

strongly seeks to paint everything in one global colour. This single-colour knowledge-system symbolizes and is celebrated as the marker of globalization. This form of oneness leads to a sense of belonging within only one type of knowledge, thus creating not just a stereotype but also taking pride in following the processes of glocalization.

Notes

- [1] Both authors share the first authorship.
 [2] ‘Glocalisation’ with an ‘s’ in it is the term in UK English. We explain later the story of ‘s’ turning to a ‘z’.
 [3] ‘Lingua franca’ means ‘Frankish language’ in Late Latin, and was originally the language of commerce in the Eastern Mediterranean. ‘Frankish’ referred to any Western European group, and the language was a pidgin of northern Italian dialects, the language of the Mediterranean coast of France (the *lingua d’oc*, distinct from the *langues d’oil* spoken in northern France, from which modern French is descended) and Catalan. The later dominance of French as the lingua franca of Europe has led to mistaken etymologies.
 [4] ‘Gross’ refers to 12 dozen or 144 units but for convenient computation, one gross is considered to be 140 units. This is a regular practice having spread since the times of colonisation.

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Problem-posing tasks and the inclusion principle

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Problem-posing (PP) tasks, if well designed and implemented, can meet several principles of effective task design, such as being accessible to students with different prior knowledge and experiences. In this short communication, I discuss an approach to design and implement PP tasks in mathematics, illustrated with a set of PP tasks designed for the Norwegian context. This discussion is a reflection on the inclusion principle, by which I mean considering students’ diverse prior knowledge and experiences *in* and *outside* mathematics. These reflections are based on my past

research on designing PP tasks with my colleagues in Iran and New Zealand.

While problem situations have been discussed in relation to designing PP tasks, and a distinction has been made between real-life and purely mathematical contexts (*e.g.*, Cai & Hwang, 2023), this short communication offers new insights on how the inclusion principle could be addressed in designing and implementing PP tasks in mathematics, particularly when task designers focus on real-life contexts. Such conceptualisation and operationalisation of inclusion in designing PP task could be considered when designing other task types.

Christou et al.’s taxonomy of the PP process

I am very much inspired by Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman’s (2005) taxonomy for designing PP tasks. Based on their taxonomy, one could design four types of PP tasks: 1) *Editing quantitative information*: Tasks where students pose a problem without any restriction based on the given information; 2) *Selecting quantitative information*: Tasks where students pose a problem that matches the given answer; 3) *Comprehending quantitative information*: Tasks that students pose a problem based on the given equations or calculations; and 4) *Translating quantitative information*: Tasks that requires students to pose a problem based on the given graphs, tables, or diagrams (Christou *et al.*, 2005). Using this framework with this simple description is a significant step toward designing PP tasks that elicit students’ mathematical thinking (see Nedaei, Radmehr & Drake, 2022). For instance, let us look at an example of *selecting quantitative information* task administered to Iranian and New Zealand students.

Task 1. Please can you pose a problem about the area enclosed between a curve and a line with any two arbitrary bounds that will give an answer of 1 (*i.e.*, the enclosed area will be equal to one) (Radmehr & Drake 2019, p. 91).

One could argue that this task has some value. It could be considered a task that has ‘wide walls’; students could consider different pairs of curves and lines that have an enclosed area of one, including pairing lines with trigonometric and cubic functions. This task also activates higher-order thinking; I observed that several students in New Zealand were challenged by it and experienced productive struggle (Radmehr & Drake, 2019). However, reflecting on the PP tasks we developed over the years and several important task design principles, I believe our previous design could be improved, as I discuss below.

Quantitative information in PP tasks

When deciding what type of quantitative information could be given to students in PP tasks, one could think about *inclusion* as one of the main task design principles (see Radmehr, 2023, for further details). Here task designers could move from *dress-up tasks to tasks with authentic context* (see Vos, 2020). But this is not all. When thinking about inclusion in terms of students’ prior knowledge and experience *outside* mathematics, having a meaningful, authentic context is not enough. We also could consider designing tasks in a cultur-

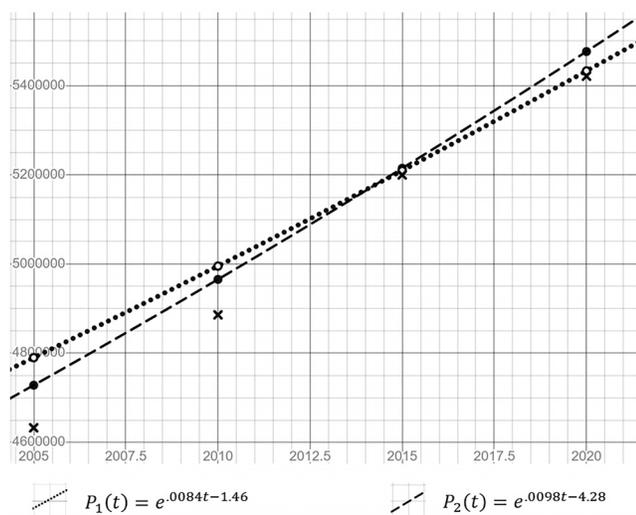
ally responsive way, meaning the authentic context chosen is relevant to students' lived experiences and cultures. This sub-principle is more important when task designers adapt mathematical tasks from other cultural contexts. Therefore, the quantitative information in PP tasks could be from authentic resources relevant to students' lived experiences and cultures. In this way, I propose integrating mathematical modelling (MM) and PP tasks. Such a proposal has been recently explored in the context of teacher education without differentiating between different types of PP tasks (e.g., Hartmann, Krawitz & Schukajlow, 2022). Integrating PP tasks with MM is different from PP that might occur during simplifying and/or mathematising a real situation in MM tasks. As in the set of tasks discussed here, students are prompted to pose problems, whereas in MM tasks, such prompts are not provided.

The order of PP tasks

Focusing on another aspect of inclusion, students' prior knowledge and experience *in mathematics*, three sub-principles could be considered: (a) low-threshold (i.e., the task is accessible to all students, including those with low mathematical prior knowledge), (b) high ceiling (i.e., the task provides opportunities for high achievers to challenge themselves), and (c) wide walls (i.e., the task can be solved using multiple approaches that is suitable for students with differ-

Task 3. Please pose a problem based on the given table and graph.

	$P_1(t)$	$P_2(t)$	Real data (×)
2005	4 789 804	4 727 939	4 632 364
2010	4 995 260	4 965 378	4 885 878
2015	5 209 529	5 214 741	5 199 836
2020	5 432 989	5 476 627	5 421 241



ent learning and problem-solving preferences). If a set of PP tasks is planned to be designed and implemented in a mathematical classroom, I would suggest starting with an *editing quantitative information* task with authentic resources relevant to students' lived experiences and cultures. This type of task typically has a *low threshold, wide walls*, and because of not having any restrictions could have a *high ceiling*.

For instance, in Task 2, I start by justifying why it is important to think about population projection and give students real data that they can use to pose and solve relevant problems.

Task 2. Population projection is important for many sectors, including government and researchers to make decisions for the future, such as planning construction, estimating basic human needs and the labor force. Pose a problem about Norway's population, based on the data available at the worldometers website [1], that would require using differential equations.

This task could be given to students in early sessions of an introductory differential equations (DEs) course because students can model the growth with a simple separable DE by assuming the rate of change of the Norwegian population is directly proportional to the Norwegian population at any time (i.e., $dP/dt = KP$, where K is some constant). The separable DEs is the first DEs class students typically learn in DEs courses; therefore, one could argue that Task 2 has a *low threshold*. For instance, if students are interested in posing the problem of predicting the Norwegian population in 2025, they could begin by first finding the general solution ($P(t) = e^{Kt+c}$), and then use the yearly percentage change on the worldometers website from the past few years (e.g., from 2016 to 2020) to estimate K ($(0.98 + 0.86 + 0.79 + 0.77 + 0.79)/5 = 0.838\% \approx 8.4 \times 10^{-3}$). Then they could use the population of Norway in one of the given years (e.g., $P(2016) = 5\,250\,949$) to find c (≈ -1.46), resulting in the prediction of the Norwegian population in 2025 to be 5 666 034. Similar to other modelling questions and solving initial value problems, if other values have been chosen for finding c , or K is estimated differently, other (more accurate or inaccurate) predictions would be achieved, indicating it addresses the *wide walls* subprinciple. This task also has a *high ceiling* for high achievers as students can go beyond a simple separable DE and focus on more complex DEs, such as the logistic model of population growth.

Both *translating* and *comprehending quantitative information* tasks can be given to help students develop further how different representations of mathematical concepts are related. These task types could also be situated in the same context as the previous task. In a *comprehending quantitative information* task, a DE could be given to students to work with. We could give a simple separable DE or continue with a more advanced model for population growth (e.g., the logistic model, $dP/dt = KP(N - P)$) depending on how students progressed in Task 2. In Task 3, I focus on the visual representations of DEs. Here, different approaches can be taken; for example, one could design a task around the slope/direction field to provide opportunities for students to explore what can be directly extracted from a DE about its solution or focus on the intervals of validity in initial value problems.

I work in the same context and data from the worldometers website. I draw two different particular solutions for the initial DE by first estimating K using the yearly percentage change from 2016 to 2020 and then considering $K = 0.0098$, based on the yearly percentage change in 2016. With the same initial value (*i.e.*, $P(2016) = 5\,250\,949$), this gives $P_1(t)$ and $P_2(t)$. I also included the real data for some years on the given graph to provide more opportunities for students to discuss the prediction provided by the DEs and its differences with the real data, *e.g.*, why are both $P_1(t)$ and $P_2(t)$ less accurate in predicting the Norwegian population in 2005 and 2010, compared to 2015? Or why is $P_2(t)$ less accurate in 2020 than $P_1(t)$ in predicting the Norwegian population? Is it the result of how K is estimated? This task could be considered to have a *low threshold* at the undergraduate level because the particular solutions are from a separable DE, not from more advanced DEs. However, it is also quite open to be considered to have a *wide walls* and *high ceiling* as students could think about what these functions represent and whether they are solutions to a DE and how they have been obtained.

This set of PP tasks could be ended by a *selecting quantitative information* task that could often be more challenging for students. In the interviews I conducted with upper secondary and tertiary students in New Zealand, I observed that some needed to change the functions and the bonds in response to Task 1 to create an enclosed area of one. This productive struggle was successful for almost half of the students who were interviewed; however, I observed that some of them gave up after a few tries, so it might be better to put this task type at the end of the set for high achievers and more motivated students.

A *selecting quantitative information* task is not always the most demanding task in a set of PP tasks. For example, here, Task 4 is not as demanding as Task 1; instead, it is designed to be interesting for Norwegian students, posing the problem of when their country's population would reach six million.

Task 4. Pose a problem that its solution would require using a differential equation and the response is approximately equal to 6 000 000.

If they use $P_1(t)$, they would find the answer 'sometime towards the end of 2031', and if using $P_2(t)$ it would be

'in early 2029'. However, the website also offers a population projection, predicting the Norwegian population to be 5 875 913 in 2030. This could trigger the curiosity of high achievers to investigate what model this website use to predict the population, which leads them to go to the United Nations website [2] and learn more about population projections.

I hope mathematics teachers and lecturers can use the suggestions provided here to integrate PP activities more effectively in mathematics classrooms/lecturers. The approach suggested here does not address all issues related to teaching and learning mathematics with PP activities but could be a starting point to make PP activities more accessible and meaningful for students.

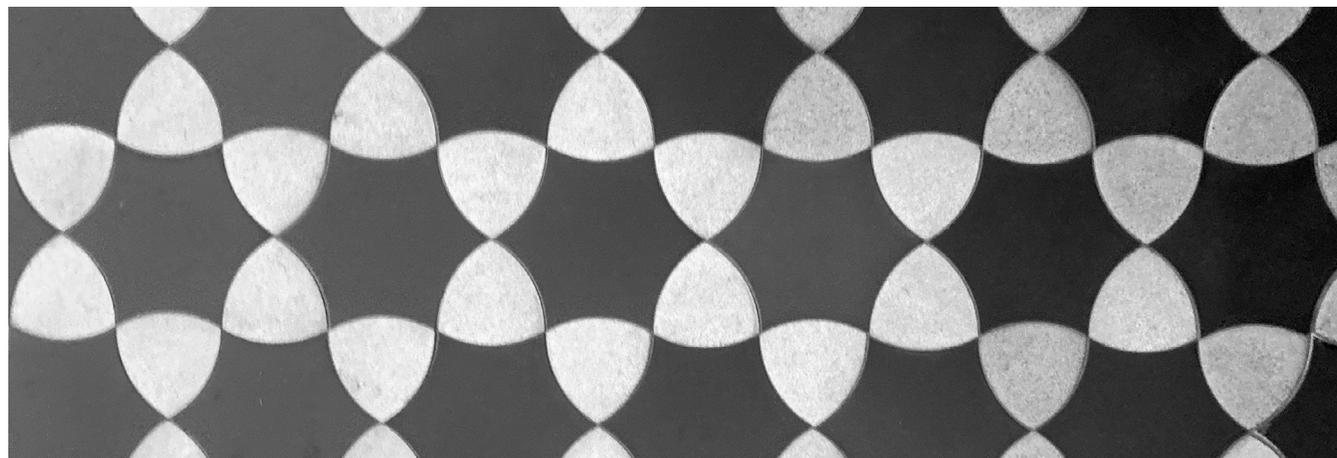
Notes

[1] <https://tinyurl.com/FLM-43-3-2>

[2] <https://population.un.org/wpp/>

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Floor tiles from the Matthias Church, Budapest.