Communications

But that's obvious: launching and persistent pitfalls

ROB WIEMAN

There are some ideas in education that are so basic that whenever we read about them they seem obvious on their face—not worthy of extended research or comment. Yet my own difficulties with teaching, learning and research come from consistently forgetting them. My experience researching 'launches' indicates I am not alone, and that what we take for granted, we easily forget when immersed in the complexity of practice.

The way mathematics teachers set up or launch demanding tasks has a powerful effect on subsequent opportunities to learn (Stein & Lane, 1996). But what is a 'good' launch? Lowering a task's cognitive demand during the launch reduces opportunities for student reasoning and sense-making during final discussions, but effective launches also include support for students working to make sense of the context and identify important mathematical relationships (Jackson *et al.,* 2013). Experienced teachers report they struggle to balance launches that provided 'just enough help' but 'not too much' (Gonzalez & Eli, 2017).

There seems to be consensus that teachers, as Sam Otten said [1], should launch the problem, not the solution. However, mathematics teacher educators and researchers have very different ideas about what kind of support, exactly, students need in order to gain the most from a demanding task, about what kind of help is 'just right'. Three examples illustrate some of the issues.

Example one: how long?

I showed a colleague a video of a teacher facilitating a rich discussion as students struggled to make sense of a problem. I considered this a model launch, but my colleague thought the video was an example of what NOT to do—it took way too long, and was not directly connected to the mathematics of the lesson. The disagreement concerned the following problem (Cameron, Dolk, Fosnot, & Hersch, 2006):

> Carol's class split into groups for a trip. Each group had some sandwiches (subs) to share.
>
> Museum of Natural History: 4 children, 3 subs
> Museum of Modern Art: 5 children, 4 subs
> Statue of Liberty: 8 children, 7 subs
> Planetarium: 5 children, 3 subs
>
> Three questions:
>
> Was this fair?
> How much did each person in each group eat?
> Which group ate the most?

Carol began the launch by showing the number of students and number of sandwiches at each location, and asked if this was fair. Almost all students first argued that it *was* fair; students would split the sandwiches equally within each group. One student argued that it was *not* fair; two different groups had five children, but one had three subs, while the other had four. The teacher then facilitated a 40-minute conversation in which students discussed whether each student got the same amount, and why.

I thought this was a wonderful launch. In order for students to focus on the relationship between the number of people and sandwiches in a group, they would have to begin with the idea that students from different groups received different numbers of sandwiches. Through the teacher's skillful facilitation, students came to the shared conclusion that students from different groups did not get the same amount, and they then focused on the mathematics of the lesson—how to apportion A sandwiches between B people, and how to compare fractions.

My colleague agreed—the teacher supported students in reasoning about a key feature of the problem. However, for her, this conversation represented ineffective planning, not effective launching. If understanding the problem required a 40-minute conversation before students could even get started, then students had less opportunity to reason about fractions and division—the point of the lesson.

Example two: discuss strategies?

Later I found myself disagreeing with a different colleague about a different video. This disagreement concerned a launch of the following problem [2]:

> At Bob's store 12 cans of kitten food cost $15.
>
> At Maria's store 20 cans of the same food cost $23.
>
> Which is the better deal?

In the launch the teacher mentioned two stores in his neighborhood that sold cat food, and wrote the stores, cans and prices on the blackboard. He then asked students how they might get started on the problem. Students suggested figuring out how much one can costs, creating equivalent fractions, or breaking the numbers of cans into bunches. The teacher then asked students to raise their hand if they had a way to get started on the problem.

Again, I thought this was a wonderful launch. The teacher had surfaced ideas around chunking, unit rate and equivalent fractions that would provide access to students without solving the problem for them. My colleague was shocked that the teacher would ask students to begin thinking about the solution of a problem during the launch, solving was definitely not part of launching.

Example three: where's the context?

The final case occurred at a conference (Wieman & Jansen, 2016). My colleague and I presented our findings about preservice teachers' understandings of launching to a packed conference room. However, instead of asking us about our findings, participants vigorously disagreed, with us and each other, about the nature of effective launches.

Our research involved a depiction of a hypothetical launch, including initial student reactions to the problem. For each student reaction we had asked our pre-service teachers to choose among different interpretations of students' thinking, including:

> This student is thinking about the mathematics in a way that will lead to a correct solution.
>
> This student has a misconception that will get in the way of them creating a correct solution.
>
> This student is confused/distracted by the context.

We then asked the pre-service teachers which of the following moves they would make in response to each student reaction:

> Facilitate a discussion during the launch in which students respond to this student's idea.
>
> Briefly explain or clarify to the whole class during the launch.
>
> Explain or clarify to individual student during the launch.
>
> Do not mention during the launch

Before we could share our data, participants engaged in a vigorous discussion about the depiction and the options themselves. Participants argued forcefully for and against several of the possible teacher moves. For instance, some stated that responding to a student's idea about the problem at length during the launch was not appropriate. Others argued that addressing misconceptions during the launch foreclosed opportunities for reasoning during the explore. Others argued that the whole depiction missed the point of a launch—students and teachers should be talking extensively about the context.

Why the disagreement?

Certainly, launching is complex. It is not surprising that groups of experts would disagree about them. However, these disagreements stem, at least in part, from three 'obvious' educational ideas that, in the heat of the conversational moment, we forgot:

> The connection between teaching and learning goals
>
> The connection between evaluation and data
>
> The need for a shared technical language of teaching

Teaching and learning goals

There is no such thing as an absolute 'best practice' in education. The same practice could have different effects with different groups of students at different times of the year, and different practices could prove effective for different goals. What is effective depends on "what learning objective, what group of students and what point in the year" (Ermeling, Hiebert & Gallimore, 2015, p. 50) or the core educational problem you are trying to solve. Perhaps differing ideas about best launching practices stem from different goals during a launch, goals that themselves sprang from different experiences.

Take, for instance, the case of the sandwiches problem. In my colleague's work with teachers, she had observed that many of them prolonged the launch and left little time for the independent problem solving that was the point of the lesson. For her, a core challenge with launches was how to limit their length so students would have enough time to grapple with the problem. When I had taught, however, my students often spent less time reasoning about the important mathematics during the 'explore' section because they did not know what they were trying to figure out. As a result, during launches I was focused on students understanding the problem and the question to be answered.

For my colleague observing the cat-food problem, discussing ways to solve the problem during the launch had often led to simply showing students a solution. The core problem I wanted to solve was to provide enough access to the problem and the related mathematics so that they had something to think and reason about during the explore.

The fiercest critics of the depiction of the sand-box problem were elementary teacher educators, especially those concerned with equity and access. In their experience, rich problems required that teachers work hard to support a common understanding of the context, especially when specific contexts might be unfamiliar to all, or some, of the students. For them, the absence of specific moves addressing these difficulties had often resulted in students being denied access to the mathematics in the problem.

In each of these three cases, what we deemed effective was connected to an instructional goal, derived from our experiences. In retrospect, given the diversity and complexity of classrooms and classroom teaching, it is not surprising that we viewed the same launch differently.

Evaluation and data

A common difficulty for novice observers of teaching is that they tend to concentrate on the actions of the teacher, rather than its effects on students. They evaluate teaching effectiveness on its effect on *them* rather than its effect on the *students*. They have not yet learned that the effectiveness of any teaching move or practice must be judged by its effect on students. Many of the arguments above, about the different launches, were not supported by specific evidence of student learning or behavior. Certainly, my colleagues were not given student data that they could use to decide if the launches were effective or not, but we still were quick to make pronouncements about what constitutes an effective launch absent this data.

Lack of a shared technical language

In 2010, Deborah Ball pointed out that teaching lacked a precise technical language, echoing what Dan Lortie had written 35 years earlier (Ball, 2010; Lortie, 1975). Without a technical language it is difficult to describe learning goals for teacher training, and performance expectations for teaching professionals (Ball, 2010). This claim seems counterintuitive: educational journals and schools teem with jargon—differentiation, professional learning communities, culturally responsive pedagogy, tiered instruction and on and on. This jargon, however, is often ill-defined, and its use masks considerable differences in underlying assumptions and visions. Although we all were using similar terms when describing launches (*e.g.,* we all agreed that we should not reduce the cognitive demand, we should support students in making sense of the problem) we had vastly different pictures of what those words meant in practice. Furthermore, we did not have a technical language that described typical student experiences in launches, common pedagogical challenges, or specific moves that teachers might make.

Conclusion

In retrospect, given the diversity and complexity of classrooms and classroom teaching, it is not surprising that we viewed the same launch differently. Divergent views on launches suggest the need for a larger framework describing:

- the most common obstacles students face when initially confronted with a particular demanding task
- factors that bring these obstacles to the fore
- moves that support students as they overcome those obstacles

Clearly, such a framework could not be perfect. It would not be able to account for all the complexity involved in launching. However, teachers know that all of their work is contingent; it is the best they can do at any given moment with the tools that they have. Professional judgment cannot be replaced by a formula, recipe, or someone's abstract definition of best practice. However, this framework might introduce a technical language that will enable teachers to plan, revise and improve their practice together. And it makes it less likely for us, once again, to fall prey to the obvious, hiding in plain sight.

Notes

[1] When interviewing Gloriana González and Jennifer Eli in Math Ed Podcast Episode 1512. Online at http://www.podomatic.com/podcasts/mathed/episodes/2015-06-24T07_53_50-07_00

[2] In the video "Joel Spengler introduces the context", Best Buys, Ratios and Rates: Developing the Context (New Perspectives Online) at https://www.newperspectivesonline.net.

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What is worth publishing? A response to Niss

ARTHUR BAKKER

Let me first thank Mogens Niss for initiating an important discussion about the nature of mathematics education research. It is important to reflect regularly on our own discipline, including its publishing practices, in particular when these practices may prevent valuable work from getting published in the main journals in our field. In this response, I first offer some general reflections before I respond as editor-in-chief of *Educational Studies in Mathematics* (ESM)—one of the journals mentioned explicitly by Niss.

Reflections on the concerns

The essence of Niss' (2018, 2019) concern, as I interpret it, is that form may have become more important than content. Articles with a particular structure—that of the classical empirical study—may indeed have become easier to publish than nonstandard ones which still communicate worthwhile content, such as conceptual, theoretical, or position papers. This form versus content issue reminds me of a controversy between Hilbert and Frege on symbolization. Frege acknowledged the importance of symbolization, emphasized by Hilbert, but also warned that progress could be stopped or delayed if formalism became too important. In a letter to Hilbert, Frege wrote:

> Ich möchte dieses [Symbolisieren] mit dem Verholzungsvorgange vergleichen. Wo der Baum lebt und wächst, muss er weich und saftig sein. Wenn aber das Saftige nicht mit der Zeit verholzte, könnte keine bedeutende Höhe erreicht werden. Wenn dagegen alles Grüne verholzt ist, hört das Wachstum auf.
>
> I would like to compare this [process of symbolizing] with lignification [transformation into wood]. Where the tree lives and grows, it must be soft and sappy. If, however, the sappiness does not lignify, the tree cannot grow higher. If, on the contrary, all the green of the tree transforms into wood, the growing stops. (Frege, 1895/1976, p. 59; my translation)

Transposing this metaphor to research, I interpret new and fruitful ideas to be the green living power, which needs some form to grow. With Frege, one may conclude that lignification is required in the development of any discipline, but there is also a risk that progress is hindered by form conventions. This is a genuine concern: In a recent analysis of the history of psychology since 1950, Flis (2018) came to the dramatic conclusion that there has been little progress in this discipline. One of the key problems in his view is that psychology research has been dominated by an emphasis on methods, in particular experimental designs. As a consequence, Flis argued, theoretical development has thus been disappointing in psychology. In Frege's metaphor this would be the consequence of an overemphasis on wood rather than the green of the tree. Of course, we should not let this happen in mathematics education research.

We shall have to resist two tendencies to which psychology, like other social sciences, has fallen prey. The first tendency is to try to be like a natural science. The second is to consider one type of research the gold standard and report it in standardized forms. The immense success of the natural sciences over the past centuries, its experimental methods, have become the gold standard of research, also for the social sciences. However, Smedslund (2009) points to the mismatch between experimental methods and the nature of psychological phenomena, and his point can be extended to many social and educational phenomena. Experimental methods tend to ignore fundamentally human characteristics such as intentionality, personal uniqueness, and locally shared meaning systems. As Flyvbjerg (2001) proposed, the social sciences should stop emulating the natural sciences, and rethink the kind of knowledge they intend to produce given that human beings are of a different nature than, say, electrons. In my view, the social sciences, including mathematics education, should have the courage to consider themselves human sciences with an eye for normativity, history, contingency, agency, and self-reflexivity (*cf.* Akkerman, Bakker & Penuel, in preparation). They should take reliability as subordinate to validity (Thomas, 2013) and privilege generativity and theoretical generalization over statistical generalization. This is hard, given the competition for resources with the natural and medical sciences, yet crucial to do justice to the nature of what we study: Human beings learning or teaching mathematics as a human activity (Freudenthal, 1973).

The second problematic tendency in psychology and educational research in the USA (*e.g.,* What Works Clearinghouse) is to consider experiments the gold standard and report them in standardized forms. The enormous increase of publications in the social sciences has created a need for easy and quick reading, hence standardization of where in a journal article particular information can be found (*e.g.,* APA, 2010). Frege's tree metaphor points to both the power and risk of this development: On the one hand, standardization helps authors and readers write and read a particular genre of research articles. Information can be easily found in predictable places in these articles. Quality is easier to assess if clear criteria are widely shared. On the other hand, standardization and so-called 'rigor' (Cartwright, 2019) may prevent new and interesting ideas—the generative life force of any discipline—from being published in our journals. I empathize here with Niss' (2018, 2019) concern.

Educational Studies in Mathematics

Right after Niss' keynote at PME-42, his main concerns were discussed among editors of ESM and during PME-43 these concerns returned to the table during a meeting of editorial board members and editors. We have talked about how to ensure that a rich variety of articles find their place in our journal that intends to represent the multifaceted nature of mathematics education research. It is true that a large percentage of ESM articles are empirical ones but there is certainly place for theoretical ones (*e.g.,* Niss & Højgaard, 2019; Pais, 2019; Scheiner & Pinto, 2019). There are examples of where the generativity or importance of ideas has been acknowledged by reviewers and editors. For instance, Konold *et al.* (2015) presented a useful framework on how students interpret data through different lenses—methodologically a nonstandard article but greatly appreciated in the community.

In line with Niss' analysis, we welcome a variety of submissions that do justice to the multi-faceted nature of mathematics education research, including its normative discussions. However, a journal is also dependent on submissions. The number of high-quality submissions of the types that Niss (2019) asks for is actually rather low. At the most recent ESM meeting two possible explanations were mentioned. The first is that it is actually hard to write and to recruit good non-empirical articles with important messages. This is a view I have heard also from editors outside mathematics education. Another explanation mentioned during the ESM meeting is that authors may hold limited views of what journals tend to publish. To remedy such self-imposed restraint, I like to emphasize that ESM welcomes any kind of submission that vitalizes mathematics education research, whether theoretical or empirical, qualitative or quantitative, standard or nonstandard. If authors are considering sending a manuscript that might not fit the typical format, they can write to the editor-in-chief for discussion. As editors we are open to continue the discussion of what is worth publishing in journals to ensure our discipline stays green and alive.

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Constructing and employing theoretical frameworks in (mathematics) education research

JINFA CAI, STEPHEN HWANG

Through his examination of the development of mathematics education research in issue **39**(2), Mogens Niss (2019) reminds us that as mathematics education continues to mature, it is perpetually necessary and instructive to take a step back and look at the big picture. By taking a broader view of our field, we can envision ways to improve our work as we move forward, not only in terms of the generation of new knowledge about the teaching and learning of mathematics but also in terms of doing research in mathematics education that has an impact on practice.

In his analysis, Niss brings particular attention to the state of theoretical frameworks in mathematics education research. He finds that the basis for such frameworks in our field is not sufficiently fully formed to support strict demands that researchers adopt them, and be bound by them. Mathematics education research has clearly evolved over time, whether in terms of the kinds of research questions that we ask, the methods that we employ to answer those questions, and the theoretical frameworks with which we work. Thus, we agree that in educational research, 'theoretical framework' is an evolving term and that the field is faced with a perpetual challenge to construct, reconstruct, and strengthen theories of teaching and learning. As Herbst and Chazan (2017) observed in their survey of theory in research on mathematics teaching, "our approach to providing an account of how theory has participated in our field's research on teaching cannot be one of contemplation of a stable object" (p. 103). Moreover, there has been an ongoing conversation in the wider educational research community as well as the mathematics education community about the role of theoretical frameworks in designing, conducting, and disseminating research (Grant & Osanloo, 2014; Leatham, 2019; Skott, Van Zoest & Gellert, 2013; Spangler & Williams, 2019).

In 2019, across a series of four editorials in the *Journal for Research in Mathematics Education* (JRME), we have argued that justifying the significance of a study requires developing a coherent chain of reasoning connecting the theoretical framework, the research questions, the research methods chosen to address the research questions, and the interpretation and discussion of the results. We therefore see the theoretical framework as a purposefully constructed structure that is essential in both the conduct and reporting of research in mathematics education. The intent of this communication is to explain how our thinking about theoretical frameworks both complements and, in one important point, conflicts with Niss' account.

A theoretical framework is purposefully constructed

Niss argues that the use of theory in mathematics education research has been quite varied, ranging from "nothing but a limited set of singular notions and terms" (p. 5) to a highly structured, logically coherent and connected set of concepts and claims. Certainly, there has been much diversity in how theory is used and positioned in mathematics education research, a fact that has attracted the attention of a number of scholars (*e.g.,* Herbst & Chazan, 2017; Stinson & Walshaw, 2017). However, Niss further positions the typical theoretical framework as a loose phenomenon—an "outline of a domain of entities, phenomena, or issues supposed to be captured by the framework, as well as […] a set of more or less connected concepts and terms" (p. 5). This sort of assemblage does indeed characterize what is called a theoretical framework in some manuscripts submitted to JRME. Other manuscripts include a more structured and tightly connected theoretical framework, closer to what Niss defines as a theory. Whether loosely or tightly constructed, though, we claim that the theoretical framework must be purpose-built to do essential work for both conducting and reporting a study in mathematics education.

We agree with Niss that the theoretical framework serves a multiplicity of purposes in current mathematics education research and moreover that researchers can and should adapt and integrate ideas and theoretical resources from multiple sources to construct useful frameworks. At the same time, given that the field has evolved to some degree over the past 50 years, it should be expected that the criteria by which we evaluate research (and theoretical frameworks) should also evolve. With respect to the theoretical framework, we stated, in an editorial in JRME issue **50**(3) that "to be useful, the theoretical framework should be constructed by the researcher as a critical part of conceptualizing and carrying out the research" (p. 219). This means that constructing a theoretical framework is a purposeful task for the researcher: "It is not simply found or chosen—ready-made, say, by searching the literature—nor can it be so generic that it provides little guidance for conducting the study or writing a report" (p. 219). Even when a theoretical framework adapts pieces from various sources, a practice that Niss encourages the field to be open to, the researcher must purposefully connect those pieces into a coherent whole that is useful in making and supporting decisions about the conduct and reporting of the study. In particular, as we will argue below, the theoretical framework is constructed for and through the justification of the significance of the research questions, the appropriateness of the chosen research methods, and the contribution of the findings.

A theoretical framework is a justified structure

The topic of theoretical frameworks has engaged the attention of the editorial board of JRME for some time, and we very much appreciate the attention that Niss brought in his article to JRME's stance on theoretical frameworks. Quoting from the guidance to authors provided on the JRME website, Niss highlights the fact that we encourage that studies be "guided by a theoretical framework that influences the study's design; its instrumentation, data collection, and data analysis; and the interpretation of its findings" (National Council of Teachers of Mathematics [NCTM], n.d., "A Coherent Theoretical Framework," para. 1). Moreover, we advise authors that "the literature review connects to and supports the theoretical framework" (para. 2) and that authors should, in reporting their study, "make it clear to the reader how the theoretical framework influenced decisions about the design and conduct of the study" (para. 3). We do not shy away from this guidance. In fact, in the editorial in JRME issue **50**(3) we elaborated our perspective on what theoretical frameworks are and what their appropriate role is in scholarly work in mathematics education. We positioned the theoretical framework as the "connecting thread that ties together all of the parts of a research report into a coherent whole" (p. 218).

Researchers have used various metaphors to describe the nature and function of a theoretical framework. For example, Maxwell (2005) used the image of a 'coat closet' to describe how a theoretical framework provides "places to 'hang' data, showing their relationship to other data" (p. 49). Eisenhart (1991) described the framework as a "skeletal structure of justification" (p. 209). Similarly, Spangler and Williams (2019) highlighted the structural role of theoretical frameworks by drawing an analogy to the role that a house frame provides in preventing the house from collapsing in on itself. Lester (2005) referred to a framework as a 'scaffold', and Grant and Osanloo (2014) have called it a 'blueprint'.

Each of these metaphors draws on the idea that the theoretical framework provides a structure that will support researchers' decision-making about each aspect of research. Like a house frame or a skeleton, its influence pervades the research, connecting the choices made by the researcher into a coherent report. In particular, the theoretical framework connects a set of research questions to the larger field, thus grounding the questions in the existing research base and making the significance of a study clear. The theoretical framework also supports methodological choices and guides interpretation of the findings. We discuss each of these functions of the theoretical framework in more detail in the 2019 JRME editorials, but here we wish to emphasize the structural role of the theoretical framework for justifying the researchers' choices.

A theoretical framework is an evolving structure

There is a key point about theoretical frameworks on which we strongly disagree with Niss (2019). In warning against the dangers of too rigidly and narrowly defining what counts as research, Niss characterizes the theoretical framework—specifically, the theoretical framework as an element of a high-quality ideal-typical research report manuscript submitted to a modern major mathematics education journal—as fixed at the start of a study and incompatible with "studies conducted in order to answer research questions that are not derived from or embedded in such a framework, but for which possible theoretical frameworks are to be chosen *post festum* in response to the research questions posed" (p. 3). This suggests that, to be published in a journal like JRME, studies must have a theoretical framework that is predetermined and all-encompassing with no room to revisit or revise it. As Niss describes it, this would essentially "force mathematics education research to be locked up in extant theoretical frameworks" (p. 6).

We wish to make our position on this point absolutely clear: We view the theoretical framework to be a purposeful construction that both informs *and is informed by* the study, not a constraint that binds the researcher a priori.

This view is a consequence of our characterization of the theoretical framework as being formed *through* justification (and not only *for* justification). That is, we view the theoretical framework as arising through researchers asking themselves a series of questions about what they are studying, why it is important to study, what they expect to find, and why they expect to find that. The process of asking and answering these questions provides the foundation of the theoretical framework—a set of educated hypotheses that connects what is new in the study to what is already known. Because we characterize theoretical frameworks in this way, we also expect that researchers will need to modify or extend their theoretical frameworks as they conduct the study and analyze their data. Indeed, we see the interpretation of the findings as a process, as we put it in the editorial in JRME issue **50**(3), of "comparing theoretically grounded predictions to actual results and then refining or extending the theoretical framework to support revised hypotheses that align with what was actually observed" (p. 222). Generating a revised framework, enriched by new findings and more educated hypotheses, is not something to be avoided if one is to conduct a study that will be reported in a mathematics education research journal. It is, in fact, an important contribution of the study to the field.

At a practical level, it would be extremely constraining to treat purposefully constructed, clearly articulated theoretical frameworks as immutable features of research in mathematics education. There are simply too many ways that unexpected findings can arise in the course of a research study. Our recognition of the importance of theoretical frameworks is not meant to calcify research, nor are we encouraging researchers to self-impose theoretical blinders when they are working in areas with scant theoretical support. As we said in an editorial in JRME issue **50**(5), we believe that explicitly constructing theoretical frameworks through and for justification "encourages researchers to be explicit and precise about how much is known in the field; it does not preclude researchers from keeping an open mind to observe the full range of outcomes" (p. 475). In fact, unexpected findings are sometimes the most interesting findings of a study, and we see them as a signal to pay more, not less, attention to the theoretical framework. This can mean deciding that the theoretical framework is still compelling and conjecturing why it was inadequate in a particular case, or deciding that the theoretical framework is flawed and revising it. In either case, the theoretical framework is not treated as an a priori constraint but rather as an evolving structure.

Conclusion

We are grateful for Niss' thoughtful reminder to consider the big picture of mathematics education research and its theoretical basis as the field continues to mature. Certainly, the theoretical basis for the domain of mathematics education is not yet the robust or universal foundation that researchers might aspire to. However, this situation does not detract from the importance of constructing theoretical frameworks and clearly communicating them in reports of research. As a structure that is purposefully built by researchers to support a study, the theoretical framework serves to justify every aspect of decision-making in research. Indeed, as the contributions to the *Compendium for Research in Mathematics Education* and other research handbooks have demonstrated, theoretical frameworks are also essential to the task of synthesizing, reorganizing, and providing insights into large bodies of research.

In summary, the theoretical framework links research questions to existing knowledge, thus helping to establish their significance; provides guidance and justification for methodological choices; and provides support for the coherence that is needed between research questions, methods, results, and interpretations of findings. At the same time, theoretical frameworks are not static. They exist in a reciprocal relationship with the work of conducting and communicating research and thus evolve with the research and its findings.

Having said that, the field of educational research in general and mathematics education in particular lacks specific guidelines for how to construct a sound theoretical framework. For example, a theoretical framework can be constructed at different grain sizes, but it is not always clear how to determine what grain size is appropriate for the study at hand. Although we have proposed some ways to think about constructing theoretical frameworks in our editorials, it seems that the construction and tailoring of a theoretical framework for a given study will always require careful and intentional thought on the part of the researcher. As the field of mathematics education continues to mature in its design, conduct, and communication of research, so does its use and construction of theoretical frameworks to support those activities. We look forward to the innovations that are sure to come in future decades as mathematics education researchers explore and expand the boundaries of theory, methodology, and practice.

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