MATHEMATICS EDUCATION: THEORY, PRACTICE & MEMORIES OVER 50 YEARS

JOHN MASON [1]

Having been invited to look back over my life in mathematics education, I take the liberty of recalling some of the most stimulating moments as they come back to me, in an attempt to analyse what mathematics education has been about for me. In particular I want to suggest that while the field has maintained and even widened the gap between theory and practice, it is incumbent upon us to remain steadfast that the purpose of our work is to understand and contribute to student learning of mathematics. One way I have consistently attempted to do this is to try to *preach what I practice* (and find effective) rather than the other way round.

One of my great amusements is that in the UK I am, I believe, seen mainly as a theorist, someone who works with ideas and tries to make them available to others. By contrast, in other countries I believe I am seen as immensely and intensely practical. In my defence in the UK I can point to numerous publications which have offered teachers practical actions to initiate when working with learners. But as I have often said to staff on arrival at the Open University, you have 12 to 18 months to establish a reputation in the university; after than it is very hard to alter. So too in the academic world more generally. For example, it seems that when people attend one lecture-presentation, they assume that that is all the person can talk about, when actually most of us are willing to engage with a wide range of issues at different phases of mathematics education.

Some historical accounts

I say 'memories over 50 years' in the title, because I started tutoring at the age of 15 at the request of my mathematics teacher Geoff Steele. It was only later that I realised just how profoundly he had influenced me through his stimulation and challenge. Many years later still I discovered that he had had no training in teaching nor much in mathematics: he worked hard to keep just ahead of me, writing out in longhand theorems in projective geometry, leading me through Hall and Knight on continued fractions, and engaging me in Susanne K. Langer's Symbolic Logic. I came to value very highly the contact I had with topics that did not appear in the formal curriculum until late university, and on this I base my recommendations that students be challenged sideways, in breadth and depth, rather than accelerated through the curriculum. It was true that I twice 'skipped' ahead, but this provided space to consolidate and explore broadly, and I am ever so grateful for it.

My first attempts at tutoring were of course completely naïve. I explained things when students I was tutoring got stuck. I tutored high school students in my first year at university, and then started supporting students in my college in the years below me. I discovered later that they used to go first to my friend across the hall who knew how to do the problems; then they would come to me and watch me struggle, asking them questions about what theorems they knew and so on! In my third and fourth years I tutored for the university. It was here that I discovered the effectiveness of *being mathematical with and in front of learners,* although I didn't formulate this slogan until much later. I would face a class of students stuck on a problem about which I knew almost nothing. So I would ask them to read the problem out loud, then to tell me from their notes what the technical terms meant, and what theorems they had in their notes concerning them. In every case I eventually 'saw' how to resolve the problem (and to my retrospective regret) I would then show them how to do the problem. At least they saw a slightly more experienced learner struggling publicly, so they could pick up some practices for themselves.

In graduate school I was shown George Pólya's film *Let Us Teach Guessing* (1965) on a Friday afternoon before teaching a semester class at 7:45 each morning starting on the Monday. As I later realised, the film released in me many of the practices used with me by Geoff Steele, and resonated vibrantly with practices I had developed spontaneously so as not to look a complete fool in tutorials. I got agreement with the class that they would work 'my way' on Mondays; on Tuesdays to Thursdays we would then work 'their way' in order to finish the chapter of the week, and on Fridays I would do revision problems with and for them. Within a week or two we were working 'my way' until at least part way through Wednesdays, finishing the chapter Thursdays, and revising on Fridays.

So what did it mean to 'work my way'? My recollection is that I would ask them questions to get them thinking. I would construct examples and generally cajole them. I would summarise in technical language what I thought they had begun to 'see' or appreciate. I felt that I was engaging them in the thinking. I am sure that an observer would have seen me getting them to 'guess what was in my mind', but the class was interactive and I hope challenging.

On my arrival at the Open University I realised that distance education was the antithesis of what I thought learning was about, but I settled down to write material. It was then that I realised how problematic teaching mathematics is, and that I had been enculturated as a structuralist through the influence of Bourbaki on many of my lecturers. One advantage was that, unlike the practices in a face-to-face institution where each lecture is an event to be survived, I frequently found myself at my desk wondering what example to use, what definition to use, what theorems to put in what order, and how best to interlace examples with abstractions and generalities.

As I was one of the few people in the faculty who had worked with Pólya's ideas, I was asked to organise the summer schools. I instituted sessions such as investigations (inspired by what I discovered going on in primary schools in the UK and based on Pólya's film), *mental callisthenics* (reproducing sessions I had experienced myself in primary school), *surgeries* (where students could come and ask for help on any aspect of the course), *tutor revelations* (in which a tutor would work through some typical questions while exposing the inner thoughts, choices and incantations accompanying the solving) as well as lectures and tutorials. I even tried sessions called *tutor bashing* in which a tutor poses a question to another tutor who then works at it from cold in front of the students before posing a question to the next tutor in sequence. This was an attempt to show students that tutors were mortal and fallible and that mathematics does not flow perfectly out of the pen in its completed form.

It took me several years to realise that what was obvious to me about how the mathematical practices described by Pólya, such as specialising & generalising, imagining & expressing, conjecturing & convincing, were not obvious to many of our tutors. Thus some tutors adopted a "don't let them leave for coffee break without giving them the formula, because they'll just get it from others anyway" approach, and recommended this to tutors new to the summer schools. On probing this it transpired that students in tutorials where the tutor did not like investigations usually came out not liking them, and students with tutors who did like them usually came out enjoying them. The stance, beliefs and attitudes of the tutor could be highly influential. This later resonated with the pioneering work of Alba Thompson (1984, 1992) working with teachers.

In 1973–74 I spent nearly a year with some 125 others in a house in Gloucestershire under the direction of J. G. Bennett, mathematician, scientist, linguist and seeker. It was a pivotal year for me, crystallizing many awarenesses and awakening me to many others. Ever since then I have been reconstructing the ideas and practices I encountered. In particular, I experienced deeply an experiential approach to enquiry, which for me extended through mathematics to every aspect of life and thought. Many years later I decided to devote time to reconstructing those practices, expressing them in the domain of mathematics education in particular, but applicable in any caring profession. I called it the Discipline of Noticing (Bennett, 1976; Mason, 1996, 2002). My idea was to provide teachers and other carers with a philosophically well-founded method and theoretical framework for researching their own practice, that is, for working on themselves.

I began my research life in combinatorial geometry, and achieved a certain notoriety within the rather small community of like-minded scholars as someone with a practical, example-rich approach to tackling really difficult mathematical problems from a structural perspective. It was after an event run by a colleague, Johnny Baker, in which teachers from secondary mathematics and science departments reported on their experience of teaching and using mathematics in schools, that I realised that although the mathematical problems I worked on were difficult, very few people cared, whereas the problems in mathematics education are essentially unsolvable, but a very large number of people care. So I turned my attention explicitly to mathematics education, and set myself three years to establish a reputation in the field. Despite this change of direction, I have never lost my interest in, no, my addiction to, working on mathematics. I always have some mathematical problem in the back of my mind that I work on in otherwise idle moments. This serves to keep me in touch with my own experience and sensitises me to struggles that others may have with different topics, concepts or problems.

It was while designing a course for mathematics teachers that I persuaded my colleagues that it would be a good idea for teachers to engage in mathematical thinking for themselves at their own level each week. But then a decision had to be made as to how to choose what problems to offer them, and how to structure that experience. We found that we had plenty of advice to offer, in my case gleaned from Pólya, Bennett and awareness of my own experience. Together Leone Burton and I decided to write a book about problem solving, in order to assist us in developing a structural framework for making suitable selections for the course. Leone introduced me to Kaye Stacey, and *Thinking Mathematically* (Mason *et al.,* 1982) was born. Surprisingly, perhaps, the only opportunity I ever had to teach a course like it, was before it was conceived, when I taught summer courses in Toronto, though others continue to use the book more than 25 years on. A new extended edition has appeared just this year (Mason *et al.,* 2010).

My non-academic activities in the 70s brought me into contact with a wide range of group activities exploring sensitivity training and body-mind connections. I encountered a wide range of practices and authors such as Abraham Maslow (1971). I immediately recognised from him that I was really interested in what is possible, what *could* be, rather than *what is the case currently.* Since at the Open University there were no students on campus to use as subjects, nor easy contact with schools, it matched my situation to be more interested in what is possible through examining my own experience while also working with teachers in in-service professional development sessions. One effect was that, with only a few exceptions, I only ever worked with teachers and students on a one-off basis rather than over an extended period of time.

Soon after I arrived at the Open University I discovered the Association of Teachers of Mathematics, and began to go to their meetings. I encountered there a group working on mental imagery and this fired my imagination, literally. I adopted and adapted practices that Dick Tahta and others used with posters and animations, to use with videotape of classroom interactions, and this was the basis for what our group constructed as an approach to using video. At ICME 5 in Adelaide I discovered this was called or at least akin to 'constructivism' which then evolved into *simple* or *psychological constructivism,* radical constructivism (much my preference) and later *social* constructivism. Combined with my experiential orientation, this set me up to resonate strongly with Ernst von Glasersfeld when I met him in Montreal in 1984.

In Adelaide I also met Guy Brousseau and extended contact with Nicholas Balacheff. Impressed by several of the constructs they used, in particular *transposition didactique* (Chevallard, 1985), *situation didactique* and the *didactic tension* (Brousseau, 1984, 1997) and *epistemological obstacles* (Bachelard, 1938), it took me a long time to appreciate even the most surface features of the deeply analytic frame that informed their impressive research. However, my own experience in working with groups of teachers experientially had shown me that offering results of research enquiries to others and expecting changes in practice is in itself highly problematic.

When I was a graduate student there were frequent calls for research on the most effective way to teach mathematics to undergraduates: 3 lectures + 2 tutorials per week, or 5 lectures per week, or 3 tutorials and 2 lectures or what? It was evident to me that the issue depended on too many factors connected with the setting, the individuals, the expectations, and the practices within lecturing and tutorials to be able to declare one better than another universally. Whereas most mathematicians that I knew were seeking a mathematical-type of theorem with definitive conclusions, I was convinced that any value system would be situation dependent. I found statistical findings deeply unsatisfying, because either they would agree with my own prejudices, in which case they told me nothing, or they would contradict those biases, in which case I would reject them as being unsuitable or irrelevant to my situation. I felt perfectly at home with the impossibility of mathematical-like theorems in mathematics education, because of the presence of human will, intention and ideals.

When I started working with teachers in the UK I soon realised that making use of ‘findings’ was problematic for others as well. Proposing a research finding (qualitative or quantitative) that is close to current practice is likely to be responded to by assimilation without noticing any subtle differences. Proposing a research finding that challenges thinking or which is not immediately compatible with practices, is unlikely to stimulate people to try the idea much less adopt it simply because it is a research finding. There has to be something which catches attention, either because it seems implausible and so motivates checking, or because it appears to match perceived current needs.

In my own case, even if I do try something, I am most likely to modify it to fit with my perspective and approach. Indeed, there is no ‘it’ as such, only my re-construction. This corresponds with my view of classroom incidents, indeed incidents and events generally: there is no ‘event’ as such, merely the stories told about it, whether at first, second, or later hand. On the other hand, if the finding fits with my experience, it is likely to seem ‘obvious’, so I am likely to pay no attention to any slight differences that might in fact be significant. Instead, I feel reassured and carry on. Thus for me a successful professional development session is one in which participants can actually imagine themselves acting differently in some situation in the future which they recognise. This statement is much more significant than it may appear at first sight. I emphasise *imagine themselves,* for one thing I learned from Bennett was the immense power of mental imagery for preparing actions to take place in the future.

What does seem to be helpful is prompting people to experience something which sheds light on their past experience and offers to inform their future choices. I see professional development as personal enquiry, stimulated and supported by work with colleagues, but essentially a psychological issue with a socio-cultural ecology. I have however always resisted pushing this as far, for example, as my one-time colleague Barbara Jaworski (2003, 2006, 2007) has done. I am content to indicate possibilities to others rather than trying to maximise efficiency and efficacy. For me, change is such a delicate matter that it must be left to the individual within their various communities, to the extent that “I cannot change others; I can work on changing myself”, and even that is far from easy!

My interests have always been in supporting others in fostering and sustaining mathematical thinking in their students. I have at various times concentrated on mental imagery, modelling, problem solving, and language, but these have been but byways in getting to grips with the nature and role of attention. I have, for example, found it convenient to shift my discourse from *processes of problem solving* to *exploiting natural powers,* finding that the same ideas (imagining & expressing, specialising & generalising, conjecturing & convincing, among others) continue to be potent as long as they are expressed in a context which resonates with people’s experience and a discourse that they recognise. Early in my reading of mathematics education books and papers I recognised that each generation has to re-express insights in their own vernacular, even though these insights have been expressed before. Indeed, each person has to re-experience and re-construct for themselves. This contrasts with mathematics in which it is possible to be directed along a ‘highway’ towards problems at the boundary without traversing all of the country in between. Mathematics education is not like that, and perhaps never will be, at least until we establish common ways of working. I take up this theme in the next section.

In the early 1980s I had the chance to attend a number of seminars led by Caleb Gattegno when he tried to re-vivify his science of education (Gattegno, 1987, 1990) in the mathematics education community in England. I found his approach attractive, with a good deal in common with what I had learned from Bennett, but leading to a rather different cosmology. I began to get a taste of what it is like when an experienced ‘grey-beard’ assembles their to-them-coherent-and-comprehensive framework or theory. Whereas when the fragments were being worked on and described there is often considerable interest amongst colleagues, once the whole is assembled, people don’t really want to know. I ran into this phenomenon again when reading Richard Skemp’s later book (1979), where again my experience was one of interest in some of his distinctions, without appreciating all of them, or the way they all fit together. Reading Jean Piaget, Zoltan Dienes, Hans Freudenthal, David Ausubel, Frédérique Papy and Humberto Maturana all had similar effects on me, partly perhaps, because I came across their work after or near the end of their careers. I found many specific distinctions of great value, but resisted taking on board their over-arching theories.

I assume that the issue is one of subordination. Philosophers are trained to suppress their own thinking in order to 'think like' the philosophers they are studying. In mathematics education, the intention is to improve the experience of learners, and this pragmatic dimension may contribute to a reluctance to let go of one's own stance in order to enter, absorb and fully appreciate the stance of someone else. Several French colleagues have given me the impression that in France they are more used to subordinating to an established theory, whereas Northern European anglo-saxon cultures appear to be more pragmatic and less theory-oriented in assembling their own personal framework or theory that 'works for them'.

The rise of mathematics education

Others are more scholarly at researching the ebb and flow, the waxing and waning of salient constructs in mathematics education. My memory is that in the 1970s and early 1980s research interest focused on students. I was, naturally, caught up in the Pólya-inspired *problem solving* discourse of *processes,* as manifested in *Thinking Mathematically.* As data accumulated, attention turned to student errors and misconceptions. I recall the breath of fresh air when Douglas McLeod and Verna Adams (1989) edited a book on affect and problem solving, and Alba Thompson championed devoting attention to teachers' beliefs as influencing both how people teach, and what students learn. As I look back now, it seems to me that one of the reasons for each generation revisiting and re-constructing classic insights and awareness is that, as well as participating in a process of personal re-construction, each generation finds itself dissatisfied with the explanatory and-or remedial power of the current discourse and foci. The discourse seems somehow to be drained of its power to inform choices, partly through over and mis-use perhaps. Each generation seeks fresh fields for explanation as to why students, on the whole, do not learn mathematics effectively or efficiently, and latterly, why teachers do not teach what they know and why they know so little of what it is necessary to know in order to teach effectively.

On the one hand we have Henri Poincaré's position of being mystified as to why perfectly rational people can fail to succeed at the perfectly rational discipline of mathematics, and on the other hand we have generations of students convinced that either fractions or algebra was a watershed for their involvement in mathematics. Clearly rationality is not the central feature of most people's psyche. One of the many things that has impressed me about Open University students over the years is that when I used to ask students on our mathematics courses why they were going to all that effort, I almost always got the reply "always liked mathematics at school; never could do it, mind, but always wanted to know more". Something touches people, even if it remains dormant.

Attention in mathematics education research has shifted variously between the structure of and inherent obstacles in specific topics, psychological aspects of learning mathematics, psychological aspects of teaching mathematics, sociological aspects of teaching and learning mathematics, acts of teaching, teachers' beliefs and how they influence learners, the historical-socio-cultural forces at work in and through institutions, and the content and format of teacher education courses, not to say the obstacles encountered by novice teachers due to weak mathematical background, and the destructive forces of school practices and government policies on the ideals and aspirations of novice teachers emerging from teacher education courses, to name but a few. Identity, agency and collaboration are currently popular, and multiple-selves may be just over the horizon. Most of what is accepted as *research* involves making observations of others (what I call *extra-spective*) whether associated with deliberate interventions or not. Observations and transcripts are turned into data by being selected for analysis. Analysis then applies a framework for making distinctions, or generates or modifies such a framework. The data becomes the object being analysed, not the original phenomenon.

It is tempting to say that we (the community of mathematics educators, scholars and researchers) have accumulated a great deal of data. We have individually, though perhaps not collectively, drawn a multitude of distinctions and formulated a plethora of constructs to analyse and account for what has been observed. But what have we really got to show for all this effort? Publications proliferate faster than I for one can read them, much less take them in and integrate them. Rarely do we get evidence that the framework has enabled teachers to modify their practice and so influence student learning. So what is the mathematics education enterprise?

The enterprise of mathematics education

On the surface, it is reasonable to expect that those engaged in mathematics education research and scholarship have as their aim the improvement of conditions for learning (and hence for teaching) mathematics more effectively, at every age and stage. Consequently evidence of effectiveness must lie ultimately in improvement in learner experience and performance, in both the short term and long term.

Of course there is an immediate obstacle, for there is little or no agreement as to what constitutes evidence of learner experience, much less evidence of improvement. Since it is easiest to gauge by scores on tests, national and international studies administer tests and pronounce on the results. Questionnaires and even interviews with selected subjects can be carried out. But there is a fundamental difficulty. Test results only indicate what subjects did on one occasion under one set of circumstances with or without specific training in preparation. Interviews at best reveal only what the interviewer probes and selects due to their sensitivities, and questionnaire responses are highly dubious indicators of what lies beneath surface reactions to specific questions. As soon as you identify an indicator of mathematical thinking or other mathematical competence or success and incorporate it into a test item, it should take a competent teacher at most two years to work out how to train students to answer those 'types' of questions. It all comes back to Guy Brousseau's notion of the *didactic contract* as manifested by the *didactic tension* and its parallel, the *assessment tension:*

> The more clearly and specifically the teacher (assessor) indicates the behaviour sought, the easier it is for the learner to display that behaviour without generating it from themselves (understanding).

Put another way in a discourse derived from Caleb Gattegno, training behaviour is important and useful, but it tends to be inflexible and even dangerous if it is not paralleled with educating awareness. It is awareness (what enables you to act, what you find 'comes to mind' in the way of actions) that guides and directs (en)action, using the energy arising from affect (by harnessing emotions). It is ever so tempting to train someone's behaviour by giving them rules and mnemonics to memorise, and quantities of exercises on which to rehearse. But real learning only occurs when these form the basis for reflection and integration so that awareness is educated (as in the Confucian culture approach to teaching and learning). Alternatively, one can work on educating awareness and training behaviour together, through harnessing emotion, and this is the approach that I have endeavoured to practice, and through practicing, to articulate for myself and others so as to make the process more efficient over time. Because I am interested in what is possible, and because the only way of directing other people's attention is through being aware of the focus of my own attention, I use *intra-spection* (between selves or between people) as distinct from *intro-spection* which garnered a negative connotation through the indulgences of people trying to develop phenomenological research methods in the early part of the 20th century.

One of the underlying tensions in mathematics education that I am aware of is that between a 'scientific stance' and a 'phenomenological stance'. Editors want their journals to contribute to the scientific development of knowledge. Journals have recently become so obsessed with theoretical frameworks that papers get longer and longer, without any growth in substance. I suspect that colleagues, especially editors, want to see mathematics education build a coherent and well-founded structure of knowledge. They like to see people building on each others' work, adding to and refining rather than starting afresh. I wonder however whether this is even possible, much less desirable, given the nature and focus of mathematics education, working as it does with human beings placed in institutional settings of various sorts, and exercising their wills and intentions through their dominant dispositions. I want to put a different case, the case of working with lived experience.

Structured awareness

I have often thought and sometimes said, that when I am engaged in my enquiries, I enjoy it most when I am at the overlap between mathematics, psychology and sociology, philosophy and religion. There is something about working on a mathematical problem which is for me profoundly spiritual; something about working on teaching and learning that integrates all three traditional aspects of my psyche (awareness, behaviour and emotion, or more formally, cognition, enaction and affect) as well as will and intention, which themselves derive from ancient psycho-religious philosophies such as expressed in the Upanishads (Rhadakrishnan, 1953) and the Bhagavad Gita (Mascaró, 1962; see also Raymond, 1972). I associate this sense of integration with an enhanced awareness, a sense of harmony and unity, a taste of freedom, which is in stark contrast to the habit and mechanicality of much of my existence. Even a little taste of freedom arising in a moment of participating in a choice, of responding freshly rather than reacting habitually is worth striving for.

One way to summarise such experiences is that, in the end, what I learn most about, is myself. This observation is not as solipsistic, isolating and idiosyncratic as it might seem, for in order to learn about myself I need to engage with others (who may, as is the case for hermits, be virtual), and I need to be supported and sustained in those enquiries. A suitable community can be invaluable, though an unsuitable community can be a millstone! I reached this conclusion through realising that when a researcher is reporting their data, and then analysing it, the distinctions they make, the relationships they notice, the properties they abstract all tell me as much about their own sensitivities to notice and dispositions to act as they do about the situation-data being analysed. Indeed I proposed an analogy to the Heisenberg principle in physics: the ratio of the precision of detail of analysis to the precision of detail about the researcher is roughly constant (Mason, 2002, p. 181).

The seeds of this observation were in working with teachers on classroom video, informed by techniques for working on mathematical animations (*e.g.,* those of Jean Nicolet and Gattegno's reworking of them). The technique is to get participants to reconstruct as much of the film as they can after seeing it just once. When they have made a good attempt, they have specific questions about portions they only partly recall, or where different people have different stories. So a second viewing makes sense, but only because there are specific questions. Applied to classroom video, we adopted a similar stance in order to counteract the common reaction of "I wouldn't let that teacher in my classroom", or "my low attainers are lower attaining than those low attainers" (Jaworski, 1989; Pimm, 1993). It seemed that teachers saw classroom video as a challenge to their identity and practices. By getting them to recount specific incidents briefly but vividly from the video with a minimum of judgement, evaluation, explaining, we found that they soon recognised incidents as being similar to incidents they had met in their own experience. So the videos became an entry into participants' own past experience, and hence gave access to their lived experience. This makes so much more sense than critiquing the behaviour of some unknown teacher whose class may already have left school, so there is no way that their behaviour could be altered!

Incidents which strike a viewer usually resonate or trigger associations with incidents recalled from the past. Describing these to others briefly-but-vividly so as to resonate or trigger their own recollections provides a data-base of rich experiences which can be accessed through the use of pertinent labels. Often sameness and difference between re-constructed incidents has to be negotiated amongst colleagues, and this is what prompts probing beneath the surface. As Italo Calvino (1983, p. 55) said, "It is only after you come to know the surface of things that you venture to see what is underneath; but the surface is inexhaustible".

I have come to recognise that Bennett (among many others over the centuries) was right when he highlighted the fundamental act of making distinctions. It is, after all, how organisms at all levels of complexity operate. Change is the experience of making distinctions over time; difference is the experience of distinction making in time. Evaluation is the experience of distinguishing relative intensities (as a ratio or scaling). Bennett went much further, amplifying Gurdjieff's observation that 'man is third force blind' (see Orage, 1930). In other words, distinguishing things, this from that, is important, but locks you into tension or evaluation, and is just the beginning of what is possible. In order to appreciate how the world works (whether material, mental, symbolic or spiritual) it is necessary to become aware that actions require three impulses: something to initiate, something to respond, and something to mediate between these, to bring them into or hold them in relationship. The product of actions can then go on to serve to initiate, respond to, or mediate a further action. Bennett continued this neo-pythagorean analysis into the quality of numbers from 1 to 12 in his monumental four-volume work *The Dramatic Universe,* which he called *systematics*, long before 'systems theory' became a slogan. Perhaps because of my structural upbringing, I found myself resonating with his approach, to the extent that I could sometimes hear in the structure of his talks the ways in which he was systematically employing systemic qualities of a particular number.

Precision & replication

There is another issue concerning transforming observations into data and the degree of precision presented in research reports. Over the years there has been an evident growth in the length and complexity of papers in mathematics education. It used to be that some detailed transcripts along with some analysis stimulated colleagues to investigate the phenomenon in their own setting. A classic example would be the paper by Stanley Erlwanger (1973, reprinted in Cooney *et al.,* 2004) about Benny's encounter with fractions. Nowadays this paper would probably be rejected by journals as failing to present an adequate theoretical framework and discussion of method and ethics. Many recent papers are so heavily theory-laden in the opening sections that by the time I get to the substance I have forgotten exactly which parts of which theory are actually being employed, and indeed sometimes it is not even very easy to detect this. It seems to me that often only tiny fragments of theoretical frameworks are called upon. Indeed, I have no problem with this at all, because of my eclectically cherry-picking approach to understanding and practice: all I can ever do is be stimulated or sensitised to notice, that is to discern details not previously attended to, and through that discernment, raise questions deserving of enquiry. But if authors are selecting fragments, why not be straight forward about it? I go so far as to suggest that experience itself is fragmentary, despite consciousness and the collection of selves that make up consciousness and personality trying to develop stories to make it look continuous and coherent (Mason, 1986, 1988). This is the one detail on which I disagree with William James' notion of a 'stream of consciousness' (James, 1890). My own observations agree with Tor Nørretranders (1998) that these stories are a fabricated illusion.

I realise that editors have a commitment to building the scientific foundations of mathematics education, but I don't see that present practices are actually furthering the field, in the main. What we do have is a plethora of distinctions, sometimes several labels for at best subtly distinct distinctions, and sometimes the same label used for different distinctions. What the field really needs is some agreement on ways of working, rather than on theoretical frames and stances. We need to build up a vocabulary for how we compare observations, turn them into data, and negotiate meaning amongst ourselves. This would then make it easier to offer similar distinctions to others including teachers, teacher educators and policy makers, and to negotiate similarities, differences and intensities. Caleb Gattegno offered his *science of education* but this is too radical for most to agree to; in the Discipline of Noticing I tried to offer a less radical and more practical foundation for ways of working; I am sure that others feel they have done the same. The problem in my view lies not in the fact that everyone discerns slightly differently, but that we don't have established ways of negotiating similarities and differences in what is noticed and in what triggers that noticing, and in what actions might then be called into play.

Despite the developments in style (I hesitate to use the word *improvement*) it is still rarely if ever possible to imagine much less actually carry out a replication of a study reported on in a mathematics education research paper in a journal. There is simply never enough detail. I happen to suspect that it would never be possible to replicate a study exactly, precisely because of the complex range of factors comprising the traditional triad of student, teacher and mathematics all embedded in an institutional environment or milieu.

If it is either impossible or not necessary to be able to replicate the conditions of a study, what is it that we are gaining by reporting on our studies? My radical response to such a question is that what matters most is *educating awareness* by alerting me to something worth noticing because it then opens the way to my choosing to respond rather than react with a more creative action than would otherwise be the case. I don't usually need all sorts of detailed data, because the more precise and fine-grained the detail, the less likely I am to pay attention to the over all phenomenon being instantiated, and so the less likely I am to recognise it again in the future and so choose to act differently.

Reprise

I am genuinely perplexed about the role and nature of structure in a domain such as mathematics education. On the one hand, with my structural background, I find it really helpful to be able occasionally to invoke one or other structure in order to inform my thinking. But I have colleagues who resist such an approach, just as I have resisted accommodating the whole of other people's structured frameworks. It is too simplistic to say that each could be expounded and then tested by experiment to see which is 'best'. I am reminded of a sequence of lectures I was required to attend in my first year at university on *the leap of faith.* My recollection is that they were about the philosophical conundrum of how you cannot investigate or enquire into what it is like to believe something without actually believing it. Put in an overly extreme form perhaps, 'if you can critique it, you haven't experienced it fully'. Of course this is anathema to many in Western society, but increasingly popular to fundamentalists the world over.

My own experience is that I do not usually use my own frameworks systematically or mechanically, because they have been integrated into how I perceive the world and how my thinking progresses. Every so often it is useful to ask myself if I have taken all aspects of an action, an activity, a potentiality, a moment, a transformation into account, and this is when it can be fruitful to remind myself of the pertinent number and its structural qualities. More specifically, the Structure of a Topic framework (Griffin & Gates, 1989; Mason & Johnston-Wilder, 2004a, 2004b), based on Bennett's 'present moment' system associated with qualities of six is particularly useful when preparing to teach a topic. The six modes of interaction (expounding, explaining, exploring, examining, exercising and expressing) arising from the qualities of three alert me to possible forms of interaction and bring different interventions to mind (Mason, 1979).

I suspect that each of us does something similar. We act in the world; when some tension or disturbance arises, we resort to accustomed modes of thinking using whatever discernments of distinctions come to mind, and then carry on. We have habitual but slowly evolving forms of activity with which we feel comfortable; we are sensitised to certain aspects of potentiality in a situation; we stress certain aspects of the present moment; and so on. Calvino (1983, p. 107) said something similar: "The universe is a mirror in which we can contemplate only what we have learned to know in ourselves", which in turn resonates with a North American shaman Hyemeyohsts Storm (1985) who phrased it as "the Universe is the Mirror of the People". The universe, whether material, imagined or symbolic, provides a mirror for seeing ourselves and so bringing possible actions to mind. Indeed, all we can see is in fact ourselves, in the sense that what we discern and relate is a reflection of ourselves. What professional development means is ongoing work to extend sensitivities, striving for a greater balance in the interplay of component features, so as to participate more fully in the evolution of awareness.

Note

[1] An earlier version of this article was included as the opening chapter in Lerman, S., & Davis, B. (Eds.) (2009) *Mathematical action & structures of noticing: studies on John Mason's contributions to mathematics education,* Dordrecht, NL, Sense Publishers. It is reprinted with the kind permission of Sense Publishers.

References

Bachelard, G. (1938; reprinted 1980) *La Formation de l'esprit scientifique,* Paris, FR, J. Vrin.

Bennett, J. (1956–1966) *The dramatic universe* (four volumes), London, UK, Hodder & Stoughton.

Bennett, J. (1976) *Noticing* (The Sherborne Theme Talks, Series 2), Sherborne, UK, Coombe Springs Press.

Brousseau, G. (1984) 'The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics', in Steiner, H. (Ed.), *Theory of mathematics education* (Paper 54), Bielefeld, DE, Institut für Didaktik der Mathematik der Universität Bielefeld, pp. 110–119.

Brousseau, G. (1997) *Theory of didactical situations in mathematics: didactiques des mathématiques, 1970–1990,* in Balacheff, N., Cooper, M., Sutherland, R. and Warfield, V. (Trans.), Dordrecht, NL, Kluwer.

Calvino, I. (1983) *Mr Palomar,* London, UK, Harcourt, Brace & Jovanovitch.

Carpenter, T., Dossey, J., and Koehler, J. (Eds.) (2004) *Classics in mathematics education research,* Reston, VA, NCTM.

Chevallard, Y. (1985) *La transposition didactique,* Grenoble, FR, La Pensée Sauvage.

Erlwanger, S. (1973) 'Benny's conception of rules and answers in IPI mathematics, *Journal of Mathematical Behaviour* **1**(2), 7–26.

Gattegno, C. (1987) *The science of education, part I: theoretical considerations,* New York, NY, Educational Solutions.

Gattegno, C. (1990) *The science of education,* New York, NY, Educational Solutions.

Griffin, P. and Gates, P. (1989) *Project Mathematics UPDATE: PM753A,B,C,D, Preparing to teach angle, equations, ratio and probability,* Milton Keynes:, UK, Open University.

James, W. (1890, reprinted 1950) *Principles of psychology* **1**, New York, NY, Dover.

Jaworski B. (2006) 'Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching', *Journal of Mathematics Teacher Education* **9**(2), 187–211.

Jaworski, B. (1989) *Using classroom videotape to develop your teaching,* Milton Keynes, UK, Centre for Mathematics Education, Open University.

Jaworski, B. (2003) 'Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships', *Educational Studies in Mathematics* **54**(2–3), 249–282.

Jaworski, B. (2007) 'Developmental research in mathematics teaching and learning: developing learning communities based on inquiry and design', in *Proceedings of 2006 Annual Meeting of the Canadian Mathematics Education Study Group,* Calgary, AB, University of Calgary, 3–16.

Mascaró, J. (Trans.) (1962) *The Bhagavad Gita,* Harmondsworth, UK, Penguin.

Maslow, A. (1971) *The farther reaches of human nature,* New York, NY, Viking Press.

Mason, J. (1979) 'Which medium, which message', *Visual Education* (February), 29–33.

Mason, J. (1986) 'Probes and fragments', *For the Learning of Mathematics* **6**(2), 42–46.

Mason, J. (1988) 'Fragments: the implications for teachers, learners and media users/researchers of personal construal and fragmentary recollection of aural and visual messages', *Instructional Science* **17**, 195–218.

Mason, J. (1996) *Personal enquiry: moving from concern towards research,* Milton Keynes, UK, Open University.

Mason, J. (2002) *Researching your own practice: the discipline of noticing,* London, UK, RoutledgeFalmer.

Mason, J., Burton L. and Stacey K. (1982) *Thinking mathematically,* London, UK, Addison Wesley.

Mason, J., Burton L. and Stacey K. (2010) *Thinking mathematically (2nd edn.),* London, UK, Pearson.

Mason, J. and Johnston-Wilder, S. (2004a) *Designing and using mathematical tasks,* Milton Keynes, UK, Open University (2006 reprint, St. Albans, UK, QED).

Mason, J. and Johnston-Wilder, S. (2004b) *Fundamental constructs in mathematics education,* London, UK, RoutledgeFalmer,.

McLeod, D. and Adams, V. (Eds.) (1989) 'Affect and mathematical problem solving: a new perspective', New York, NY, Springer-Verlag.

Nørretranders, T. (1998) *The user illusion: cutting consciousness down to size,* Sydenham, J. (trans.), London, UK, Allen Lane.

Orage, A.(1954) *Essays & aphorisms,* New York, NY, Janus.

Pimm, D. (1993) 'From should to could: reflections on possibilities of mathematics teacher education', *For the Learning of Mathematics* **13**(2), 27–32.

Pólya, G. (1962) *Mathematical discovery: on understanding, learning, and teaching problem solving, combined edition,* New York, NY, Wiley.

Pólya, G. (1965) *Let us teach guessing* (film), Washington, DC, Mathematical Association of America.

Raymond, L. (1972) *To live within,* London, UK, George Allen & Unwin.

Rhadakrishnan S. (1953) *The Principal Upanishads,* London, UK, George Allen & Unwin.

Skemp, R. (1979) *Intelligence, learning and action,* Chichester, UK, Wiley.

Storm, H. (1985) *Seven arrows,* New York, NY, Ballantine.

Thompson, A. (1984) 'The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice', *Educational Studies in Mathematics* **15**, 105–127.

Thompson, A. (1992) 'Teachers' beliefs and conceptions: a synthesis of the research', in Grouws, D. (Ed.), *Handbook of research in mathematics teaching and learning,* New York, NY, MacMillan, 127–146.