A Cerebral Basis for an Ontology of Mathematics and Physics

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1. Functioning of the cerebral hemispheres

There are differences between the cognitive functions of the left and right cerebral hemispheres of the human brain. Various dichotomies have been suggested to distinguish between the functions of the hemispheres. [Bogen, 1969] We shall limit ourselves to two such dichotomies: the analytic-synthetic dichotomy of Levi-Agresti and Sperry [1968] and the temporal-spatial dichotomy of Carmon and Nachshon [1971].

Ben-Dov and Carmon [1976] have suggested that all the hemispheric dichotomies can be derived from the analytic-synthetic dichotomy, which they renamed "extraction-integration" They constructed a multi-stage information theoretical model for the brain's functioning. In this model they derived the temporal-spatial dichotomy from the analytic synthetic one. According to this model the function of the left hemisphere is to extract single items of data one after another temporally, while the right hemisphere processes several data simultaneously and creates a new whole out of them

According to the model, data are processed by both hemispheres, in several stages, and the output of each hemisphere after every stage is available to both hemispheres for additional processing. For example while non-musicians listen to music, individual notes are processed one after another by the left hemisphere Afterwards the right hemisphere integrates a melody from the totality of notes. The left hemisphere can then treat the whole melody, which is integrated by the right hemisphere, as a new individual item Therefore professional musicians identify melodies by the left hemisphere while non-musicians do it by the right one [Kimura, 1964; Gordon, 1970; Bever and Chiarello, 1974]

The authors suggest that the first stage of sensory data processing by the left hemisphere is the presentation of the data individually to consciousness, one after another temporally, while the first stage of its processing by the right hemisphere is the spatial presentation of the same data They conclude that subjective time and space are the results of these presentations. Thus the authors derive the subjectivity of the Kantian modes of perception, time and space, from neurology (Kant's modes of perception and their subjectivity are explained in section 4 below)

The left hemisphere extracts one object after another. Between any two such extractions it is impossible for there to be an additional extraction. From this observation the authors derive the fact that time cannot be continous and must be quantisized.

2. Ontological approaches

There are two extreme, opposite, ontological approaches: platonism and nominalism, and an intermediate one, conceptualism.

According to platonism individual objects have no significance Only universals, which are sets of all elements having a certain property, really exist. The universals and the related properties are virtually identical They exist as real ideas.

According to nominalism only individual objects, which are concrete phenomena, exist. These objects can have properties. But universals, or sets of all elements having these properties, are only names and have no real existence.

According to conceptualism universals exist in the mind but do not have a real non-mental existence.

3. The hemispheres and ontology

We may apply the analytic-synthetic dichotomy of Levi-Agresti and Sperry to formulate a hypothesis which explains the nominalistic-platonistic dichotomy According to this hypothesis individual objects are extracted one at a time by the left hemispheric mechanism while universals are integrated from individual objects, as new comprehensive entities, by the right hemispheric mechanism Thus the existence of the two extreme ontological approaches is explained by the existence of two functionally different data processing mechanisms in the human brain

4. Kant's approach to mathematics

According to Kant the source of mathematics lies in the dual modes of perceiving experience: space and time [Kant, 1964; 1977] Geometry is derived from spatial intuition and is Euclidean. The arithmetic of ordinal numbers is derived from an intuition of "one after another" in time.

Kant describes several paradoxes regarding space and time. Kant concludes from the existence of the paradoxes that space and time are subjective and not necessarily related to things as they are in themselves. Space and time have only mental existence

Therefore we may obtain from Kant's view that mathematics, which is constructed from subjective spatial and temporal raw materials, must be a mental structure

Therefore Kant's approach to mathematics should be classified as conceptualism

According to Ben-Dov and Carmon [1976], Kant's modes of perception, space and time, are related to the right and left hemispheres respectively. Therefore we conclude that geometry and ordinal arithmetic are related to

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the right and left hemispheres respectively Thus we identify the neural mechanisms which according to our hypothesis construct Kant's mathematical structures.

5. Platonism versus nominalism in mathematics

According to Quine [1953] logicism is mathematical platonism while formalism is mathematical nominalism.

Logicism was established by Frege. He invented the predicate calculus in order to obtain cardinal arithmetic from logic Frege conceived the number 5, for example, as a property common to all the sets, concrete or abstract, having 5 elements [Frege, 1960] This property is equivalent to the property of being a member of the set of all sets having 5 elements. Therefore the last set can be defined as the number 5

The belief that the last set (the number 5) and similar concepts exist was Frege's motivation for formulating his axiom of abstraction: For every property P exists the set p of all sets having the property P. Properties can be represensted by predicates Therefore the axiom of abstraction can be formulated as: For every predicate $P(x)$ exists the set p of all the sets x which fulfill the predicate $P(x)$. This axiom has a platonic character. Moreover, Frege ignored the possible existence of elements which are not sets. Thus he followed the platonic view of denying the importance, or even the existence, of individual elements.

Logicism is an analytic *a priori* approach to mathematics

On the other hand Hilbert's formalisms is a synthetic *a posteriori* approach to mathematics, i e it establishes mathematics on concrete phenomena According to Hilbert [1964] we can infer from modern physics that infinity does not exist in nature. From Einstein's closed and finite universe he inferred that infinitely large quantities do not exist. From the existence of elementary particles, minimal electrical charge, and quanta of energy, he inferred that infinitely small quantities do not exist.

Hilbert conceived the universe as a finite limited space which is swarmed by a finite number of indivisible particles. This view is very close to that of Democritos (see section 6) In this world only concrete individual objects exist. This view is nominalistic

Hilbert stated that the domain of mathematics is composed of the concrete objects which are used as letters of a logical and mathematical calculus. Mathematics is the concrete formulas themselves.

The concrete formulas may have meaning But this meaning is not part of mathematics If the meaning is that of an infinite set, then it is a Kantian idea, which is not part of experience and so is not in the realm of mathematics. This meaning is not a real platonic idea

If the concrete calculus is consistent then there are no contradictions in the meaning. Therefore Hilbert defined mathematical truth as consistency of the concrete calculus. Thus Hilbert's mathematics treats concrete individual objects which serve as symbols Therefore in Quine [1953] it is considered to be a nominalistic approach to mathematics

Logicism and formalism use the same predicate calculus. The difference is that the logicists consider mathematics to be the platonic ideas which are the meaning of the formulas, while the formalists consider mathematics to be the concrete formulas themselves, the meanings of which can be Kantian ideas.

Our hypothesis is that the dichotomy of nominalism versus platonism is related to the dichotomy of the analytic versus synthetic data processing of the brain which was suggested by Levi-Agresti and Sperry [1968] According to our hypothesis the left hemispheric mechanism perceives concrete individual phenomena analytically one at a time, while the right hemispheric mechanism synthesizes a new whole out of several individual phenomena. This new entity is not a new concrete individual phenomena. It may be a set, a universal, or a platonic idea The right hemisphere "considers" its creation to have a real existence which is inconceivable by the left hemisphere, at least for some time

Thus the nominalist-platonist controversy may, perhaps, be explained by the two modes of human thinking: one with a left hemispheric dominance and the other with a right hemispheric dominance, respectively

6. Platonism versus nominalism in physics

6.1 *The Eleatic school and Einstein*

An early version of the argument between logicism and formalism about the nature of mathematics happened in ancient Greece, though at that time in connection with the nature of the physical world This was the discussion between the Eleatics and the atomists

The Eleatic school was founded by Parmenides. He based his doctrine on the statements that "what is thinkable can exist" and "the thought and the subject of the thought are identical". Another belief of Parmenides which influenced Plato is that if there is a contradiction between what our logic and our senses tell us, the inference from our logic is the correct choice.

The basic logical law, according to Parmenides, is the law of identity: $A = A$. According to this law "what there is" equals itself, or rather has the properties of itself. Therefore it has the property of existence. On the other hand, by the same argument "void" has the property of non-existence.

"What there is" is the entire universal physical being.

"What there is" is not created, because otherwise it would have to be created from void, which does not exist It does not disappear, because otherwise it would become void, which cannot exist.

"What there is" does not change, because a change means a lack of something. But lacking something is a property of "void" and not of "what there is"

For the same reason "what there is" is homogeneous, has no parts, is indivisible, and has no directionality.

"What there is" has spherical symmetry, or even perhaps the shape of a sphere

There is no motion because there is no void into which a moving body can enter. Time is meaningless in this Parmenidean world because it is permanent, without movement or change.

"What there is" is an everlasting, homogeneous, changeless, motionless, indivisible, full geometrical space. According to Parmenides it has a spherical shape, while according to his disciple Melissos it extends indefinitely.

We observe that the Eleatics disregarded concrete individual phenomena and were concerned only with the totality which can be integrated from all of them. The existence of this totality and its properties are inferred logically.

Concrete individual phenomena are not conceived of by the Eleatics as real but as sensory illusions. The totality of all of them, which is a logical idea, is conceived of as real

This is a platonic approach. Indeed, Plato was influenced by Parmenides

A modern approach which is similar to the Eleatic approach is Einstein's philosophical approach to physics. Einstein considered the world to be a four-dimensional geometrical space. Time is only a fourth-geometrical coordinate. There are no motion and no changes in Einstein's world because there is no external time during which changes can occur in this four-dimensional space

The four-dimensional space is full with the force lines of a gravitational field. Concrete objects are only local changes in the metric of the geometrical space, or in the density of the force lines of the gravitational field.

Einstein believed that all the forces of nature can be obtained from one unique universal force field, but he did not succeed in constructing a mathematical model for his theory. Einstein beleived that the world is a finite closed four-dimensional space full with the force lines of a single force field. The only difference between this space and the Parmenidean sphere is the homogeneity of the latter.

However according to the second law of thermodynamics (the law of increase in entropy) the closed system of the cosmos will be homogeneous in the future. Therefore we may infer from the principles of conservation and symmetry in physics (which were first formulated by Parmenides) that the cosmos is also homogeneous at present. The fact that we perceive the cosmos as non-homogeneous can be explained by the method by which the existence of the neutrino was inferred: there are force fields undetected by our senses and our instruments such that the total sum of them and the observable force fields is equal at every point in the cosmos.

We can conclude that Einstein's doctrine of a unified field supplied with the second law of thermodynamics results in a Parmenidean universe.

Einstein did not perceive the world as a void inhabited by discrete and concrete individual particles. On the contrary, he perceived it as the meaning of mathematical formulas. This meaning is not intuitive and cannot be perceived by our sensory imagination. But it has a real physical existence

This approach is platonic: the world is not a world of concrete phenomena but of the real ideas which the phenomena incompletely represent. These real ideas are mathematical ideas. This view originates in a source which influenced both Parmenides and Plato, namely Pythagoras.

6.2 *The atomistic school and elementary particles theory*

Democritos, who founded the atomistic school in ancient Greece, preferred the evidence of the senses to logical inference. The senses show us that motion exists, therefore Democritos concluded that empty space, into which moving bodies penetrate, must also exist

The world according to Democritos consists of an empty space which is swarmed by a multitude of atoms. Each atom is a very small Parmenidean "what there is" which is not created and does not disappear. Each atom possesses an initial oscillation. If two atoms collide their oscillations cause a linear motion. These linear motions cause the atoms, through additional collisions, to create large cosmic swirls from which worlds emerge.

The view of the Greek atomists is similar to the modern theory of elementary particles. As a matter of fact each elementary particle is conceived as a Democritean atom, i.e. a small concrete indivisible particle. The atomistic and the modern views are, in fact, ontologically nominalistic. They acknowledge the existence of only discrete and concrete individual objects. This view is similar to that of Hilbert, classified by Quine as nominalistic

6.3 *Zeno's paradoxes and the duality in physics*

Zeno was a disciple of Parmenides and his purpose was to prove his master's theory by countering the views of the opposition. However Democritos used at least one of Zeno's paradoxes, the paradox of dichotomy, to support his own theory

Democritos' version of this paradox is: a finite physical rod can be divided into two equal parts. Both parts can be further divided into two equal parts, and so on indefinitely. Let us consider the actual infinity of all these divisions. There are two possibilities:

a) The resulting parts of the totality of divisions will each have a finite length (we may consider them all to be equal). In this case the sum of their lengths will be infinite, contrary to the finiteness of the initial rod.
b) The resulting parts will each have length zero. In this case we cannot understand how the initial concrete rod can have consisted only of parts which are each nothingness.

Democritos concluded that there is a finite limit to the number of physical divisions of a rod. He called the indivisible particles which are thus obtained "atoms".

Zeno's original paradox was not about the physical rod but about the finite geometrical segment which is its length. Zeno's divisions are not physical divisions but geometrical divisions. Zeno's conclusion negates the assumption that every geometrical segment can be divided into two parts. Thus Zeno supports Parmenides' idea that "what there is" cannot be divided and has no parts.

Two paradoxes against the existence of motion which were formulated by Zeno are the runner paradox and Achilles and the tortoise paradox. In these paradoxes Zeno divided a motion along a finite path into a potentially infinite number of parts which decrease in magnitude.

There is no last part, and therefore Zeno claimed that the related motions will never end, contrary to experience

According to some modern writers [Vlastos, 1967] the atomists used these paradoxes as proof of the atomization of motion, i e that motion involves atoms of length or atoms of time. According to the same modern view the paradox of the three moving blocks (the stadium paradox) was formulated by Zeno as an answer to the atomistic explanation of the first two paradoxes The anti-atomistic version of this paradox is:

> Three equal blocks with a length l are placed one on top of the other. The lowest block is stationary while the two upper blocks move in opposite directions with equal constant velocities. The time of passage of the upper blocks over the lower one is t.
>
> Suppose that l is an atom of length and $t/2$ is an atom of time (time is not necessarily atomized) Then the two upper blocks pass one over the other during the atom of time $t/2$ But during this time each block passes over half of the lower block, i e over half an atom length. This contradicts the atomization of length.
>
> Suppose that the time t during which the two upper blocks pass over the lower one is an atom of time and the length l of a block is an atom of length (length is not necessarily atomized). Then the two upper blocks pass one over the other during half an atom of time, $t/2$. This contradicts the atomization of time.

According to modern writers a possible answer by the atomists is that there are no intermediate states, and the two upper blocks pass over the lower one is one "jump". These modern writers consider Zeno's arrow paradox to be an answer to such an atomistic view. The anti-atomistic version of the arrow paradox is:

> Suppose that an arrow is moving according to the atomistic view by jumps. During an atom of time the arrow is stationary at one point and then it "jumps" to another point It is inconceivable how and why a stationary arrow would perform such a "jump"

We see that there are two possible theoretical explanations to Zeno's paradoxes. One is atomism which is ontologically nominalistic. The other is the platonist Eleatic explanation. These two ontological approaches are, perhaps, related to the analytic-synthetic dichotomy of Levi-Agresti and Sperry, i.e to the left and right cerebral mechanisms respectively.

The atomistic Eleatic controversy was only one episode in the discussion between the discrete and continuous conceptions of nature. Another was the discussion between Newton and Huygens regarding the nature of light: particles or waves. Maybe that more modern duality in physics is another version of the ancient conflict.

It is possible that the duality in our perception of the physical world is a result of the existence of two cerebral mechanisms. One is the left hemispheric mechanism which perceives phenomena as discrete and concrete Our hypothesis is that this mechanism is responsible for the nominalistic ontological conception of the world The other is the right hemispheric mechanism which perceives the same phenomena as spatial, continuous, holistic, but not discrete This mechanism is responsible, according to our hypothesis, for the platonic conception of the world.

7. Kant's modes of perception and the duality in physics

According to Kant [1964], [1977], space and time are the two modes by which we perceive phenomena. In section 4 we saw why Kant considered space and time to be subjective and did not consider them to belong to things as they are in themselves. Let us see how duality in physics is related to the Kantian modes of perception.

Kant described the relation between the spatial mode of perception and physics in Kant [1977] pp 30-32. He described the speculations of mathematicians who were also philosophers that physical space may have properties which are different from those of mathematical Euclidean space Then he argued:

> They did not recognize that this thought-space renders possible the physical space, i.e. the extension of matter itself . for the space of the geometer is exactly the form of sensuous intuition which we find *a priori* in us, and contains the ground of the possibility of all external appearances (according to their form); and the latter must necessarily and most precisely agree with the proportions of the geometer, which he draws not from any fictitious concept but from the subjective basis of all external appearances, viz. sensibility itself.

If we apply the same principle also to the temporal mode of perception, i e the one-after-another in time, we shall find that it forces the external appearances to be discrete phenomena which can be regarded one after another in time. We may infer that both nominalistic atomism, which considers nature to comprise discrete particles, and the platonist Eleatic approach, whcih considers nature to be a geometrical continous real being, can be obtained from a Kantian approach. The Kantian theory can be considered as the basis for both.

The nominalist-platonic dichotomy as well as the temporal and spatial Kantian modes of perception are derived, according to the model of Ben-Dov and Carmon [1976] from the analytic-synthetic dichotomy of Levi-Agresti and Sperry, i.e., from the left and right hemispheric mechanisms respectively. In section 1 we saw that according to Ben-Dov and Carmon [1976] the derivation of the temporal mode of perception from the left hemispheric analytic mechanism causes time to be quantisized. Thus we arrive again at our hypothesis that duality in physics is related to the existence of two functionally different cerebral mechanisms.

8. The Kantian mathematical schools

Two opposing schools emerged from Kant's approach to mathematics. The first school is Brouwer's intuitionism. [Brouwer, 1913]

Brouwer argued that since non-Euclidean geometry exists, maybe physical space is non-Euclidean. If this is the case, Euclidean intuition of space contradicts physical experience, contrary to Kant's view. Therefore Brouwer suggested that mathematics should be established on the intuition of time alone, i.e. the intuition of one-after-another in time.

Intuitionist mathematics accepts ordinal numbers and series but not cardinal numbers and sets. In particular it accepts potentially infinite processes, but not actually infinite sets.

Intuitionism considers mathematics to be a mental construction. In Quine [1953] it is defined as a conceptualist approach. However the conceptualist character of intuitionism follows from the subjectivity of Kant's temporal mode of perception having only mental existence. Therefore intuitionist mathematics, which is constructed from temporal raw material, also exists only mentally.

The second school which emerged from Kant's approach to mathematics is Frege's geometrical approach, which we shall call "geometrism" [Frege, 1979] After the undermining of Frege's logicism by Russell's paradox, Frege returned to Kant's approach as a safer basis for establishing mathematics. However Frege considered time as a linear continuum and not as one-after-another. Therefore time is essentially no more than an additional geometrical line and is superfluous in establishing mathematics. So Frege established mathematics on spatial intuition alone.

Frege returned to the ancient Greek method of establishing the real numbers directly from geometry and extended it to define the complex numbers. He defined the division of two complex numbers through the relation of similarity of triangles. Thus geometrism considers numbers to be neither ordinal nor cardinal but a geometrical relation.

Frege considered an actual infinity of points to be intuited directly from the intuition of space. Thus the axiom of infinity has the same *a priori* status as the axioms of Euclid.

Since Frege's geometrism is a pure Kantian approach like Brouwer's intutionism, its ontological approach is also conceptualist.

According to Ben-Dov and Carmon [1976] the temporal and spatial Kantian modes of perception are related to the left and right hemispheres. Therefore intuitionism and geometrism should also be related to the left and right hemispheres, respectively.

9. Experimental evidence

Several experiments were conducted to test the hypotheses mentioned above regarding the relation between the cerebral hemispheres and schools in philosophy of mathematics and of physics. The experiments were conducted with students of a course in the philosophy of mathematics and with students of first year mathematics at the Technion, Israel Institute of Technology, Haifa.

The students were tested for the development of their cerebral hemispheres. A detailed description of the hemispheric tests is given in Fidelman [1985]. The scores of the students on the hemispheric tests were correlated with their scores on questions in the examinations on mathematics or philosophy of mathematics, and were compared with their preference for philosophical approaches.

We define "dominance of the right hemisphere" as the difference between the standardized scores on the right hemispheric and left hemispheric tests. In Fidelman [1985] it was found that students with a dominant right hemisphere preferred logicism to intuitionism while students with a dominant left hemisphere preferred intuitionism to logicism. The latter result was repeated in Fidelman [1986b] where it was also found that dominance of the right hemisphere is related to preference for logicism over formalism and for Frege's geometrism over Brouwer's intuitionism.

In Fidelman [1986c] it was found that dominance of the right hemisphere is related to preference of the Eleatic (and Einsteinian) explanation for the paradoxes of Zeno rather than the atomistic (and elementary particle theory) explanation. In Fidelman [1984] it was found that students with a dominant right hemisphere were more successful, relative to the average of the class, in Frege's cardinal arithmetic than in Peano's ordinal arithmetic. The results regarding students with a dominant left hemisphere were the contrary. In Fidelman [1986a] it was found that students with a dominant right hemisphere were more successful, relative to the average of the class, in the non-standard approach to infinitesimal calculus than in the standard approach. Contrary results were obtained regarding students with a dominant left hemisphere. Thus a relation was found between the right hemisphere and actual infinity (N S analysis) and between the left hemisphere and potential infinity (standard analysis).

In conclusion: the preferences of students for philosophical schools are in line with the hypothesis. The same is true regarding their scores on items which are related to supposed right hemispheric philosophies (cardinal numbers and actual infinity) and to supposed left hemispheric philosophies (ordinal numbers and potential infinity).

10. Conclusions

If a person considers himself as a perceiver of the phenomena which appear in his consciousness and detaches himself from his physical body and brain, then neuropsychological and other experiments cannot furnish answers to metaphysical questions. However a person can consider himself and others to be phenomena which his consciousness perceive. Then he can compare one kind of phenomena, namely opinions about mathematical and philosophical problems which are expressed by himself or others, or grades in examinations about such problems, with other phenomena, namely the brain structures of the same persons. He may discover a causal relation between them.

In our experiment he must infer from the relation between the two hemispheres and the two aspects of mathematics in which these mechanisms specialize that mathematics is a subjective creation of the brain. Therefore his ontology must be conceptualist. According to this view the temporal and spatial modes of perception as well as the

analytic and synthetic data processing functions of consciousness are related to the hemisphere mechanisms. Therefore all their creations have conceptualist ontological status. Both nominalism and platonism should be considered as partial approaches of a comprehensive conceptualist approach.

The situation regarding the ontology of physics is more complicated. If we consider ourselves as part of phenomena we must agree that the modes of perception are subjective. The same is true regarding the representation of phenomena as concrete and discrete individual phenomena (the nominalist picture of the world) on the one hand, and as real comprehensive logical ideas (the platonic picture of the world) on the other. The representation of physical phenomena in consciousness is indeed subjective and is related to the two hemispheric data processing mechanisms. Nevertheless they may represent things in themselves.

Maybe the two hemispheric data processing mechanisms evolved because of a duality in things as they are in themselves. We cannot be sure.

There is an intrinsic relation between the mathematical and physical ontological approaches. The pictures of the world of Democritos and of Hilbert are virtually identical. Both accept the evidence of the sense that concrete and discrete individual objects exist. These objects are also indivisible. Therefore the nominalist approaches to mathematics and to physics are two aspects of the same philosophical approach.

On the other hand the platonic approach to physics disregards the evidence of the senses that concrete and discrete individual objects exist. Only logically inferred entities, like the Parmenidean "What there is," exist. Similarly Frege's logicism disregards atomic elements (which represent concrete and discrete objects) and acknowledges only the existence of the set of all elements having a logically defined property. Therefore the platonic approaches to both mathematics and physics are two aspects of the same approach: "What is thinkable can exist."

The intrinsic relation between the nominalist mathematical and physical approaches may be caused by the relation of both to the left hemisphere. Similarly the relation between the platonic mathematical and physical ontological approaches may be a result of the relation of both to the right hemisphere.

In conclusion: the ontological problem is "What exists?" The answer regarding the part of consciousness which is related to the left hemisphere is: only individual discrete objects exist. These objects are regarded one at a time. The answer regarding the part of consciousness which is related to the right hemisphere is: only comprehensive entities exist. Each such entity is comprehended simultaneously and sometimes spatially.

If a person considers consciousness to be a phenomenon which is related to the phenomenon of the functioning of the brain, then whatever exists is represented by two different contradictory subjective mental pictures as described in the two previous answers.

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