STUDENT THINKING AS A CONTEXT FOR HIGH EXPECTATIONS

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> There's something very implicit about the whole thing. So part of it is hard to describe. Part of it is dealing with the student in a way that focuses on or taking for granted that the student can learn what I'm giving to him or her. And when I say taking that for granted, I mean dealing with a student in a way that *assumes* that as opposed to having to on a daily basis reinforce that the student is not learning...Whether a student comes in with a U [failing grade] or has no intention to go on anywhere, I still will work with that student to the same degree – having the same expectations as I would for a student that is highly motivated [and] totally college bound. (interview 4/7/98 [1])

The quotation above illustrates the expression of high expectations of a high school algebra teacher, Matthew, for his urban, Latino/a students. Given that the students were enrolled in a low-achieving mathematics class and were largely non-native speakers of English, this teacher's expressions of high expectations are noteworthy, as they contrast with those often experienced by students in similar circumstances (*e.g.*, Allexsaht-Snider and Hart, 2001; Atweh, Bleicher and Cooper, 1998; NCTM, 2000; Oakes, 1995; Zevenbergen, 2003). The purpose of this article is not simply to share this teacher's belief in his students' capacity to learn mathematics. Rather, we seek to illustrate the goals that he held for his students and the specific instructional strategies that he used to instantiate those high expectations.

Background

Matthew was a mathematics teacher in an urban high school in the Midwestern United States. The school served a large Latino/a population (approximately 60%) with smaller populations of African American, Asian, and white students. The student population of the high school was predominately of low socioeconomic status, with 65% of the student body eligible for free or reduced lunch.

Matthew was one of four Spanish/English bilingual mathematics teachers at the school. He deliberately chose this context as a student teacher, electing to commute 1.5 hours (one-way) to an urban school district so that he could be placed in a Spanish-speaking classroom. Upon completion of his student teaching, he was immediately hired by the school to fill a mid-year vacancy. The classroom episodes described in this article took place during Matthew's second year of teaching at the school.

Like many entry-level teachers, Matthew was assigned to teach some of the lowest-level mathematics classes offered at the school. All students in the school were required to pass a course called *Algebra I* in order to graduate, as part of the district's participation in a special mathematics initiative. In response to this requirement, the school had created special classes for students who had previously failed *Algebra I*. Matthew had been assigned to teach two sections of this 'repeat' course. The classroom episodes included in this article are taken from one section of these courses. The students in the class ranged from sophomores to seniors and all were bilingual Spanish-speaking students, mostly of Mexican or Puerto Rican descent. English and Spanish were readily interchanged during instruction, with Matthew often repeating statements in both languages for the benefit of students who were dominant in one language over the other.

Matthew's daily instruction fell into a consistent pattern of presenting students with a real-world problem, placing students in groups to analyze the problem, assisting students individually and in groups as they worked on the problem, and following up with a whole-class discussion of students' solutions and the results. The interactions that occurred during the whole class discussions were central to his pedagogy. He stated on multiple occasions his explicit intention to engage students in sharing their thinking and to recognize this as an important part of learning. His goal was to help his students develop the habits of

> being able to express themselves, wanting to express themselves, but even more than that, accepting [other] students' responses as valid and meaningful, you know, and not just waiting for the teacher. (interview, 1/12/99)

Addressing student needs

Matthew was one of four bilingual mathematics teachers participating in a research project [2] intended to study the teaching of algebra for understanding. Both authors of this article served as university researchers on the project. Our early classroom observations of the teachers took a primarily cognitive approach and focused almost exclusively on how the content was taught. However, from these initial observations, we noted that Matthew did more than just teach the content. He also seemed to put significant effort into addressing the affective needs of his students – students with a history of failure in mathematics.

From this point, we expanded our research perspective to examine further the interactive nature of Matthew's dual goals of teaching algebra and attending to students needs. As explained below, whole-class discussions in which Matthew elicited student thinking became a viable and pertinent site to examine his affective and cognitive goals.

For the Learning of Mathematics **25**, 2 (July, 2005)
FLM Publishing Association, Edmonton, Alberta, Canada

Matthew viewed eliciting student thinking as a means of serving multiple goals. These were:

- to build understanding
- to build self-confidence
- to build self-image as a scholar.

The first is "just good *Standards* stuff" (interview, 4/7/98) with emphasis on learning mathematics with understanding. The latter goals reflect Matthew's attempts to increase his students' participation in the classroom. He distinguished the latter two categories by providing examples of two types of students – one who lacked confidence and one who lacked engagement.

To illustrate the need for building confidence, Matthew described one female student who was always on task but viewed herself as weak in mathematics. He felt that her mathematical self-esteem might have been diminished because of her placement within a group of highly successful students in her previous algebra class. One of his goals was to increase the confidence of students such as the one he had described – students who had lost the belief in their abilities and were often reluctant to share their thinking.

Matthew's third goal, to build self-image as a scholar, was used to address students who were not engaged in learning mathematics, *e.g.,* Matthew described a student who was a gang member and had been hospitalized for a gang-related shooting, as a student who did not expend much energy in his schoolwork. Matthew elaborated that he wanted this student to see himself as "a scholar as opposed to just a gangster" (interview, 4/7/98). In this sense Matthew recognized that some students may not necessarily lack confidence but perhaps lack investment (McKay and Wong, 1996) in learning.

Matthew used whole-class discussions as one of the most visible means of treating students as scholars, attempting to increase their motivation by interacting with them in a way that makes clear the expectation that they can learn. Simultaneously, these discussions, which elicited student thinking, also served as sites for learning. He describes this multipurpose approach in the following way:

> Exploring the students' understanding, beginning where the student is coming from, and then taking the student to where you are at, rather than, bringing yourself to the student, or just, giving the student the information. And, I think, in doing that, it is both curricularly sound, and, at the same time, emotionally sound because you are giving the student the feeling that "Hey, what I think is important. This guy respects me. This guy thinks that I know something. This guy is here to show me that I know a lot." (interview, 4/7/98)

Strategies

Matthew made explicit in an interview some of the strategies he used to accomplish the above goals of building understanding, building self-confidence, and building a self-image as a scholar. He indicated that the ways in which he attended to student thinking during whole-class discussions were intended to meet one or more of his goals for students. In other words, these were purposeful, rather than unintentional, patterns in his interactions with students. This information influenced the subsequent analyses of the classroom episodes. We searched for strategies commonly employed during whole-class discussions as Matthew attended to student thinking. Four strategies emerged.

We label the first strategy 'CGI' since Matthew used this term in reference to the program *Cognitively Guided Instruction* (Carpenter and Fennema, 1992; Fennema, Franke, Carpenter and Carey, 1993). CGI is a professional development program that is based on the same constructivist theories of learning that are represented in the most recent set of recommendations of the National Council of Teachers of Mathematics (2000). Specifically, the program is built upon the premise that understanding of mathematics grows as the teacher builds upon students' prior formal or informal knowledge. Thus, the use of the CGI label to represent Matthew's strategy is intended to reflect this attention to student thinking. Evidence of this strategy was found when Matthew required students to explain and justify their solution strategies. In almost every instance when eliciting a response, Matthew would not simply be satisfied by the answer to the problem but also asked, "How did you do it?"

A second strategy, 'multiple solutions', was evidenced when Matthew elicited alternative solutions or strategies and/or affirmed an alternative solution or strategy. This approach was also quite common. He readily asked for "different" ways of solving the problem. Furthermore, at least in his teaching of rates and linear equations, this strategy was part of his pedagogical plan:

> The other thing I like to do is once we get one answer find if there were other answers. The rate of change is an excellent example because, depending on what points you choose, you are going to get a different answer. So at least take one or two other answers from other people and compare those to make sure that we all see that it is really giving you the same thing. (interview, 3/17/98)

A third strategy, 'wrong terminology/answer', represents Matthew's attempts to scaffold [3] an incorrect solution into a valid mathematical thought. This strategy often occurred when Matthew would prompt a student for further explanation of his/her thinking even after the student did not produce the mathematically correct response or would "put on hold" a solution until something valid could be made of it.

For example, Matthew had requested that his students give him examples of quantities that are sold as rates. One student suggested that shoes are sold by rate. Matthew never directly stated that the response was not appropriate but rather posed an argument for more thought:

> The problem with doing this by branding is that we need a specific unit here. Let's put a question mark for shoes? (observation, 2/17/98)

The last pattern that we identified in the classroom observations and that Matthew described as an intentional strategy is one that we call 'positive affirmation'. This strategy reflects any attempt by Matthew to validate a student's solution, affirm a student with a compliment ("very nice"), or attend to a student's comment even if it is off task or inappropriate. Matthew describes this latter scenario when students sometimes may offer a response that is somewhat off-task:

I am not hearing you goofing off. I am not hearing you doing those things. What I am hearing is an intellectual piece of information there somewhere and that is valid and I respect you for that and I am going to reward you for that by incorporating it somehow into the discussion or at least validating it and saying, well, whatever, as opposed to just, no, it is not right. (interview, 4/17/98)

Classroom episodes

We present three classroom episodes to illustrate the strategies Matthew used to elicit and respond to student thinking. The first episode, taken from a class discussion on 2/17/98, illustrates the use of all four strategies and highlights interaction with two students. In the context of teaching students about rates, Matthew asked the class to consider a situation in which a car is travelling 40 miles/hour. He told the students to make a table of 10 values, find a pattern, and make a graph. There was a short discussion about the label of the two columns – hours and miles. Prompted by Matthew, many of the tables had sequences going up by 2 in the 'hours' column (2, 4, 6, 8), and students found the corresponding mileage by adding 80 each time.

Marvin offers a different solution:

Matthew: Marvin added numbers in between, he did it one by one. How did you figure that out Marvin? [CGI]

Matthew elicits Marvin's table and copies it on the board. The conversation continues as Matthew helps students build additional understanding by relating the changes in the table to the relationship between the number of hours, the rate, and the distance travelled.

Matthew: OK, so, he has an even more complete table for numbers. Is there another way to do this? [Positive affirmation/multiple solutions]

George: Multiplying, Mr. [4]

Matthew: Multiplying, so far we have, add eighty, this time, and forty for each hour, and here's the same for two hours... how do you multiply, George? What did you mean? [Multiple solutions; CGI]

The conversation continues as Matthew elicits that multiplying 40 by the number of hours will produce the number of miles. Matthew then changes the numbers to make the computation more difficult, but with the intent that the solution strategy (i.e., multiplying) is the same.

Matthew: How about that instead of forty miles per hour I say he was doing thirty seven point five... miles per hour? Would that make this problem any more difficult?

George: Yes.

Matthew: Why? Because you add, what? George, why is the… problem harder if you did thirty seven point five? [CGI]

George shrugs his shoulders in a 'I don't know' fashion.

Matthew: Maybe you don't think it does. How would you do it if it were thirty-seven point five miles per hour, instead of forty? So, just because the number of miles per hour changes, you have no idea? I think you do George. [Wrong terminology/answer; positive affirmation]

Matthew: Think about it. How far would you go in one hour?... What is your speed? George, pay attention. I changed the problem, instead of forty, we have thirty seven point five, thirty seven point five per hour. If it was ten, in an hour, what's the distance? [Wrong terminology/ answer]

At this point, Matthew begins to scaffold [3] with George asking him how far you would travel in one hour.

George: Thirty seven point five.
Matthew: OK, and in two hours?
George: Double.
Matthew: And in four?
George: Times four.
Matthew: And in six?
George: Times six.
Matthew: So, do we change the mechanics of solving the problem if the number changes?
George: No.

The excerpt begins with Matthew eliciting student thinking by having students describe how they solved the problem – a CGI strategy. From here, Matthew can now ask for different solution strategies – using a 'multiple solutions' approach. George then gave a response that Matthew did not agree with (that 37.5 miles per hour is harder than 40 miles per hour). When asked why this is the case, George gestured that he did not know. Instead of accepting this, Matthew challenged him to "pay attention" and refuted the implications of the gesture by responding, "I think you do know", offering him some 'positive affirmation'. To make this point further, he began a dialogue that scaffolded George's responses to the correct one that Matthew was looking for by changing the context to one with easier numbers (10 miles per hour), indicative of a 'wrong terminology/ answer' strategy. The excerpt illustrates all four strategies, indicating how those strategies are used by Matthew to meet his goal of increasing students' understanding of mathematics.

As in the previous episode, the following excerpts also show Matthew's use of multiple strategies. In these exchanges, we highlight the interactions with particular students and map these interactions to the other goals that Matthew holds for students. The following excerpts are taken from a whole-class discussion on 3/17/98. The purpose of the lesson was to understand the rate of change and corresponding equation given a table of values that represented a newspaper-selling job. The commission earned was 20 dollars plus 10 cents for each paper sold. The independent variable (x) represented the number of papers sold and the dependent variable (y) represented the amount of money earned. Matthew's content-related goal was for students to construct an algebraic equation that models the situation.

The following table was on the classroom chalkboard:

X	*Y*
0	20
10	21
20	22
30	23
40	41
50	25

The first segment involves an exchange between Matthew and Isabel. Isabel was a student who, in Matthew's assessment, lacked confidence in her mathematical abilities. She did not appear to lack investment in the learning of mathematics, as she attended class regularly and consistently exerted effort in her studies (two characteristics that were not universal in Matthew's low-achieving class). However, she appeared to be hesitant to offer solutions publicly, for fear of being incorrect.

After time spent working in groups on finding the rate of change and the algebraic equation to model the situation, Matthew begins a whole class discussion.

Matthew: Let's start with the rate of change. Isabel, can you tell me what you got for that? How did you do it? [CGI]
Isabel: It was twenty-two minus twenty-five, over twenty minus fifty.
Matthew: Can you tell me where did you get those numbers from? [CGI]

Isabel explains that she chose two ordered pairs from the table.

Matthew: So, these are the two points that she chose to get the rate of change. Could she have chosen any other two points? [Multiple solutions]

A male student replies in the affirmative. Matthew asks Isabel to do the subtraction. She responds with three over thirty (technically, the response should have been negative three over negative thirty).

Matthew: If I subtract twenty minus fifty, I should get negative thirty, so, how do you get three over thirty? [Wrong terminology/answer]
Isabel: Because I just subtracted the twenty minus fifty.

Isabel is subtracting in absolute terms, which is technically incorrect.

Matthew: OK, but if the answers were actually negative how did you end up getting positive 3 over 30? [Wrong terminology/answer]
Isabel: Oh…
Matthew: Well, what happens with these two negatives? If you divide a negative number by a negative number what do you get? [Wrong terminology/answer]

The students begin to respond, "positive", when Isabel calls the teacher.

Isabel: I also divided it.
Matthew: So, you simplified it and what did you get?
Isabel: 1 over 10.
Matthew: 1 over 10. Does everybody agree with that? [Positive affirmation]
Students: Yes.
Matthew: OK. Nice job. [Positive affirmation].

The classroom episode illustrates Matthew's attempts to build understanding and confidence through a whole-class discussion. He begins with a CGI strategy, eliciting Isabel's solution process with the questions, "How did you do it?" and "Where did you get those numbers from?" He used a 'multiple solutions' strategy to allow for the possibility that an alternative strategy (picking other points from the table) is acceptable. He was initially unsuccessful in getting Isabel to recognize that her subtraction, according to her explanation, would produce negative values.

Notably, he does not dwell on the incorrect response but searches for a way to scaffold the response he was looking for by drawing the class's attention to the result of dividing a negative by a negative. This scaffolding is indicative of a 'wrong terminology/answer' strategy. We argue that this strategy is of particular importance in interactions with students who lack confidence (a point we will return to later in this article). Lastly, he expresses 'positive affirmation' of Isabel's thinking in two ways. First, Matthew revoices (O'Connor and Michaels, 1993) her response to the class ("Does everybody agree with that?") and thereby positions her contribution as valid. He then closes the exchange by affirming her efforts with a compliment ("Nice job"). Again, this kind of positive affirmation is likely to be of particular importance for students with low confidence in their mathematical abilities.

Ronnie

The second classroom excerpt involves a student named Ronnie. It is likely that many teachers would describe Ronnie as disruptive. He was frequently off-task in class and often spoke in a tone that was much louder than other students and certainly louder than was necessary for others to hear. The most disruptive behavior observed occurred on 4/15/98 when Matthew dismissed him from class for expressing a physically threatening behavior to a female student. Matthew believed that Ronnie's behavior indicated that he lacked investment in learning mathematics.

The commission problem continued. Matthew attempts to elicit a linear equation to model the situation. One female student, Selena, offered a correct equation which Matthew wrote on the board: $y = 20 + (1/10)x$. As Matthew verbally confirms this solution, Ronnie interrupts:

Ronnie: Erase it. [*referring to Selena's solution*]
Matthew: Erase it?
Ronnie: Erase it.
Matthew: What do you want me to erase it for? [Positive affirmation]
Ronnie: Just erase it.
Matthew: Can't I just go like this? [Positive affirmation; *wants to write Ronnie's solution on the side of Selena's solution*]
Ronnie: Just watch. OK, put point ten.
Matthew: Where are you getting that from? [CGI]

Ronnie: [*ignores question*] And then put one under it. Can it be like that?
Matthew: Can it? [*to class*; Positive affirmation]
Matthew: Now, if you're sitting there with one over ten, what can you do arithmetically to get point ten over one? [Multiple solutions]
Selena: Divide.
Matthew: You can...
Selena: Divide... divide...
Ronnie: Break it down.
Matthew: You can divide, you can break it down, however you want to call it, or you can just divide it. You can take one and divide it by ten, you will get point ten, so let's put money, newspapers... so, what do you want the sentence to say then? For every one, or for every ten, or for every...? [Wrong terminology/answer; multiple solutions]
Selena: For every ten newspapers you should make a dollar.
Matthew: You like that one better?
Selena: Yeah.
Ronnie: But the last one's more accurate, anyway, 'cause it's exactly, for every newspaper ten cents.
Matthew: Ah, more accurate. [Wrong terminology/answer]
Ronnie: …work with me, dude.
Matthew: I think you have a point, Ronnie. I think you have a point. You might be interested in knowing how much your salary goes up for each newspaper you deliver, however, there's another point for this one, is that you might also be interested in knowing how many more papers you have to sell for each dollar, 'cause a lot of times, when we think about money we think of per dollar, right? So, both of them were pretty good. Yours is good and the one that... is also good …Very nice. [Positive affirmation; multiple solutions]

In this excerpt, Matthew allows for 'multiple solutions' and presses Ronnie to justify his solution as indicative of a CGI strategy. However, 'positive affirmation' was also central to this exchange. Matthew expressed 'positive affirmation' in multiple ways. First, acknowledging the interruption and going along with the "Erase it" sequence, Matthew allows for the possibility that Ronnie's seemingly inappropriate demands will lead to a valid mathematical contribution.

In contrast, given Ronnie's past disruptive behavior, a more controlling teacher may not have allowed the interruption, may have immediately refused the "Erase it" request, or may have demanded that his direct question of "Where are you getting that from?" be answered. That Matthew allowed Ronnie to pursue his line of thought, even in a potentially inappropriate manner, gives evidence of Matthew's efforts to treat Ronnie as a scholar, to *assume* that the student will give him something worthwhile that can be used to further the mathematical discourse.

Another way to acknowledge the 'positive affirmation' is evident when the teacher shifts the dialogue between Ronnie and himself to Ronnie and the class by revoicing Ronnie's question ("Can it be like that?") to the class. Similarly to the second classroom excerpt, this shift in dialogue now publicly positions Ronnie's contribution, and in essence Ronnie, as a worthy contributor to the production of knowledge and as a scholar of mathematics.

In a third instance, Matthew pursues Ronnie's response that one solution is more accurate to extract some valid mathematical thought, indicative of a 'wrong terminology/answer' strategy. Instead of assessing whether or not "more accurate" was correct or incorrect, he goes along with Ronnie's request to "work with me dude". In the end, Matthew validates both solutions and benefits of each. Hence, his consistent use of 'multiple solutions' allowed him to expressive 'positive affirmation' for student thinking.

These two brief vignettes are intended to illustrate how Matthew uses the various strategies to meet the goals that he has set out for his interactions with his students. In each of the vignettes, the four strategies were used to address his goals of understanding mathematics, building self-confidence, and self-image as a scholar. In the next section, we examine how these strategies can operate on multiple levels simultaneously.

Cognitive and affective dimensions of Matthew's strategies

Potari and Jaworski (2002) have argued that teachers engage in different domains of activity as they teach. One of these domains includes what the authors refer to as *sensitivity to students*. They define this domain as

> knowledge of students and attention to their needs; the ways in which the teacher interacts with individuals and guides group interactions. (p. 353)

According to Potari and Jaworski, this sensitivity to students can also operate in different domains. In particular, teachers can exhibit both cognitive sensitivity in the form of questioning and seeking clearer conceptual explanations, as well as affective sensitivity through praise and encouragement of students.

As indicated earlier, Matthew recognized both the cognitive and affective dimensions of instruction as important and described them both as motivations for engaging students in classroom discussion. We focus here on the distinction between cognitive and affective in order to examine not only his larger purposes in eliciting student thinking, but also to consider how these two domains might be reflected in his actual strategies.

At first glance, the categorization of strategies with respect to these domains might appear straightforward. It would be easy to see the CGI, 'multiple solution', and 'wrong terminology/answer' strategies as indicative of cognitive sensitivity. Certainly, these strategies are consistent with the tenets of teaching and learning mathematics with understanding.

In contrast, the 'positive affirmation' strategy would appear to reflect more directly Matthew's attention to the affective domain. However, we argue that the separation into cognitive and affective is not as simple as it may appear at first glance. We will attempt to illustrate the dualistic nature of Matthew's strategies by examining his approach to wrong answers.

Alrø and Skovsmose (1996) have argued that the approach taken by the teacher to student incorrect answers is important insofar as it reflects the teacher's philosophy of mathematics and shapes students' understanding of what mathematics is all about. In this particular case, Matthew's description of his approach to students' incorrect answers reflects a philosophy of mistakes as sites for learning and indicates both cognitive and affective purposes. He describes not only accepting students' wrong answers without judgment, but also taking those answers and building upon them in his instruction. In the following quotation he indicates the circumstances under which he will take a wrong answer offered by a student and work with it in the classroom discussion, using it as an object of reflection:

> If I find the strategy, if I think that it's fairly common, and I think a lot of other students might be seeing it that same way, if it's faulty, I like to take that and bring it to everybody's attention, because I know I can catch a lot of errors that way. If it's something that's really strange, and I know that it's going to be very difficult to understand and probably the only person who might have done it that way, but it's excellent, I mean it shows like tons of insight, that's another time I'll take it and do it [build on the wrong answer].
>
> Other than that, you know if I think the student needs a self-confidence booster or needs some kind of validation, that's another time I'll use it [this strategy]. I enjoy doing that rather than me just telling everything. I think it's really helpful to take answers that aren't quite right and make them right, because then you can bring it back to the class and ask somebody else to make it right. You can say "Hey, this is great, boy, but I think there's one thing missing. What's actually missing here?" something like that. (interview, 9/24/98)

Matthew uses students' wrong answers as opportunities both to build toward greater mathematical understanding and to address students' affective needs by increasing their self-confidence (such as in the example with Isabel).

Students' statements regarding their experiences in Matthew's class illustrate the significance of this approach, particularly for students who have experienced low achievement in the past. For example, Ana was a student who had not had positive experiences in mathematics. In particular, she attributed her failure in *Algebra I* the previous year to her lack of comfort with the teacher. She was afraid to raise her hand to either ask a question or give an answer:

> You know, you want to say something and you are like afraid to say it because probably… I say… the wrong answer. (interview, 4/25/98)

However, she contrasted this lack of comfort the year before with her experiences in Matthew's class, asserting that she was much more willing to ask questions and share answers than she had been previously. In part, she attributed this increased confidence to Matthew's response to student answers. In particular, she noted that even wrong answers were still "counted like good."

Moreover, as Alrø and Skovsmose (1996) suggest, students learn something about a teacher's philosophy of mathematics from his response to student mistakes. Ana reveals the following insight regarding Matthew's approach to student answers:

> Like if I say a question, he will say, well, you are kind of close or you are kind of far, but this is good where you said this and that… So, we *learn from our mistakes*. (interview, 4/25/98, emphasis added).

She does not feel afraid to "say something dumb" in Matthew's class because she knows that he will take what she said and work with it to build to a correct solution.

We chose to focus on the 'wrong terminology/answer' strategy in this section in order to illustrate two points. Firstly, we sought to demonstrate that the same strategy operated for Matthew in two different domains. Building on students' thinking (correct or incorrect) clearly serves a cognitive purpose and is consistent with the tenets of mathematics reform. However, both Matthew's description of his intent and his students' responses to the strategy indicate that the acceptance and use of incorrect answers or strategies can simultaneously serve an affective purpose of increasing student comfort and confidence in sharing their thinking.

The second point that we sought to make with this example is the relevance of this affective sensitivity for students who have not achieved in mathematics in the past. As indicated in Chazan's (2000) description of his low-achieving algebra class and in Ana's description of her own mathematics history, the negative experiences that students have had in the past can impact students' self-confidence and shape their response to instructional practices, such as classroom discussion, that have the potential to make public their misconceptions.

The kind of affective sensitivity demonstrated by Matthew appeared to make a significant difference in Ana's response to instruction. This case suggests, therefore, that the goal of engaging *all* students in practices associated with mathematics reform may take more than cognitive sensitivity to students' prior knowledge of mathematics. It may also require affective sensitivity to students' prior experiences with school mathematics.

Such sensitivity requires knowledge of students as individuals. As evidenced in the examples of Ronnie and Isabel, Matthew appeared to possess this knowledge and to use it in his instruction. He did not take a 'one-size-fits-all' approach to engaging students in discussion. Instead, he based his instructional strategies on what he understood to be the needs of the particular student.

In another incident, for example, Matthew asked a student to come to the chalkboard and work through a problem publicly to discover a mistake. When asked about this interaction, Matthew clarified the individualized nature of this approach:

> Well, it *depends on who the student is*. I mean someone like him, I'd be glad to [have him] come up and

> say, this is what I did, this is what happened. But there are definitely situations where I don't know if it's warranted, [for example] if another student is extremely shy. (interview, 9/24/98, emphasis added)

Throughout our interviews with Matthew, he displayed the knowledge of his students necessary to support this type of individualization. For example, he described one of his students in the following way:

> Sometimes he's really sharp. Other times [he has] very low confidence… He tends to think that he's wrong and he typically has the right answer. He says "Check this out. Check this out. I don't know if it's right". (interview, 12/10/98)

Gutierrez (2002) found similar knowledge and understanding of individual students in a study of high school teachers who were successful in instantiating high expectations for their Latino/a students. Although Gutierrez focused on language use in the learning of mathematics, her findings on the importance of teachers' knowledge appear relevant here:

> When teachers have solid knowledge of their students (respecting their individual identities) and their specific learning needs, they are more likely to develop strategies that will support such students in complex ways that align with local strengths and constraints. (Gutierrez, 2002, p. 1083)

We assert that the example of Matthew's pedagogy offers further evidence of the importance of such knowledge and the strategies that it supports.

Conclusion

This case is significant in several respects. First it contributes another example to the literature of a teacher who held high expectations for his traditionally underserved students and engaged in strategies to instantiate those expectations. [5] It offers an additional existence proof that such high expectations can and should translate into practice for *all* students, including students who have previously been unsuccessful in mathematics.

These expectations contrast with those that we often find expressed for underserved students in similar circumstances (*e.g.*, NCTM, 2000; Oakes, 1995). One example of this contrast can be seen in comparison with the statements made by students in Zevenbergen's (2003) study of low-achieving mathematics classes in Australia. One student in the study described his teachers in the following way:

> You know that they don't like being in our class. They think we are the dummies and treat us like that. (p. 6)

Another student stated,

> The teacher thinks we are really dumb and goes really slow. (p. 7)

These characterizations of low-achieving classes differ markedly from Matthew's stated expectations for his students and his students' statements regarding his interactions with the class. In this way, Matthew's case provides another image of what is possible when high expectations are translated into practice.

Matthew's case also suggests the need for attention to both cognitive and affective sensitivity in the instantiation of high expectations and the involvement of students in mathematical discussions. This is a second important aspect of this case. Matthew's goals for his students and his specific strategies for achieving those goals had both cognitive and affective dimensions. Moreover, the responses of his students suggest that both types of sensitivity were important for their participation in the mathematical discussion and willingness to share their thinking.

In a previous issue of this journal, Civil and Planas (2004) pointed to the impact of student self-concept on participation in classroom discussion, again giving evidence to the significance of both cognitive and affective issues in the construction of participatory structures. Matthew's case extends this conversation by illustrating particular strategies used by one teacher to promote student participation in classroom conversation. Given his students' past failure in mathematics, the engagement of his students in sharing their thinking is significant, as it points to the potential for teachers to overcome some of the inequities in participation described by Civil and Planas.

This case suggests that through the teacher's attention to both cognitive and affective factors, even students who had previously been unsuccessful in mathematics can be engaged in classroom discussion.

Finally, we would argue that this case raises important questions regarding the preparation of secondary teachers, particularly those who will work with students such as Matthew's who were both from traditionally underserved populations and had previously experienced little success in mathematics.

If the engagement of students in classroom discussion is facilitated by cognitive and affective sensitivity, what are the implications of this finding for teacher education? How do we prepare pre-service teachers, or engage in professional development with in-service teachers, in ways that promote the cognitive and affective sensitivity demonstrated by Matthew?

Moreover, both this study and previous research by Gutierrez (2002) point to the importance of teacher knowledge of their students for the instantiation of high expectations. How do we develop teachers' capacity to seek out and use such knowledge of their students? A key component of the Equity Principle articulated by the National Council of Teachers of Mathematics (2000) is the pressing need to raise expectations for all students. As we envision a future for mathematics education in which equity becomes a reality, we must consider the instructional practices and forms of teacher knowledge that will be necessary to achieve that vision. We believe that the results of this study contribute to the ongoing conversation of what that vision might look like in the classroom.

Notes

[1] 4/7/1998 is the 7th April, 1998.

[2] This research was supported in part by a grant from the US Department of Education, Office of Educational Research and Improvement to the National Center for Improving Student Learning and Achievement in Mathematics and Science (R305A60007). The opinions expressed do not necessarily reflect the position, policy, or endorsement of the supporting agencies.

[3] We use the term "scaffold" to describe a type of exchange between teacher and student(s). In this case, such exchanges were typically initiated when a student did not offer a correct or complete answer. Matthew would often ask the student different (often simpler) questions or direct the student's attention to a specific aspect of the problem solution. This redirection or additional questioning had the effect of providing the support ("scaffolding") necessary for the student to reach a correct solution.

[4] It is quite common for Spanish speaking students to address their teacher as Senhor or Senhora in Spanish. When speaking English they can simply address their teacher as Miss or Mister, which is abbreviated as Mr. On the other hand, most English speaking students in the USA would address their teachers as Mister (Mr.) Cabral, with a surname. So, in this case, the student is simply responding directly to his teacher.

[5] Other examples include Gutierrez (2002), Gustein e*t al.* (1997) and Tate (1995).

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