ACCOMMODATING THE INSTRUMENTAL GENESIS FRAMEWORK WITHIN DYNAMIC TECHNOLOGICAL ENVIRONMENTS

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For the purposes of this article, we concentrate on particular dynamic software environments including dynamic geometry environments (*e.g.,* Geometer's Sketchpad®, Cabri II+) and simulation software (*e.g.,* SimCalc MathWorlds®, a dynamic algebra environment). Such software allows mathematical representations to be dynamic through particular embodied features. These include:

- Navigation – ability to move around the screen, move mathematical figures, scroll and zoom coordinate systems, scroll around simulation worlds,
- Interaction – click and hold and drag or manipulate objects,
- Annotation – marks, literals or numerals can be added (and adhere to) parts of figures and diagrams,
- Construction – mathematical figures or diagrams can be built up in parts through specific tools,
- Simulation – allow objects that are part of, or associated to, the figures or diagrams, to be animated, or model data and observe a simulation of these data, and
- Manipulation – constructed figures or diagrams can be changed by interacting with particular features of the construction, while preserving mathematical rules within the construction.

One of the key infrastructural pieces of these software that allow many of these features to operate is the existence of "hot-spots." These are points that can be used to construct mathematical figures, *e.g.,* join two points with a segment, or construct a piecewise graph, and then used to dynamically change the construction. Hot-spots depend on the environment they are constructed within. We shall now discuss the existence of such phenomena from a theoretical perspective with respect to user-environment interaction, using examples to describe how such interaction is a sustainable *bi-directional* process that has the potential to ground and develop certain mathematical concepts.

Our purpose in analyzing such dynamics of use in an educational technology paradigm is to introduce the notion of *co-action,* which examines the nature of the relationship between the user and the environment (in our case a software environment). We propose that as users (or students) we redefine the exploration space through new forms of action and interaction, and so we need to enhance the framework of instrumental genesis to accommodate the changing form of technology; in particular, the changing actions of the users with such technologies as a cultural dimension. Users are actively changing the status of the object of inquiry through the examination and executability of the environment (because of the features described above). We describe how our theoretical constructs build on and enhance the framework of instrumental genesis. Through the provision of three exemplars, we conclude that the space of the learning environment dynamically changes to be one where we can stretch the limits of the tools or artifacts of our environment and hence drive the Zone of Proximal Development of the Artifact (ZPDA) itself through co-action.

Links to existing theory

Our discussion is centered on the examination, and use of, tools and artifacts within software environments but also situates itself in the perspective of broader cultural environments. It aims to parallel and offer an enhanced perspective of the complex construction of *instrumental genesis* (Verillon & Rabardel, 1995) but within dynamic environments. Related work investigates the use of Computer Algebra Systems (Drijvers, 2000; Guin & Trouche, 1999) to distinguish between instrumentation – how tools affect and shape the thinking of the user – and instrumentalization – where the tool is shaped by the user. Instrumentation is developmental since mental schemes (Vergnaud, 1996) emerge as users execute a task. As the task is completed the uses of a certain tool become internalized. Instrumentalization is related to the way of shaping the artifact in use. It is a psychological process which develops ways of using, manipulating, and shaping the artifact in use, an organization of use-schemes, a personalization and sometimes transformation of the tool, and a differentiation between the complex processes that constitute instrumental genesis and those which are critical for teachers to master (Guin & Trouche, 2002). Rabardel (2002) clearly defines the difference between the instrumentation and instrumentalization processes:

> *Instrumentalization processes* concern the emergence and evolution of artifact components of the instrument: selection, regrouping, production and institution of functions, deviations and catachreses, attribution of properties, transformation of the artifact (structure,

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> functioning etc.) that prolong creations and realizations of artifacts whose limits are thus difficult to determine;
>
> *Instrumentation processes* are relative to the emergence and evolution of utilization schemes and instrument-mediated action: their constitution, their functioning, their evolution by adaptation, combination coordination, inclusion and reciprocal assimilation, the assimilation of new artifacts to already constituted schemes, *etc*. (p. 103)

Our work focuses on the impact of students' actions with representational tools that are infrastructural and consequently ignite co-action processes. Whilst we agree with the constituent parts and definition of the process of instrumental genesis, we aim to accommodate the notion of co-action within the framework, a symmetric notion, which stresses the importance of the role of the environment in which the tool is being used and the dialectic process between the user, the tool and the environment.

Static tools and infrastructural tools

We believe that it is important for the reader to realize that our work is also positioned in a broader cultural context even though we mainly focus on exemplars from mathematics education.

Our ancestors used stone tools to modify the surrounding environment to their advantage. Perhaps their original intentions were to obtain an edge in their search for food and to enhance living conditions. Increasingly, the stone tools (and others that came later) were producing a subtler change as well. In fact, the *mediated activities* that began to take place under the presence of these stone tools began to saturate the natural environment and this reflected human *intentionality* and eventually generated a new environment that emerged as the "natural" space for our development. Such saturation of the natural environment is a mirror-image of our intentionality.

Consider a traditional use of a tool: the hammer. It is used to bang nails into a form. In this context, a hammer is a static tool or object, which can be environmentally coupled with other objects to produce similar actions. You can use other objects to function as a hammer albeit in an inefficient manner. For example, a brick can be used to strike something. Also, hammers can be used inefficiently. You might use a hammer but bang your thumb. There is nothing in the fixed, static, functionality of the hammer that can help the user hammer "better." We propose that digital tools or artifacts are more integrated into the environment's infrastructure and embody knowledge that can be executed through co-action. In this context, a hammer is an infrastructural tool for a carpenter.

A hot-spot is infrastructural because its existence offers an "invisible hand" that projects the intentionality of the designer of the environment into the actions of the user. One important definition of infrastructure for us is an invisible yet highly configured (some might say constrained) set of rules – modes of action-reaction that continually redefine the agency between the environment and the user. To change the hammer, you need to change the environment but this is independent from the hammer. But hammers do not suddenly appear in our environment. They have been continually modified through changes in the environment AND the cognitive human that develops new ways of making a hammer from discovering new artefacts. This is a form of co-action. The stone tools of our ancestors were not just amplifiers of physical activity; more importantly, those tools were *crystallizations* of intentional actions whose purpose was to transform the environment. We wish to use the metaphor crystallize not encapsulate, as we want to suggest essential properties of a crystal, particularly its stability and its possibility for change and growth (Moreno-Armella & Hegedus, 2008). Those tools were symbols. The biface axe was a two-sided reality: a material tool and, at the same time, embodying a crystallized action with its crystallized intentions (Moreno-Armella & Hegedus, 2009).

Today, whilst similar actions exist in digital environments, we propose that static tools do not always exist particularly in the world of dynamic figures. This is because the functionality of a tool within a digital environment is subject to co-action. Of course selecting some text and pressing Command-C or Ctrl-C will copy text or pictures to a clipboard in most modern word processors – a good stimulus-response software action.

We wish to focus on the role of "hot-spots" in our chosen software environments, but extend the definition of them as tools or instruments. The "hot-spot" in our chosen software environments is not an artifact of the environment but infrastructural that allows "true" mathematical figures to be built. Dragging a "hot-spot" is not the same as "using a hammer to try to hit a nail" – note the verb use. A hot-spot will always be used for dragging (in various forms and for various purposes); a hammer will not always be used for hitting well. A hotspot will always be dragged and a hammer is never hit but instead used to hit. Will they ever be the same? Well, the hammer is still as effective as the hitter. The hitter hits a particular point. The action is directed by the actor. The local environment does not help with the accuracy or efficiency of the tool use; it resides with the user and practice. In addition, the action of dragging a hot-spot leads to the software environment reacting in some way. It is also true that hot-spots could be used in an ambivalent way, dragging without any understanding of what the hot-spot- environmental coupling is constructing or preserving, a form of catachresis.

We propose that in a dynamic environment with "hot-spots" the action is not owned – in fact, agency is a collaboration between the user and environment, both are actors and re-actors. Both the user and environment act and re-act on each other. Basically, a co-action is always in effect. It is because hot-spots are infrastructural that our focus is made more pertinent and is the main thrust of our essay. But let us be clear, even though our paper focuses on digital environments especially ones with dynamic representations we do not believe that co-action cannot occur in natural environments. Quite the opposite it exists in many aspects of nature and human-artifact interaction. For the purposes of this paper, we will now present three exemplars to illustrate our analysis just using mathematics education technology. We aim to present these exemplars as existence proofs to enhance the framework of instrumental genesis. We believe that many more examples can be generated from these three exemplars. In fact, Rabardel (2002) questions whether the theoretical constructs within the instrumental approach are also relevant for ICT.

Exemplar 1: Dynamic geometry

Consider a construction of an equilateral triangle in a dynamic representational media such as Dynamic Geometry Software (DGS). Constructions that do not use measuring tools are called Euclidean constructions.

Euclid, in the first book of his *Elements,* presents the proposition (#1; Book 1) that the construction of an equilateral triangle (a 3-gon) based on a line segment AB, where two circles are constructed with radii AB (A and B are centers for each of the two circles respectively), and the third vertex (C) of the triangle is where the two circles intersect. Euclid's assumption (often debated) was that the two circles do in fact intersect (in fact twice). Now, a paper and pencil construction, with straightedge and compass, can be used to construct the triangle. But we only create one triangle. We have actively engineered the object, and our actions are *crystallizing* that object into something that can be symbolic. It can be given to someone else since there will be a shared sense of meaning and use. The shared existence of these mathematical entities is among the communities that take them as shared.

We choose to focus our attention on dynamic geometry software (DGS) environments (particularly Geometer's Sketchpad® and Cabri II+) as they aim to develop spatial sense and geometric reasoning by allowing geometric postulates to be tested, offering "intelligent" constructivist tools that constrain users to select, construct or manipulate objects that obey mathematical rules (Mariotti, 2003). Empirical work shows how these features lead to improvement in student engagement through aesthetic motivation (Sinclair, 2001), enhances students' ability to generalize mathematical conjectures (Mariotti, 2001) and aid students in developing theoretical arguments (Laborde, 2000, 2001; Noss & Hoyles, 1996).

Each DGS offers an environment where point-and-click Euclidean tools can be used to construct geometric objects that can be selected and dragged by mouse movements in which all user-defined mathematical relationships are preserved. In such an environment, students have access to conjecture and generalize by clicking and dragging hot-spots on the object which dynamically re-draw and update information on the screen as the user drags the mouse, and in doing so, efficiently tests large iterations of the mathematical construction. Figure 1 below attempts to illustrate this dynamism through a snapshot of such a physical action. As in Euclid's first proposition in Book 1, we construct a triangle based upon a line segment AB. Two circles are defined with Centers A then B with equal radius AB. The sides of the triangle have been marked to leave a trace. The center *B* "hot-spot" is the right-end point of the original line segment (AB) and is dragged from left to right (from *B* to *B*, in doing so the circles enlarge, but the triangle's properties appear to be preserved in an array or family of similar equilateral triangles. Dragging the hot-spot illustrates how the Euclidean construction of an equilateral triangle has been correctly implemented in this sketch.

Indeed, we have discretized this 'physical' motion of

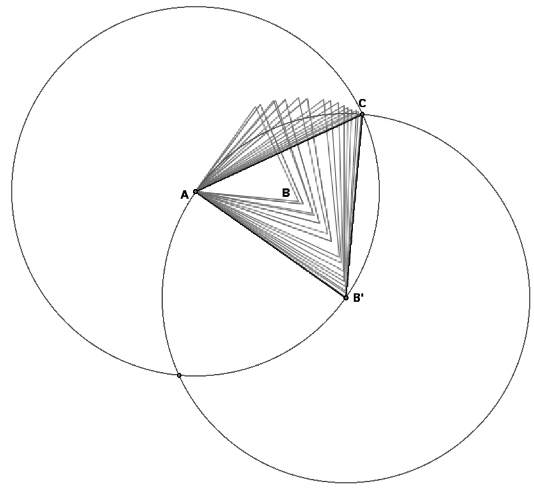

Figure 1. A trace of equilateral triangle constructions

grabbing (A) and dragging the hot-spot. But what we have here is an illustration of where the user has not only actively constructed the triangle, but has the affordance of a flexible media where the diagram can be deformed, but the engineering preserved, through one dynamic action. The dynamic action allows a series of constructions to be instantly created as an embedded environmental automated process, and the medium can keep a trace of such constructions and actions, but more so, *co-actions* between the user and the environment.

The "hot-spot" is a critical part of the construction. It is not just a spot (or dot) that can be moved (although the user started it off as such originally) but it fuses pieces of the geometric figure and becomes essential to the figure – deleting it would delete pieces of the figure. In moving the hot-spot, the figure dynamically re-constructs, so the hot-spot now has ownership of the figure, and in fact, the hot-spot is intimately bound up with the mathematics of the figure, i.e., that such a construction will always produce an equilateral triangle. By marking the triangle, we can see a discrete trace of the mathematical constructs inherent in the figure.

Reflection on what is occurring in dynamic figures

Here is the critical point: the hot-spot is no longer directly owned by the user. It is an infrastructural piece of the environment from which the user is now receiving feedback. In fact, it goes further than the existing theories of instrumentation and instrumentalization. The actions of the hot-spot and the figure being dragged by the user are now environmental (belonging to the software) in terms of visual feedback. So, the genesis of this figure goes from something personal – user actions – to environmental in terms of feedback. Following a construction, the diagram becomes more quasi-independent of its creator. Colette Laborde (2004) has made the point that the artificial realities of the diagram obey the rules of geometry that are preserved in the elements of the diagram, just as world objects obey the rules of physics in nature. But when an element of a diagram is dragged, the resulting re-constructions are developed by the environment NOT the user. So what becomes important is that the environment provides useful feedback. We continue to use this point when we reflect on instrumental genesis later.

The tool – in this case a hot-spot – that the user once defined as part of a construction, becomes re-shaped by the environment and the Euclidean rules that govern it. The actions of the user co-exist with the response or actions of the environment. The tool is also highly efficient (unlike our hammer example at the start) as it continues to fulfill its role without fault, as pre-determined in the construction, and so one might conjecture that this becomes an extremely useful learning tool precisely for this reason.

We can also discuss the physical use of hot-spots as a method to test the validity of geometric constructions (Mariotti, 2001). The dragging of well-constructed objects to establish whether the mathematical constructs that underlie their engineering can be preserved upon manipulation offers another dynamic perspective on geometric diagrams and is referred to as a "drag test." For example, the construction of the equilateral triangle in Figure 1 is a "true" Euclidean construction as illustrated by the drag test. Such embodied actions of pointing, clicking, grabbing and dragging parts of the geometric construction allows a semiotic mediation (Brousseau, 1997; Pea, 1993) between the object and the user who is trying to make sense, or induce some particular attribute of the diagram or prove some theorem.

Exemplar 2: Dynamic simulations

We now offer another example from a second type of representationally-rich software. Our example focuses on the coordination of piecewise linear position functions and stepwise constant velocity functions represented as graphs and motions in a software environment called SimCalc MathWorlds®, hereon called "MathWorlds." [2]

MathWorlds supports the creation of graphs, which are visually editable by clicking on hot-spots as well as being algebraically editable. These motions are simulated in the software so that users can see a character move whose motion is driven by the graphs they, or someone else, have constructed. Students can step through the motion, examine tables of values, and perform other operations in order to help them make qualitative and quantitative inferences about the motions represented by the graphs; all representations are linked.

Software runs on hand-held devices (for example, the TI-83/84+ graphing calculator or the Palm) as well as computers (as a Java Application). Figure 2 illustrates a screenshot from the computer version of MathWorlds. An actor A is depicted by a red dot in the world (horizontal in the top third of the screen). This actor's motion is driven by the piecewise function graphically visible in two forms in the lower half of the screen.

Observing the Position-Time graph in the window to the right you can see hot-spots on the end of each segment, coordinates (5,10) and (8,13) as well as two hot-spots on the

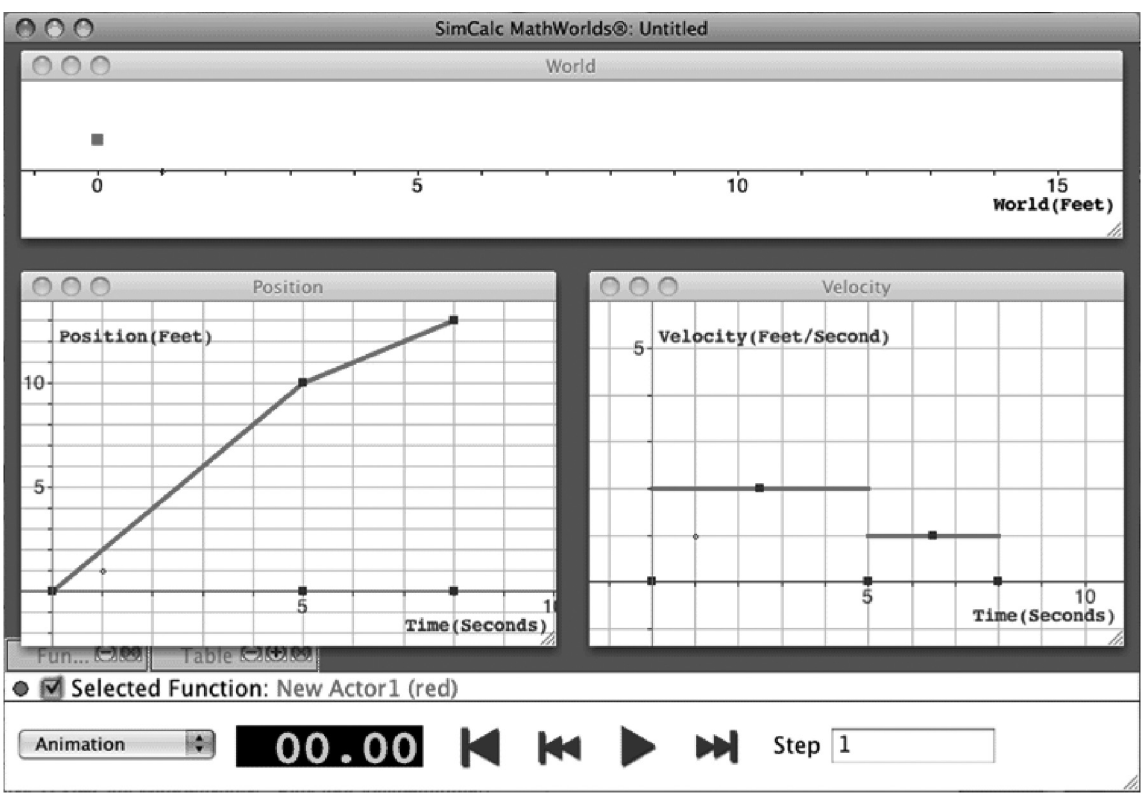

Figure 2. SimCalc MathWorlds® – Rate and Accumulation

time axis (5,0) and (8,0). We have parsed the two actions of vertical dragging, to change the slope of each piece (2 then 1 foot per second), from horizontal dragging to change the duration of each piece, to allow students to examine each covariate separately. So the hot-spots here are tools for the user (to change the data that drive the actor's motion) but the software environment offers feedback (every time), which is consistent mathematical feedback. If the slope of the first piece is changed by the user dragging the hot-spot at (5,10), the first segment of the Velocity-Time graph also changes. So if the slope increases, the constant rate piece increases by the same amount, fusing the relationship between position (accumulation) and velocity (rate) – a fundamental Calculus principle that is being made accessible through executable representations in MathWorlds. Also, the actor cannot disappear for a moment of time and so the position pieces are continuous (and are forced to be so in the software). Once again the hot-spots are infrastructural and once the user has used the tool, this tool or instrument, which is embedded in the environment, executes a series of actions on the representation. As in the geometry example, the mathematics of the construction are axiomatic to how the environment behaves and are part of the environment.

Kaput (2000) highlights how hot-spots are embedded in two of five innovations that constitute a representational infrastructure for the SimCalc environment. These include definition and direct manipulation of graphically editable functions, and direct, hot links between graphically editable functions and their derivatives or integrals. Others include connections between representations and simulations, the ability to import physical motion, and re-animate it, and the use of hybrid physical/cybernetic devices embodying dynamical systems. These are realized in a new media for carrying representational infrastructures.

So such "tools," as hot-spots, are actually instantiated at an infrastructural level and are a product of new, dynamic medium.

Whilst this example highlights the functionality of such a tool in MathWorlds, actual classroom activities that we have devised make use of one or more actors that we have either pre-defined in an activity document and that the student has to interact with or are the product of the student's work. An example of the first would be to make a motion for Actor A graphically that matches the motion for Actor B, except we previously hide the graph for Actor B. Here the use of the "hot-spot" undergoes a shift in utility from being a tool for the user to an executable representation in MathWorlds.

Exemplar 3: An extension to networked classrooms

Related work (*e.g.,* Hegedus & Kaput, 2003) has combined the use of MathWorlds with the latest advances in classroom connectivity, where multiple functions constructed on hand-held devices can be aggregated into the computer version of MathWorlds and projected onto a whiteboard. Now multiple representations can be executed in a social context, where students' personal contributions make an interesting gestalt in terms of their collective motion or as a family of functions. These can be hidden and displayed as needed so that the teachers can focus the attention of the students' work in meaningful ways. Varying constructions across naturally occurring groups in the classroom give rise to a suite of interesting mathematical activities, beyond the scope of this paper, but an important emerging example of how the new ingredient of networked classrooms (now including hand-held devices to computers) is leading to a new emerging environment characteristic, new representations of mathematical objects are being shaped and formed by multiple contributions. So the teacher can choose to interact with an aggregation of mathematical objects for a variety of pedagogical purposes because the environment now allows the interaction of multiple constructions.

Co-action is enhanced in such contexts where the environment is structured not only by the executability of the visual representations but also by the presence of a public space where the work of every student can be viewed and analyzed. In such a space the presentation can be controlled and the teacher can ask questions about expectations before a set of graphs or motions are displayed. In such an environment the executability of action leads to various forms of expressivity (Hegedus & Penuel, 2008) in terms of gesture as well as speech. The executability of the actions embedded in the digital environments entails various forms of expressivity. Students aim to explain "what they see" and consequently, they express themselves in terms of gestures as well as speech. Then, co-action extends into the social space between the user (student) and the whole set of contributions from all students. We can affirm that co-action, by this extension, becomes embedded in a social structure. Again, there is an "invisible hand" that can both guide the conceptual structure of the task and the flow of argumentation in the classroom. This is structured by the very nature of the mathematical concepts being explored yet, more so, the predominance of expressivity (through gesture and speech actions) that can occur as one moves from a private to a public space, and is sustained by the flow that networks can afford. Co-action becomes a relationship between a student, other students and an executable space within the technological environment.

A different perspective on instrumental genesis

We are offering a different perspective on instrumental genesis, which adheres to the existing process in the utilization of tools as a relationship between instrumentation to instrumentalization as summarized at the start, but which extends it with respect to the environment that the user interacts with. In thinking of hot-spots as not only tools or instruments but infrastructural to the software environment, they are intimately bound up with the mathematics that is preserved within the software (in the routines of the program), *e.g.,* continuity, multiple representations of functions, Euclidean constructions. As such, tools-as-instruments can be perceived in a slightly different way. For instrumentation, we additionally define it as *how co-actions with a tool shape the user's actions and understanding of the use of such a tool within, and with respect to, an environment.* Instrumentalization is extended to *how the tool is shaped by the user (user's knowledge) and the environment, i.e.,* when the tool is manipulated by environmental factors following a user-input. So an instrumental genesis can be extended to include:

- simultaneous co-actions between a user's use of a tool and a software environment's use of a tool,
- the feedback and reaction of a user being a certain process of utilization, and
- internalization of how the tool is manipulated, used by the environment, and then re-used by the user.

Let us situate this within a broader cultural argument. The role played by mediating artifacts has been key in our fate. Humans do not, and did not, interact directly with the environment. Human cultures appeared as an instrumental medium of adaptation and artifacts played a central role in all these processes (Moreno-Armella & Hegedus, 2009). These artifacts have not been just amplifiers of physical activity; they have been *crystallizations* of intentional activity whose outcome was us, a species inextricably linked to its cultural and technological environment, as we suggested previously. Along these complex processes, artifacts have undergone complex transformations as well. And this is what we want to emphasize in this paper: the *co-action (in this case with the artefact)* as the main characteristic of the mode of existence of human activity. This type of *co-action generates a zone of proximal development (ZPDA) for the artifact* that will be realized (in human activity) as long as the artifacts are stable and full of visibility in the cultural/technological medium. Mediated activity eventually leads to reflecting on the medium itself, the artifact. The *stable presence* of the artifact within an activity medium, its continued visibility, works as a guarantee that that technology will have the opportunity to evolve. Now we are not talking about biological evolution but, instead, we are talking about history.

The ZPDA is "felt" when we approach the limits of the artifact's capacity whilst the artifact is being guided through a purposeful activity. In fact, there is no other kind of activity. Writing systems, for instance, provided the ZPDA of incised bones or of another previous technology of memory. We must make clear the profound cultural meaning of the ZPDA because it is within the culture that we develop a conception and a way to deal with any artifact. There is *purpose* crystallized in an artifact and this opens its symbolic dimension that is crucial in its ZPDA.

We mentioned that co-action (of an artefact) is a creative process that began creating our human media. Technologies have been formatting societies for a long time. We live, in varying extents, in technologically saturated societies that do not allow us to think about a fixed process of appropriation of a technology (that happens, mediated by an artifact, as a phenotype of that technology). Culture cannot be factorized from a technology appropriation process. There is a space of artifactual intervention that is culture defined.

However, this does not mean it cannot be transformed. Scientists, artists, writers permanently alter these cultural spaces and in some cases, they even alter the spaces in a permanent fashion.

Notes

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[2] See http://www.kaputcenter.umassd.edu/products/software.

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