Research in Native American Mathematics Education

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My involvement in ethnomathematics originated with a startling and unsuspected research result. In 1973 I had been a mathematics professor at Northern Arizona University for thirteen years. Nearly every class I had taught had included Native American students, most of them being members of the Navajo tribe. At that time I had never detected any appreciable distinction between the thought processes of the Indian students and the general student population.

While pursuing the doctorate degree at the University of Michigan, I became interested in the developmental concepts of Jean Piaget. Subsequently I prepared a study designed to determine the degree to which the students in the general student body at Northern Arizona University had integrated into their problem solving strategies the concept of conservation as described by Piaget.

The conservation demonstration

A set of six demonstrations was developed, each designed to test the student's understanding of a different conservation concept. Those concepts were:

(1) Conservation of substance, shape varied.
(2) Conservation of weight, shape varied.
(3) Conservation of displacement, shape varied.
(4) Conservation of displacement, weight varied.
(5) Conservation of area, arrangement varied
(6) Extinction of conservation of weight.

Of possible significance is the format of the demonstrations. At the start of each demonstration, a class of students was presented with a particular physical situation; e.g., two balls of molding clay of equal size and weight, two metal cylinders of equal size and shape but of different weight, etc. A transformation was then made on one of the physical objects and the students were then asked what the result would be "if" the other object were treated in the same way. The students then checked an answer on a response sheet and wrote out their explanation for the answer. The demonstrations were presented to 284 students ranging from freshmen to seniors.

At the time the research was conducted I had no hypotheses whatsoever concerning our Indian students and their development with respect to conservation concepts. However, a cursory examination of the first few sets of completed response sheets indicated a significant difference and this observation provided a rationale for the null hypothesis that our Indian students have no more difficulty answering questions concerning conservation tasks than those of the general student population.

In order to test the hypothesis, two classes consisting entirely of Indian students, mostly Navajos, were identified and those students were presented with the same set of demonstrations and answered the same set of questions. The percentage of incorrect responses were computed and compared with those of the general student body tested earlier. That comparison is exhibited in Table 1.

Comparison of Incorrect Responses to the Conservation Demonstrations

Task No.	Indian	General
1	.33	03
2	.64	17
3	.67	.25
4	.33	.16
5	.20	06
6	.94	74

Table 1

For each conservation concept, the hypothesis that Indian students make no more incorrect responses than the general student on that conservation task was rejected at the .01 level of confidence. It should be noted that the general student body consisted of both non-Indian and Indian students and about ten percent of the students on campus were Indians. Worthy of note, further, is an observation made when studying the answer sheets of the Indian students. Those who responded correctly had written explanations that were entirely as correct, rational, and lucid as the correct responses from the general student population. In subsequent interviews with those students, they appeared to be amazed that not everyone had answered all of the questions as easily as they had. Although there were differences between the Indians who had answered correctly and those who had not with respect to their thoughts concerning the physical transformations of the demonstrations, those differences had apparently never been detected by the two groups of students who were acquainted socially. It appeared that some Native American students had achieved a certain threshold with respect to scientific observation and, having done so, difficulties other than individual differences were minimized. This tended to suggest that differences between the two groups were superficial and could possibly be removed through teacher understanding and appropriate modification of methods of teaching basic mathematical concepts

Follow-up research

Although the proportion of incorrect responses to the conservation task by the sample of the general student body was not out of line with results of conservation research at other universities, they did seem rather high. It was thought that possibly the poor proportion of correct responses resulted from the students never having tested the concepts on a physical level. Follow-up research was conducted during the subsequent year to test the hypothesis: "If students have conducted tests on the conservation tasks under question on physical materials with mechanical apparatuses, they will thereafter have a firmer grasp of those concepts." [6]

Fifty sets of balance scales were constructed. The individual members of classes of undergraduates were each provided with a balance scale and a quantity of modeling clay. They performed experiments in which each personally tested the concepts under consideration and together with the principal investigator they identified, stated explicitly, and agreed upon each conservation concept they had verified; e.g. "If two quantities weigh the same and one of them is changed only in shape then they will still weigh the same." Thus, the principal investigator knew the students had empirical experience with the conservation concepts to be tested.

After a period of two weeks the original six demonstrations were presented to those same students. The results showed no improvement over results obtained from students who had not had this experience. The hypothesis was rejected and it was concluded that the source of difficulty with conservation concepts was rooted somewhere in the students' earlier childhood environment and associations.

Implication of conservation concepts

The implications of a poorly developed concept of conservation for the learning of mathematics are not difficult to appreciate. We may write on the board an equation such as

$$ax^2 + bx + c = 0.$$

We ask the students to believe the two sides of the equation are equal when they do not even look equal. The two balls of clay were weighed before the eyes of the students and they could see that the balls of clay were equal and they themselves decided when they were equal with respect to weight. Nevertheless they experienced difficulty believing that the pieces of clay were still equal in weight when one ball was transformed in shape. Now assume that we make a series of transformations on the equation mentioned above such as:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

We ask students to believe that the equality is conserved through each transformation based upon a set of logical principles and that the student should find intellectually satisfying our conclusions that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It is almost ludicrous to expect the students to believe that an equality, which is no more than a mathematical assertion in the beginning, is conserved through a sequence of algebraic manipulations based upon a set of logical principles when they do not even have a firm grasp of the concept of conservation applied to a single physical transformation. Consequently, instead of our students learning mathematical thought (conclusions based upon the deductive process), they learn to accept those conclusions and assert belief in those ideas they perceive as being palatable to the professor.

If the foregoing indictment of our method of teaching the scientific method is valid to any degree, the situation is manifestly worse for any group of students for whom the conservation concept may constitute a special problem.

Description of the demonstrations

Two of the conservation demonstrations will now be described in greater detail.

Demonstration No. 3 Conservation of displacement, shape varied.

Two balls of clay were shown to the students and their weights were compared on a simple balance scale. The students were asked if the balls weighed the same. If they said one was heavier, a bit was cut from that ball. The process was continued until *the students asserted* that the balls were equal in weight.

Two identical glass cylinders were then shown to the students, each containing a quantity of colored water. The students then directed the principal investigator to pour water from one cylinder to the other until they agreed that the two glass cylinders contained the same amount of water.

One of the clay balls was then lowered into one cylinder and the level to which the water rose was observed. The other clay ball was then kneaded and pulled until it formed a rough rod about one foot long. The clay rod was then formed into a pretzel shape. The students were then asked, "If the "pretzel" is lowered into the other glass cylinder, will the water rise to (1) a lower level, (2) the same level, or (3) a higher level than the water in the cylinder containing the ball?" They checked one response and wrote out the reason for their choice.

Demonstration No 4. Two metal cylinders, one steel and the other aluminum were machined on a lathe in the physics machine shop to the same diameter (one inch) and the same length (two and one-half inches). Again two identical glass cylinders containing colored water to the same level were prepared as in demonstration No. 3. An eye-screw was attached to the end of each metal cylinder and each could be suspended by a thin nylon thread.

The investigator held up the two cylinders at the same level by the threads and the students discussed them. They usually could tell by the color that one was steel and the other aluminum. They then said that they knew that the steel one was the heavier but they didn't know by how much. The weights of the cylinders were then compared by placing the two on balance scales and the students' belief was confirmed. The steel cylinder was indeed heavier; other items including the investigator's knife and wallet were then placed on the pan with the aluminum cylinder until balance was achieved. The students were somewhat surprised to learn that steel is almost three times as heavy as aluminum.

The aluminum cylinder was lowered into one of the glass containers and the rise in the level of the water was noted. The students were then asked, "If the steel cylinder is lowered into the other glass container, will the water rise to (1) a lower level, (2) the same level, or (3) a higher level than the water in the glass container with the aluminum cylinder?" One answer was checked and the reasons written in spaces provided. The results can be observed in Table 1.

The search for answers

My research and observations up to 1980 had indicated the existence of problems with respect to the preparation of Navajo students for a study of mathematics. Still to be determined was whether there existed aspects of the students religion, culture, or heritage that could conceivably constitute a factor contributing to those problems. I began my study by reading widely on the history, philosophy, and religion of the Navajos.

In 1981 the National Institute of Education awarded me a grant which enabled me to travel to educational centers throughout the Navajo reservation. On those trips I asked questions of, and listed carefully to, Navajo school administrators, mathematics teachers, and students. Following is a quotation from my report to the NIE indicating conclusions based upon those experiences

> I wish to state at the outset that as a result of extensive reading and discussions with my Navajo students, both former and present, and also as a result of talks with educational leaders at centers of education on the Navajo reservation, the basic concepts and objectives of the philosophy underlying the Navajo culture do not appear to be incompatible with, but appear to be entirely consistent with, those views of the greatest contributors to the development of mathematics. For example, assuming that the philosophical views of the Navajo singers are representative of the Navajo people as a whole, Reichard concluded: "All chanters share the same assumptions, a common belief in the universal order, assurance that man has or may obtain power to fit into the world securely and smoothly, and faith in their power to correct when it becomes necessary to reduce the friction generated by ignorance of the universal machine." [9]
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> Through both a study of the literature and discussions with Navajo acquaintances, I find the quest for unity, harmony, order, and beauty to be a powerful influence upon which their daily living is based. Descartes, working to unify the concepts of algebra and geometry, or Kepler, searching for a mathematical model which would be in harmony with observations of the movements of the planets, or Einstein striving for a unified field theory which would eliminate apparent inconsistencies between the action of objects moving at everyday speeds and those moving at velocities approaching the speed of light, would have felt entirely comfortable with the views of the chanters as described by Reichard.

The relation of language to mathematics education

The continuing investigation led me to the writings of Benjamin Lee Whorf. Whorf was a linguist who made extensive studies of the languages of the Indians of the southwest in the 1920s. He assumed and asserted that the characteristics of an individual's native language impacts upon the manner in which that individual interprets the environment and approaches a problem solving situation. The assumption is known as the "Whorfian hypothesis". The hypothesis was not universally accepted by linguists in the early 1980s. It seemed to me to hold promise at that time, and now in the early 1990s researchers continue to support the premise. Benjamin Whorf wrote of the Hopi language, "Hopi, with its preference for verbs, as contrasted to our own liking for nouns, perpetually turns our propositions about things into propositions about events . ." [12]

Kluckhohn and Leighton state in regard to the Navajo language: "The structure is too different. The language of The People represents an importantly different mode of thinking and must be regarded as such." [2]

The idea appears to be persisting. In the December 1990 issue of *UME Trends*, Claudette Bradley *et al.* quote Pinxten as saying, "Navajo tend to speak of the world in terms of process, event, and fluxes, rather than parts or wholes or clearly distinguishable [static] entities." "The Navajo world," she says, "stresses the dynamic rather than the [static] aspects of reality." (The brackets are mine). It occurs to me now that an acceptable level of mastery of the concept of conservation as defined by Piaget is based upon a belief that certain physical attributes of an entity can remain unchanged, i.e. static, when related attributes are varied. Consequently it should come as no surprise that students whose thought processes are rooted, to a significant degree, in a language that "stresses the dynamic rather than the static aspects of reality" should experience difficulty in applying the logic of conservation when faced with an apparently contradictory situation.

Space does not permit a detailed description of other aspects of the Navajo language that potentially impact a student's learning of mathematics. The lack of words for "if", "multiply", and "divide" are examples. These are discussed in greater detail in [5]. I shall, however, describe briefly an item of informal research.

David Witherspoon, in his book *Art and language of the Navajos* [13], discusses the importance of the order of the nouns in a sentence in Navajo. The rule is that the first noun mentioned plays the dominant role. As an example he considers the two sentences:

(1) The girl drank the water.
(2) The water was drunk by the girl.

To the native English-speaking person, the two sentences appear to convey essentially the same information. Witherspoon's wife, a Navajo, remarked that the second sentence was ridiculous because it is impossible for the water to drink the girl.

In an attempt to test the impact of the order of the nouns, I showed the same pair of sentences to five groups of Navajo university students. Each group consisted of two to four students who happened to be in my office.

First I wrote on the board, "The girl drank the water." The students read the sentences and exhibited a "so what?" attitude. Next I wrote, "The water was drunk by the girl." In each case the students laughed at the second sentence. When asked to explain what was funny about the sentence they said that the second sentence implied that the water drank the girl, or some similar absurdity.

Those students knew how to read English and they were aware that the person writing the sentence knew no Navajo, yet the structure of their native language had a notable impact upon their interpretation of the sentence. I have since studied the Navajo language formally. Had I done so earlier the result described in the preceding paragraphs would not have seemed so surprising.

It is difficult to believe that these students would not consider initially ridiculous a problem that is introduced with the statement, "A house is built by John in twenty days." At least three levels of absurdity can be identified: (1) a house building a man, (2) a house making a decision, and (3) a house not yet built making a decision.

The students said that the word order used in the statement of problems in mathematics texts was a source of difficulty not experienced by their non-Indian counterparts. Textbook writers and teachers writing test items might benefit from awareness of the importance of word order to their Navajo students.

Holism and mathematics learning

As a philosophical concept holism is defined as "the theory that whole entities, as fundamental components of reality, have an existence other than as the mere sum of their parts." Certain observations tend to suggest that holistic concepts may warrant serious consideration as a component of Native American mathematics teaching and further constitute a source of hypotheses upon which potentially fruitful research could be based.

Robert W. Rhodes describes the process used by a native painter who was commissioned to paint a gymnasium-length mural for a boarding school. The painter

> first spent several hours simply looking at the blank wall. He then proceeded by applying all of the red paint to the sections which were to be red, from one end of the mural to the other. Next came the application of browns, then blues, and so forth. The shape of what was being portrayed did not become apparent until well into the process. [10]

The painter did not commence executing the details of the painting until he had the entire (whole) mural firmly fixed in mind.

Gladys Reichard wrote of the mental feat of Navajo ladies who weave rugs possessing beautifully symmetric and intricate design elements without observable visual reference to a pattern.

> The weaver must keep the composition of the entire rug surface in her mind, but she must see it as a huge succession of stripes only one weft wide. It matters not how ideal her general conception may be, if she cannot see it in terms of the narrowest stripe, meaning a row, of properly placed wefts, it will fail of execution. [8]

Similar observations have been made concerning the work of native basket makers and potters. The basis of the holistic approach is to have a firm grasp of the "big picture" before proceeding to details.

An awareness and sensitivity to the holistic method of thought may impact positively upon the approach of mathematics teachers of native American students. I have made some initial informal approaches utilizing the holistic philosophy in teaching concepts in algebra. I shall describe the process with respect to the introduction of the topic of quadratic equations.

Prior to becoming aware of the possibility of utilizing holism in the mathematics classroom I began writing on the board everyone's favorite quadratic equation:

$$x^2 - 5x + 6 = 0.$$

Following that, the students were encouraged to factor the trinomial and then to "set each factor equal to zero." Sound familiar?

Since I was aware that I had several Navajo students in class I decided to try to utilize a holistic approach. I attempted, with the cooperation of the students, to develop the "whole picture" first. We discussed problem situations which might have more than one solution. For example, I posed the following problem. "If you throw a baseball straight up, when will it be fifty feet high?" There were some sneers and snickers and responses such as, "Well, how fast was the ball thrown?" I endured these remarks and persisted with my initial question until one student observed: "But there will be two times when the ball is fifty feet high, one with the ball going up and a second when the ball comes down." We then discussed models of equations with more than one solution. Straight line graphs can cross the x-axis at only one point but if the line were allowed to bend it could cross at two points. Of course, it would no longer be a straight line and some students noted that they were familiar with the parabola which had that property.

I tried to build a total (whole) picture of the problem situations requiring quadratic equations as a model before getting to $x^2 - 5x + 6 = 0$. It was only from Anglos that I noted certain individuals roll their eyes suggesting a "why doesn't he get on with it" frustration. If a mathematics teacher is anxious to get on with the details he too is likely to feel uncomfortable with the holistic approach. The native students seemed comfortable with the leisurely paced holistic introduction. The technique needs more study, observation, and refinement. Then, perhaps, meaningful research on holism in mathematics education can be designed and conducted.

One more observation: In almost all of my mathematics classes the students are required to keep a notebook of all class notes, homework, tests, quizzes, solution sheets and handouts. The notebook is handed in for evaluation at mid-semester and again at the end of the semester. Neatness, completeness, and organization are important factors. Before learning of holism I had observed that the Indian students uniformly handed in admirable notebooks. Even students getting a poor class grade made beautiful notebooks. Possibly their liking to see all their work together in one (whole) neat package is a further manifestation of the holistic tendencies of the painter and the weaver.

Mathematics and culture

As we continue to explore and define the subject of ethnomathematics, it is proper for us to pose the question, "Is mathematics culture-free?" My answer must be that mathematics is definitely not culture-free.

Each of us is born into a group We learn the behaviors, attitudes, values, etc., of that group. The sum total of those attitudes, values, skills, and acquired knowledge is the individual's culture. The mathematics of the group into which we are born becomes our mathematics Consequently each of us possesses a culture of which the mathematics of the group is a part. Putting it more precisely, Leslie A. White insists that we not only possess a culture but we are possessed by it. [11] Consequently it is extremely difficulty for an individual born into one culture to feel comfortable when attempting to integrate into his own thought processes the values and thought processes which are an integral part of an even slightly different culture.

Understanding well this basic concept of the paramount importance of culture, no mathematics teacher could even contemplate seriously taking only the values of his culture and a textbook which is a product of his culture and imposing both himself and the textbook upon individuals possessed by a culture that diverges from his in any significant area. The differences in culture are so vital that it behooves every mathematics teacher to make the necessary effort to understand any cultural diversity that may exist between himself and his students

The Outdoor World Science and Mathematics Project

A writing project at Northern Arizona University was based upon the need to integrate relevant cultural elements with subject matter. The concept of the Outdoor World Science and Mathematics Project was developed by Alice Killackey, a science teacher at Zuni, New Mexico The objective of the project was to write a set of science and mathematics learning modules to serve as the basis for classroom study. The modules were to be related to: (1) the immediate outdoor environment, (2) the cultural heritage of the community, and (3) the economic outlook for the community The science modules and mathematics modules were bound separately The writing project was funded by the National Science Foundation and the materials were developed and written in 1980-1982.

It fell to me to write the mathematics modules and I received willing and much appreciated help from knowledgeable sources. Ark Husky, a mathematics student and Navajo builder, provided me with the structural details to form the basis of a module entitled "Hogan Geometry". Executives of the Black Mesa Pipeline made available documents with information leading to the module, "Pipeline and percents." (Many Navajos are employed by Black Mesa Pipeline.) Dover Publishers gave permission to use details from their books dealing with string figures. A module was then written showing the relations between the making of string figures, an ages-old, and still popular, recreation of Native Americans to mathematical thought. A Navajo acquaintance volunteered information on how to determine when a person is qualified to be listed on the tribal rolls. A related module was entitled, "Computing tribal blood degrees". Twenty-two mathematics modules were bound and others have been prepared. [4]

Teachers from reservation schools attended workshops on the university campus to learn to teach from the modules. They then field-tested the modules and each was evaluated by both teachers and students.

There are still other writing projects located in the United States, Canada, and Alaska resulting in culturally-related mathematics materials to be used by Native American students. A catalogue of all such materials, if one were published, could provide helpful sources to all mathematics educators sensitive to the relation between their subject and the special problems precipitated by cultural diversity.

Mentors in an atmosphere of cultural diversity

In conclusion I would like to draw attention to the role to be played by mentors as educational institutions attempt to deal with the challenge of culturally diverse populations. In an address to the members of the Stony Brook conference on "The role of the faculty in meeting the national need for African America, American Indian and Latino scholars", James E. Blackwell described the aims and objectives of mentors as personal as well as academic advisors to minority students [1]

The mentoring program at Northern Arizona University is directed by Dr Frank Dukepoo, a Hopi geneticist. The faculty mentors meet for lunch on the first Friday of each month. The meeting opens with a sharing of mentoring experiences Invited officials from various departments of the university then describe the functions of their programs that pertain to minority students to help the mentors to better advise their "mentees". Observations indicate that the mentor program is a success and that the time has arrived for research to be conducted to determine the degree to which mentoring programs are helping minority students achieve their educational objectives.

The many incidences of excellent Indian mathematics students noted in mathematics classes, along with an increase in the number of Native American mathematics majors, indicate that the research that has been conducted and the programs that have been developed are producing positive results. Perhaps research studies could be designed to quantify the effectiveness of mentoring programs.

If present efforts are continued and all concerned continue to be alert to the indications for needed research there is reason to believe the future will be bright for both Native American mathematics students and teachers alike

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We can see now how the belief that mathematical truths and realities lie outside the human mind arose and flourished. They *do* lie outside the mind of each individual organism. They enter the individual mind as Durkheim says from the outside. They impinge on the organism, again to quote Durkheim, just as cosmic forces do. Any mathematician can see, by observing himself as well as others that this is so. Mathematics is not something that is secreted like bile; it is something drunk, like wine Hottentot boys grow up and behave, mathematically as well as otherwise, in obedience to and conformity with the mathematical and other traits in their culture. English or American youths do the same in their respective cultures. There is not one iota of anatomical or psychological evidence to indicate that there are any significant innate, biological or racial differences so far as mathematical or any other kind of human behavior is concerned. Had Newton been reared in Hottentot culture he would have calculated like a Hottentot. Men like G.H. Hardy, who know through their own experience as well as from the observation of others, that mathematical realities enter the mind from the outside, understandably—but erroneously—conclude that they have their origin and locus in the external world, independent of man. Erroneous, because the alternative to "outside the human mind", the individual mind, that is, is not "the external world, independent of man", but *culture*, the body of traditional thought and behavior of the human species

Leslie A. White
