 6
Determining the volume of an oblique quadrilateral prism

Spatial competencies as they pertain to mathematical concepts cannot be adequately developed simply by analyzing drawings or writing descriptions of three-dimensional objects. The construction and handling of objects by the students is essential "if an internalized reproduction of perceptive-exploration acts is to be accomplished" [Aebli 1966].[3]

The central role of transposition in the shift from analytic to synthetic activities

Transposing a written explanation of a problem into a model or a projective drawing (analytic ◄━► synthetic) illustrates the main difficulties that students encounter. First of all, they have virtually no geometric vocabulary.[4] There is no doubt that terminology is a major obstacle to teaching the generation of two- or three-dimensional geometric figures.

In other types of transposition, which are essentially visual, the transition is much simpler. For example, the correspondence between a central projection and an orthogonal projection (synthetic ◄━► analytic) of an object was grasped quickly. Visual transpositions would therefore seem to facilitate the development of this intellectual operation as opposed to transpositions of a linguistic or even a physical nature.

For all intents and purposes, it seems evident that the intellectual operation of transposition from one medium to another—whether the medium is physical (e.g., a model), linguistic (e.g., text), or geometric (e.g., a projective drawing)—is an essential component in the transition from an analytic activity to a synthetic one, and vice-versa, and from determining spatial relationships to visualizing spatial structures, and the reverse. Transposition is not, however, a single-faceted or definitive operation. Various transposition activities must be used, whether the transition is from a linguistic medium (text) to a physical one (a model)—a transposition that largely involves an auditory context—or from a physical entity to a geometric representation (a drawing)—a transposition that primarily involves visual representation.

The secondary role of determination in the development of deductive reasoning

Problems that involve the intellectual operation of determining—i.e., isolating elements or parameters that are defined by the geometric limitations of a spatial structure—provide a fruitful means of making students aware of the various considerations that are involved in deductive mathematical reasoning: equivalences that allow for substitutions, reasoning about various information, analogies, the appropriate use of geometric properties, etc.

In problems concerning the "determining" operation, the concept of indefiniteness, randomness or open-endedness (expressed by "*a*" or "*any*") can be problematic (synthetic ◄━► analytic). It is, however, at the heart of nearly all deductive reasoning, whether in geometry, algebra, or analysis. With respect to planes, the intersection of "any" two planes is already problematic; however, when students are required to consider any three planes, the problems intensify. Familiarity with contextualized examples (e.g., the model of an open book: see Figure 4b) led the subjects to consider a specific set of planes with a line in common as an open-ended or random example. They did not consider the common line shared by all the planes as enough to divest the set of planes of its open-ended or random properties (in contrast to parallel planes, for instance).

The exploration of geometric transformations by means of hands-on work with three-dimensional objects enabled the subjects to arrive at several equivalences that are fundamental to deductive reasoning (synthetic ◄► analytic). For instance, one of the younger students primarily used rotations and reflections of polyhedrons in exploring the transformation of an oblique rectangular prism into a parallelepiped, while the other chose to use translation (see Figure 6). The equivalence between a translation and a 180° rotation became evident naturally. In a situation in which informal deduction was used, while one of the youngsters used translation to conclude that two three-dimensional figures resulting from the trisection of a half-cube into three tetrahedra were equivalent (had the same volume), the other one applied reflection symmetry (see Figure 7).

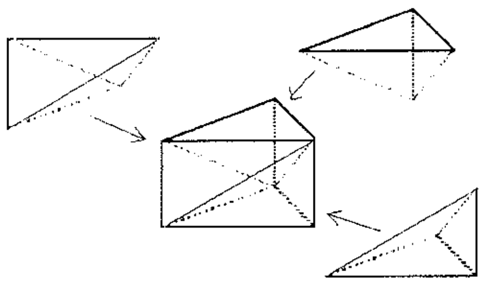


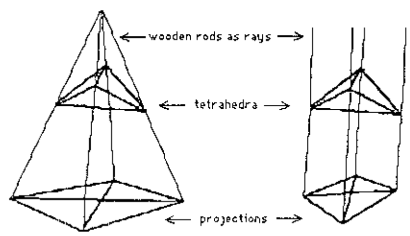
Figure 7
Trisection of the half-cube

Students sometimes draw analogies intuitively, which puts them on the track of informal reasoning. For example, an analogy may be drawn between the surface area of a triangle (base × height ÷ 2) and the volume of a pyramid (base × height ÷ 3), in which the number of dimensions of the objects are taken into consideration (two and three, respectively; analytic ◄► synthetic). The discovery of other geometric properties in the process will sometimes facilitate students' reasoning—for instance, the fact that three distinct, non-collinear points can identify a single plane.

The development of non-euclidean geometric properties
A study of non-euclidean properties allows for the exploration and reinforcement of geometric properties that are generally considered to be easily mastered through the explorations of conventional geometry—for example, the fact that a projection of a straight line generally remains a straight line (except in the specific case in which it is represented as a point) and a projection of a plane generally remains a plane (except in the specific case in which it is represented as a straight line),... (analytic ◄► synthetic). The same applies to certain topological properties—for example, the point of intersection of any three planes proves to be easier to visualize than the line of intersection of two planes (see Figure 4).

In one of the activities (see Figure 8), the students were asked to find the various configurations of the projections of a tetrahedron onto a plane, first by means of central projections and then by parallel projections (synthetic ◄► analytic). The activity concerned the construction of a model consisting of a tetrahedron and long wooden rods (the rods go through the vertices of the pyramid, like rays), the identification of the different projections (central and parallel) of the tetrahedron, drawing them, and writing some conjectures.

The subjects easily discovered two general projections of the tetrahedron: in the first, the image of one vertex of the tetrahedron fell inside the images of the other three vertices arranged in a triangle; in the second, the projected configuration was a quadrilateral. The second case proved particularly instructive for the students as it highlighted a projective property of which they were not fully aware—i.e., the fact that the projection of a figure whose faces are all triangular can result in a non-triangular figure, in this instance, a quadrilateral (analytic ◄► synthetic).



Central Projection   Parallel Projection

Figure 8
Single-plane projections of three-dimensional figures

The subjects—especially the two youngest ones—had a great deal of difficulty with the other two types of projection, which are less common and more complex. The first type occurred when three vertices projected onto a straight line—i.e., one of the faces projected into a straight line rather than a region of the projective plan. The second case, which was even more unusual, occurred when two vertices projected onto a single point—i.e., when an edge of the tetrahedron projected onto a point rather than onto a straight line. The students eventually realized that the projection of a triangular face of a tetrahedron could be a straight line, but they still seemed doubtful—or perhaps they did not judge the situation to be normal (analytic ◄► synthetic).

On the other hand the homothetic transformations needed in order to enlarge or reduce a projection, as well as the conditions needed to do so, were much easier to visualize in three dimensions, with the help of the model, than in analogous two-dimensional problems (synthetic ◄► analytic). For instance, it is much easier to grasp instructions such as "move the geometric object (i.e., the tetrahedron) closer to the center of projection" in order to enlarge the size of the projection than an equivalent activity in a necessarily two-dimensional situation. This type of activity, proposed at the end of the sequence, enabled the students, in most cases, to function at the second level in the Van Hiele model [Gutiérrez, 1992], very seldom above, except for some informal deductions (level 3 of Van Hiele) about the volume of tetrahedron.

Finally, the subjects did not necessarily apply generalizations about central projections to parallel projections even though working with wooden rods in the model enabled them to find the desired solutions relatively quickly (synthetic ◀▶ analytic). Moreover, the subjects did not often apply parallelism properties—for example, to section a half-cube into tetrahedra (see Figure 7) or to generate, by means of either a model or a drawing, an oblique triangular prism.

Conclusion: the role of alternating activities

The advantages of this alternate use of analytic and synthetic activities for the development of spatial competencies have already been pointed out several times. In the case of generating a tetrahedron by means of combined symmetry operations (see Figure 9), the students were able to discuss the role of rotation and reflection as well as the properties of the object that was created by means of those operations. The activity was based on written instructions and drawings, using combined symmetry to construct a triangular pyramid FGHI (a rotation with center A orthogonal to π and a reflection in π where the square BCDE is the intersection of the tetrahedron FGHI and π), and completing the drawing on the basis of the created model. The students' analysis was based on the object they constructed.

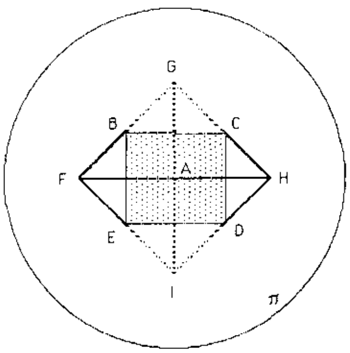


Figure 9
Creation of isometric three-dimensional figures by means of combined symmetry

Another good illustration of the ability to analyze after carrying out activities of a synthetic nature is the situation in which the students were asked to create tetrahedra by sectioning a cube (see Figure 10). The activity was to truncate a cube (made of polystyrene, with a 12-cm edge) in order to obtain the fewest tetrahedra possible, to draw and to explain the solution. For the 14-year-old students, the concrete operation of truncating a polystyrene cube made it easier to draw their results. However, the 12-year-old students consistently found difficulty in interpreting the drawing.

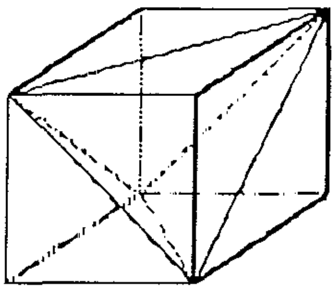


Figure 10
Creation of tetrahedra from a cube

In the creation of tetrahedra from a semi-cubic triangular prism (see Figure 7), the analytic and synthetic competencies overlap. Only when the construction process was completed were the students able to recognize three tetrahedra and analyze them two at a time (same base and height in each case). Without the earlier construction activity their observations remained superficial.

The development of spatial competencies in geometry by means of alternating analytic and synthetic activities has produced a number of results that could enrich the teaching of geometry.

One of the problems encountered during our activities pertains to the use of physical models. All too often the students were required to use teacher-provided tools which did not necessarily correspond to their perceptions of reality. For example, the use of pieces of paper to represent infinite planes may not mean a great deal to students. Even an adult who uses such a model to discuss intersecting planes may have a great deal of difficulty representing them in two dimensions (see Figure 4). Students who are more familiar with two-dimensional representations of space find three-dimentional representations difficult (see Figure 5 and the photograph).

Concerning the questions we raised before the study began, our research has provided some answers. In the creation or generation of spatial representations, hands-on work with physical media is certainly important, but it is also important not to create new obstacles to learning. The materials used must therefore be chosen very carefully. Our findings have also highlighted major gaps concerning topological and projective figures, which are often neglected in academic programs. In addition, our results suggest the importance of having students carry out activities involving the intellectual operation of determining—i.e., defining the elements or parameters that are determined by the geometric limitations that apply to a spatial structure—in order to lead them gradually to develop deductive reasoning. Last, we have shed light on the importance of diversifying intellectual operations in the activities used with preadolescents and adolescents, even the youngest ones, while aiming towards a constructive alternation between analytic and synthetic activities.

Notes

[1] This is a revision of a paper delivered at the Seventh International Conference on Mathematics Education (ICME-7), Québec City, August 1992.

[2] The research on which this paper is based was funded by the Québec government's Fonds pour la formation de chercheurs et l'aide à la recherche (FCAR).

[3] Translation ours. Page numbering refers to the French edition.

[4] Idem.

[5] Translation ours.

[6] For instance, they associate the French word for "plane" (*plan*) with the plans for a house (the singular of which is also *plan*).

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Bibliography

Aebli, Hans [1986] *Didactique psychologique*. Delachaux et Niestlé

Alsina, C , C. Burgués, and J. Fortuny [1987] *Invitación a la didáctica de la geometría*. Barcelona: Sintesis

Baracs, J., and R. Pallascio [1981] Le développement de la perception spatiale. *Processus de géométrisation et de visualisation Les Actes de la 33ᵉ rencontre de la CIEAEM* Pallanza, 37-49

Baracs, J., D. Dion, R Pallascio, and V Papillon [1983] Vers une définition opératoire de la perception spatiale, *Les Actes de la 5ième rencontre annuelle du PME-NA* Montréal, vol. 1, 314-321

Baracs, J. [1988] Spatial Perception and Creativity In Senechal and Fleck (ed.), *Shaping space, a polyhedral approach*. Boston and Basel: Birkhäuser, 1988

Bishop, A J. [1983] Acquisition of Mathematics Concepts and Processus In R. Lesh & M. Landau (eds), *Space and geometry*, Academic Press, N-Y, 176-203

Gutiérrez, A [1992] Exploring the links between Van Hiele Levels and 3-dimensional geometry. *Structural Topology*, 18, 31-49

Mongeau, P [1989] *Analyse et évaluation géométrique et psychologique de la représentation spatiale* Doctoral dissertation (unpublished), Université de Montréal

Mongeau, P , R. Pallascio and R. Allaire [1991] El desarrollo geométrico de la representación espacial. *Suma*. Spain

Pallascio, R., R. Allaire, and P. Mongeau [1992] Spatial Representation and the Teaching of Geometry *Structural Topology*, Montreal, 19 71-82

Pallascio, R., Allaire, R. et Mongeau, P [1991] Pour une géométrie générative. *Bulletin A.M.Q.*, XXXI (2)

Pallascio, R., V. Papillon, et D. Dion [1985] Typologie des habiletés perceptives d'objects polyédriques. *Séminaires du CIRADE sur la Représentation*, 107-121

Parzysz, B [1988] "Knowing" vs "Seeing" Problems of the plane representation of space geometry figures *Educational Studies in Mathematics*.

Piaget, J., et B Inhelder [1948] *La représentation de l'espace chez l'enfant*. Paris: Presses Universitaires de France

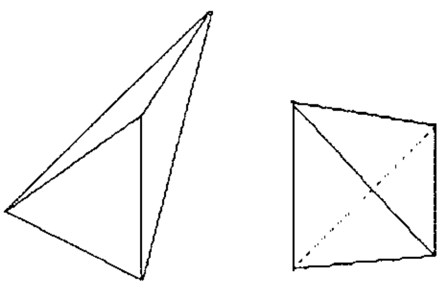
Traversi, R. [1979] Remarques su la didactique de la géométrie à l'école primaire. *Math-école*, 88, 2-11

Appendix

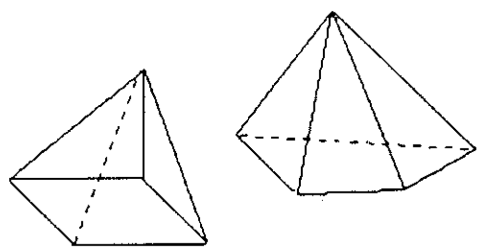
Examples illustrating various points on the matrix of the development of geometric spatial competency.

Spatial relationships	Operation: classifying	Geometric mode: topological

Three-dimensional objects can be grouped together according to the number of vertices, edges, and faces they have.



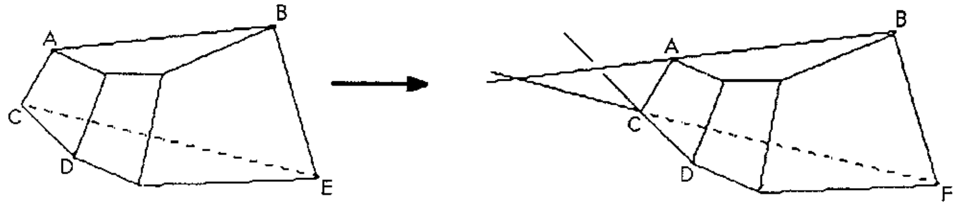
4 vertices, 6 edges and 4 faces



5 vertices, 8 edges and 5 faces

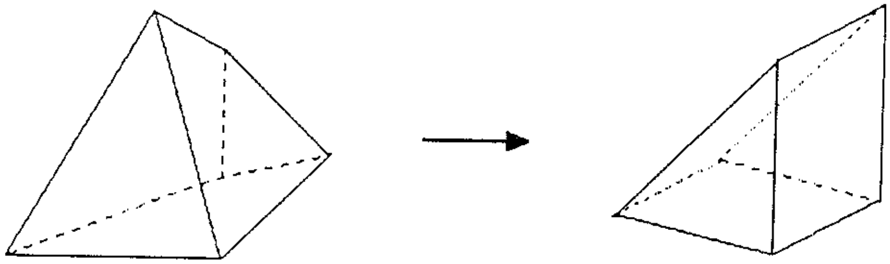
Spatial relationships	Operation: structuring	Geometric mode: projective

If the edges of the following three-dimensional object are extended, some of them will intersect each other, while others will never meet For example, although edges AB and CD do not meet, AB will meet CE at a certain point.



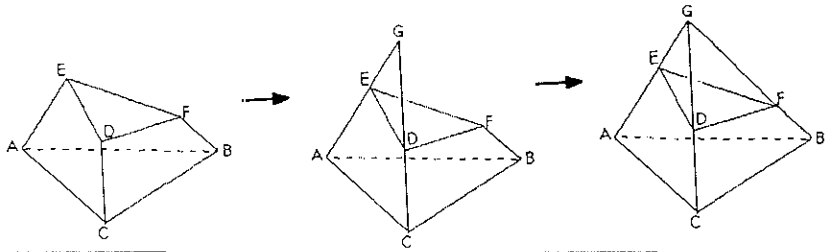
Spatial relationships and spatial visualization	Operation: transposing	Geometric mode: affine

The following drawing has been modified so that the figure it represents will have as many parallel edges and faces as possible.



Spatial relationships	Operation: classifying	Geometric mode: topological

If the faces of the three-dimensional object illustrated below are flat, then edges AE and CE intersect at point G when they are extended in space; therefore, point F will necessarily be located on edge GB.



Spatial relationships	Operation: classifying	Geometric mode: topological

Certain three-dimensional objects can be sliced so as to produce two pieces that are identical in size and shape