The Interpretation of Graphs Representing Situations

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This work arose out of a concern for those aspects of mathematical competence which are of general value and usefulness to the majority of people We were aware of the widespread use of graphical representations of situations in the sciences, and indeed, in everyday economic affairs; some of these uses in other school subjects appear in Ling [1977]. We felt that the treatment of this topic in the mathematics curriculum was generally underdeveloped, and related too much to specialised mathematical techniques, such as the solution of equations by reading off points of intersection of two graphs. This is particularly true of the secondary school curriculum British primary schools often do exploit some of the practical uses of graphs. Indeed, one of the first guides produced by the Nuffield Mathematics Project in Britain was entitled Pictorial Representation It described work in which data from various everyday situations — such as the takings at a cinema box office over the course of a week — were represented by block or line graphs, and the story told by the graph was then discussed and written The graph was used to expose features of the situation not immediately obvious from the numerical data. By contrast, the graphical work in currently popular secondary school courses consists mainly of relatively brief treatments of travel graphs, mainly composed of line segments, and, in some courses, more extensive work on the rather specialised use of graphs for optimisation in linear programming situations

It might perhaps be considered that the ability to use graphs as a language for these purposes is sufficiently well developed in primary schools, and that further specific teaching is not required in the secondary school; that is, until, say, the introduction of the technical apparatus of calculus for determining maxima and minima and for sketching curves from equations. The work to be reported in this paper will show that, in fact, most secondary pupils are weak in the ability to interpret global graphical features so as to extract information about many everyday and scientific situations. In undertaking this work, we had the aim of developing some teaching material to fill this gap. The material was used initially in exploratory interviews, to clarify the nature of the difficulties experienced by pupils in this field, as well as to see how readily they were able to learn the necessary skills and concepts A full account of this work appears in the second author's Ph.D. thesis [Janvier, 1978a] Discussion of one aspect of the work — the effect of personal experience of the situation underlying the graph — has appeared elsewhere [Janvier, 1981]. A description and discussion of the *teaching experiment* (see below) has appeared in conference proceedings [Janvier, 1978b]. The present article attempts to give an outline of the work as a whole and its main outcomes.

Graphs in the curriculum

We shall begin by reviewing in a little more detail the incidence of graphical work in current British school courses, and the outcomes of previous general surveys of graphical understanding In the primary school, in spite of the considerable encouragement to emphasise graphical work, which has resulted in increased activity, it appears that block and bar graphs predominate over line graphs in general use, and discussion is usually based on *point readings*, with a little comparison, but rarely treats global features, such as the *general shape of the graph, intervals of rise or fall*, or of *maximum increase*. These "global features" became the focus of interest in our work [Bettis and Brown 1976; Read 1970; Ward 1979.] An example from the APU Primary Survey of 11 year olds [Foxman *et al*, 1980] shows a typical piece of work. (See table on next page.) 90-95% of pupils here could identify the greatest and least heights of the bars, and 45% could compare increases for the same girls between the graphs. Another question from the same section of the survey shows the well known difficulty of interpolating between the numbered grid-lines on a conversion graph connecting old and new prices.

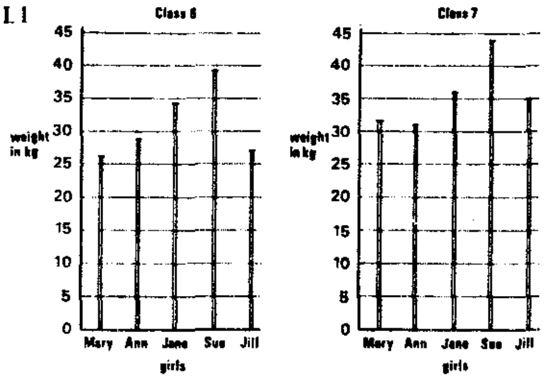
In the extensively used SMP secondary school course (Books A to Z), the use of Cartesian graphs to represent functions is somewhat delayed, on account of the wish to begin the study of functions with relations and arrow diagrams. The graphs of relations such as $y = x + 2$, and of the 'slide-rule' graph $y = 2^x$ come in the second year, well before graphs of practical situations, such as the weight and cost of minced meat, the petrol left in a tank and distance travelled, or the travel graphs of pupils' journeys to school.

The earliest work is on plotting points and on identifying regions such as $x > 2, y \leqslant 3$. In the third year, regions such as $x + 2y \geqslant 6$ are considered in connection with linear programming problems and their solution sets. When formulae such as $A = 3r^2, y = x^2 - 2x$ are graphed, also in the third year, it is in order to solve equations, not to study the form of the function Later, in the fourth or fifth years, more realistic graphs appear, but this is for the purpose of *calculating* gradients and areas under the graph. In the statistics chapters, more real-life data is used, but still the graphs are used to *display* information, not for the elaboration of the properties of the underlying situation. Thus the definition of function as a set of ordered pairs, the technicalities of using graphs to solve equations, and the rather difficult and highly specialised use of graphs for linear programming occupy the bulk of the time, and there is little or no attention to global graph-reading and interpretation

The South Nottinghamshire Project material for ages 11-13 [Bell, Wigley and Rooke, 1978] is one of the few courses presenting a substantially different approach, and

Item	Response analysis		Item facility
L1	a) Incorrect	8%	90%
	Omitted	2%	
	b) Incorrect	3%	95%
	Omitted	2%	
	c) Incorrect	51%	45%
	Omitted	4%	
L2	a) Incorrect	36%	50%
	Omitted	14%	
	b) Incorrect	57%	23%
	Omitted	20%	
	c) Incorrect	64%	18%
	Omitted	18%	

L1



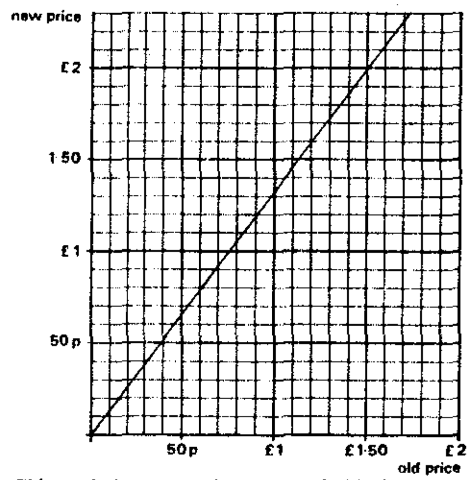
5 girls made a chart to compare their weights when they were in Class 6 and again a year later when they were in Class 7.

a) Who was the *lightest* girl in *Class 7?* . . . .

b) Who was the *heaviest* girl in *Class 6?* . . . .

c) Who *gained* most weight during this year? . . . . . .

L2



This graph shows new prices compared with old prices.

a) What is the new price of something with an old price of £1.50? . . . .

b) What was the old price of something with a new price of £1 20? . . . .

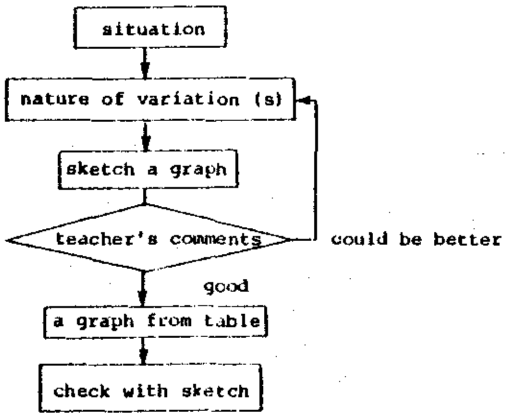
c) What is the old price of something with a new price of £1? . . . .

one more in accord with the need we are arguing. The teachers' notes introduce this topic thus:

> A conventional approach is to construct a table of values and then to plot a graph. . . . . Much less easy is to anticipate the type of variation to be expected. The main objectives of this topic therefore are to identify and describe the way the numbers will vary in different situations and to relate this to a sketch graph . .

Let us emphasise the fact that, here, functions are more than a subset of ordered pairs but "a definite connection between two things which change, also called variables".

After an introduction to prerequisite notions in MF1, MF2, MF3 and MF4 (to be described later), MF5, 6 and 7 bring in worksheets built on the following scheme:



MF6

1 A family goes to the fair with £3 to spend. Dodgem rides cost 10p and rides on the big wheel cost 20p. The family decides to spend all their money on these types of ride. Suppose d is the number of dodgem rides and w is the number of rides on the big wheel which they can take.

Think about how w varies with d.
Describe the variation and sketch a graph.

Check by constructing a table of values and plotting an accurate graph.

MF4A

Here is a list of phrases describing types of variation. You may find it helpful to select the appropriate ones from the list when making your comments on MF 4, MF 5 and MF 6.

1 As x increases,
(a) y increases
(b) y decreases

2 As x goes up by equal amounts,
(a) y goes up by equal amounts,
(b) y goes down by equal amounts,
(c) y goes up by increasing amounts

3 When x is large
(a) y is large
(b) y is small,
(c) y becomes zero

7 y is decreasing with
(a) constant rate of change
(b) increasing rate of change
(c) decreasing rate of change

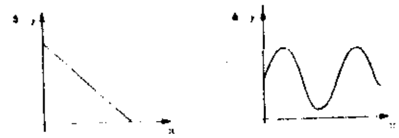
The idea of using this notion of variation together with sketching a graph allows the pupil to go *directly* from a situation to its graph and the feedback from his teacher is the way of ensuring that the translation process (situation↔graph) really takes place. On the other hand, MF7 presents the pupil with a variety of graphs illustrating different types of variation. This exercise, and MF8, help to develop the basic subskills on which the particular translation skills — graph↔situation, graph↔equation — may be built later on.

MF7

Do you notice any connection between the different kinds of variation and the pattern of the graphs?

Write your comments.

Describe briefly the types of variation illustrated by some of these sketch graphs.



MF8

In the first example on MF 2, apples cost 4p each. The two variables were [illegible]

x number of apples bought
y total cost (in pence)

A formula connecting y with x [illegible]
$x \times 4 = y$ [illegible]

Go back to the examples you did on MF 2, MF 3, MF 5 and MF 6.

In each case [illegible] formula [illegible] the two variables [illegible]

Extension

[illegible]

Pupils' conceptions and difficulties

By far the most extensive study of pupils' concepts of graphs, prior to the present work, is that of Kerslake [1977] for the project Concepts in Secondary Mathematics and Science based at Chelsea College, London [Hart, 1980]. This tested a representative sample of some 1400 pupils, of the 2nd, 3rd and 4th years in British secondary schools. It showed that

(1) reading and plotting points was successfully done by 90-95% of pupils

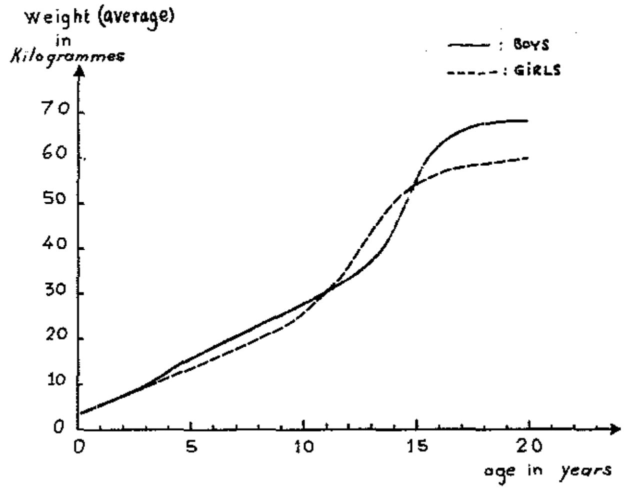
(2) interpolation and the use of decimals reduce facility to 70% and 30-35%, while the infinity of possible points on a line was appreciated by only 10-20%

(3) with regard to gradients, the link gradient←congruent triangles (for the steps) is grasped by 30-35% and the relationship equal gradient↔parallel by only 5-10%

(4) the relation between straight lines and their equations was understood by 5-30% (depending on age)

(5) graphs representing situations are readily misinterpreted whenever pictorial aspects conflict with correct meanings

In the present study, the first task used for the exploration of pupils' graph reading capacities was based on a pair of graphs of height and weight increases of boys and girls from the ages of 0 to 20. The finally developed written version of this task is shown. (See next page.)



1. The average weight of boys at age 9 is ....
2. The average weight of girls at age 17 is ....
3. From what age do the boys on average weigh more than 55 kilogrammes? ....
4. From what age do the girls on average weigh more than 20 kilogrammes? ....
5. When (at what ages) do the girls weigh more than the boys? ....
6. By how many kilogrammes does the average weight of girls increase between age 3 and age 8? ....
7. At what age do the girls put on weight most rapidly? ....

The questions begin by requiring simply point-reading, then three types of interval-reading (specifically, an open ended x-interval corresponding to a given open-ended y-interval, an x-interval defined by a comparative condition ($G > B$), a y-increase corresponding to a given (two-ended) x-interval), and finally a point of greatest rate of increase.

In the form of this task used in the exploratory interviews, after initial familiarisation by point-reading, the main question was put, "During which one year period do we have the largest increase in the boys' weight?" If this was not understood, the pupil was asked leading questions about what were the increases between age 5 and 6, and between age 6 and 7. Two main observations were made. First, the greatest increase question was often first answered by (15,20) or (17,20), modified after a reminder about a one year period being required to (19,20), or (17,18). The greatest increase, it seemed, had to be connected with the greatest value. This "increase *vs* value" distraction reappeared constantly in this and other tasks. The second observation was that increases, when needed, were generally obtained by referring back to the axes, reading off the values and subtracting them. More sophisticated methods, such as reading the difference directly from the scale on the axis, without subtraction, or reading it even more directly from the grid in the body of the graph, were rarely used.

This example illustrates most of the "global features" which we found to be of interest and the understanding of which was studied in further situations. In addition to these, some other aspects of understanding also emerged as important in these further situations. It may be helpful to list all these aspects now, before presenting the situations which embody them. They are, (repeating the first for completeness),

(1) the recognition of *global features* by a progression from point reading to interval and to gradient reading;

(2) *measuring* intervals or gradients and *comparing* intervals or gradients;

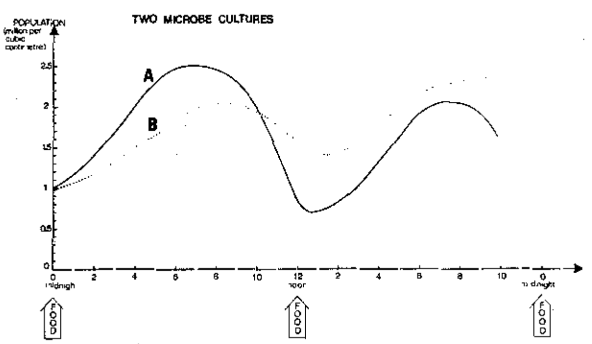
(3) *interpolation*, both within the set of integer points, and also when it is necessary to extend the number system to include fractional points;

(4) various *distractors*, in particular pictorial distractors, when the shape of the graph is confused with that of the hill being climbed or the race track being traversed; and *situational* distractors, when experience of the situation interferes with attention to the meanings of the abstract features of the graph;

(5) the process of *interaction* between graph and situation, in which in the course of extracting meaning, the situation itself gradually becomes less dominant as the graphical features themselves become the embodiments of the meanings acquired.

To illustrate these aspects, other tasks must be described.

Microbes presented the graphs of the numbers of two populations of microbes in a laboratory culture, in relation to their times of feeding



(This question was given to first year pupils as part of the teaching experiment post-test.) Two of the questions asked about this situation were particularly discriminating. The first was, "When is population B greater than population A?" This was answered wrongly at first by 11 out of 17 of the pupils, though three of these quickly corrected themselves when their answer was queried. The main difficulty was in giving an interval rather than a point response; the point given was generally the maximum of B, i.e. about 8 or 9 pm. There were also comparisons between the value of B being greater than that of A, and the *difference* B − A being greater at some points than at others; the presence of minima for A and B was also a distraction. The following extract shows some of these difficulties.

INT: *'And when is population B larger than population A?'*

KER: *Well. at 8 o clock at night "*

INT: *"But is it still larger let's say at noon. population B?'*

KER: *'Yes'*

INT: *'Over which period of time is population B larger than population A?"*

KER: *'Er well in the afternoon Oh. not afternoon 8o'clock at night. Er ... ."*

This pupil, even with further prompting, never succeeded in giving an interval. The second most discriminating question was, "Which population is growing the faster between (a) midnight and 6 am, (b) 1 pm and midnight?" Responses here showed fully the confusion among *rate* of increase (gradient), *amount* of increase (interval) and greatest value. The following extract illustrates this.

MIL: *'A grows faster ... no it s B (quickly) not A "*

INT: *How do you explain that?'*

MIL: *'Well it looks. . that's trying to put you off it s not growing faster but it s growing more .. that is steeper . you know. . it s fairly straight .*

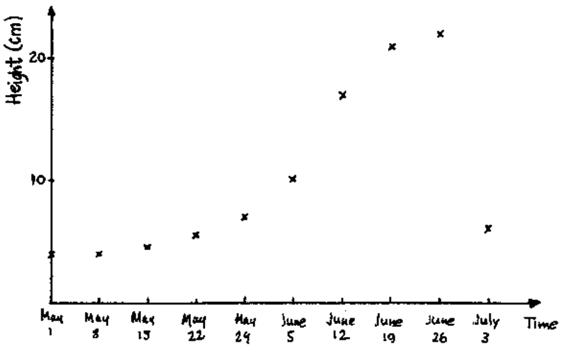
INT: *It means . when it s steeper. it grows more rapidly."*

MIL: *Yes But, when it s higher it's got more growth. It grows more... that's (A) growing rapidly but that s growing a lot (B) "*

Here the attempt at verbalisation develops confusions in what was apparently a correct initial insight. "Growing faster" and "growing more" are in conflict. This question was answered correctly by 2 out of 10 pupils, and, after a prompt, by a further 3. As well as being asked these specific questions, pupils were asked, "How do these populations vary over this period. How do they react to this food diet?" This led to some cases of misinterpretation of the graph as an "eating curve", the rises representing the consumption of food. This task provides two questions which, along with others to be described, determine the rough hierarchy of difficulty of the various global feautres; it also gives the above example of *situational* distraction.

Interpolation was not required very extensively in the tasks used. The main occurrence was in *Flower* (see below), where the height half way through a week is requested. This requires estimating the y-value of an unmarked point, between grid lines. This item was used in pre- and post-tests for the teaching experiment and showed a marked increase in both taught classes, from 8 to 13 (out of 22) in the "graphs" group and 11 to 16 out of 22 in the "tables" group. This receives comment below. Here we note that most of the errors gave the end-values 7 or 10 instead of the mid-way value; and that this item is of the same order of difficulty as the "when is B > A" item discussed above.

Jane planted some flowers in her garden, and measured one particular one every week. This is a graph of its growth:



Did it grow all the time at the same rate? ....

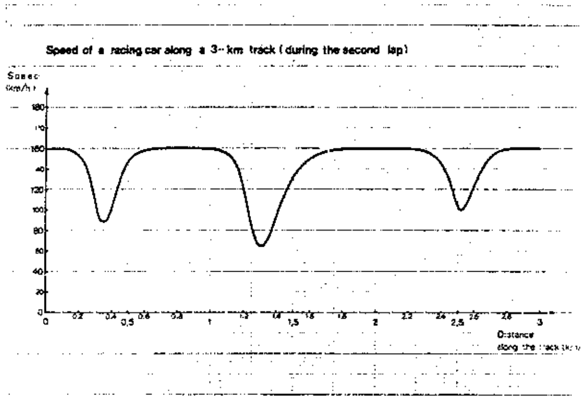
During which week did it grow fastest? The week ending ....

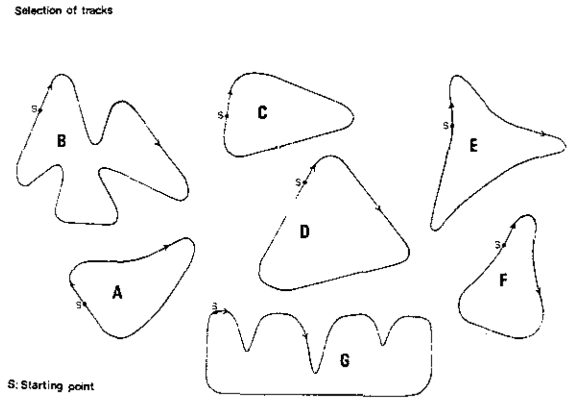
Estimate the height of the flower half way through the week ending on 5 June ....

What do you think happened in the week ending July 3? ....

....

Situational distraction has been referred to; this and pictorial distraction are both illustrated by the *Racing Car* task. In this, the graph represented the speed of a racing car going round a track; the questions were to decide the number of bends, and the shape of the track. Several forms of this task were used, some involving drawing speed graphs for given simple shapes of track, as well as tracks for given graphs. The form we discuss here is shown below; the correct track had to be selected from those given.





Track G was a favourite first choice (pictorial distraction) When the 3-bend shapes had been selected, the choice involved recognition of the relevance both of the depths of the dips in the speed graph and also of their placing in relation to the start. (A fuller discussion of this situation is contained in the article [Janvier, 1981] mentioned above)

The task *Vending Machine* shows a number of features — the development of grid-reading, and difficulties associated with discontinuities (the graph disappears); but it is described here mainly for the way it illustrates the progressive unravelling of the story the graph has to tell. The graph represents the amount of mineral drunk in a factory vending machine at times during the course of a week

Successful responses were classified according to two criteria. Some pupils were very *systematic* in their reading, going from 8 till 5 and describing all the drops, while others only picked up the dinner drop and had to be asked additional questions. However systematic they were, they could read each drop *more or less absolutely*, namely: each drop could be "measured as a number" (12 litres drunk . .) or compared to the others (they drink more, less . .).

Let us give a few examples

(1) systematic, absolute

BAR: *"From 8 till 9.30 there are 40 litres in the machine. At half-past nine, the workers start to drink They drink. . . er . . 35.5 . . 4.5 litres. Afterwards they don't drink till . . er 12. And then at 1.30, there is 16.5 litres in the machine So, they drank (a calculation) 19 litres . . "*

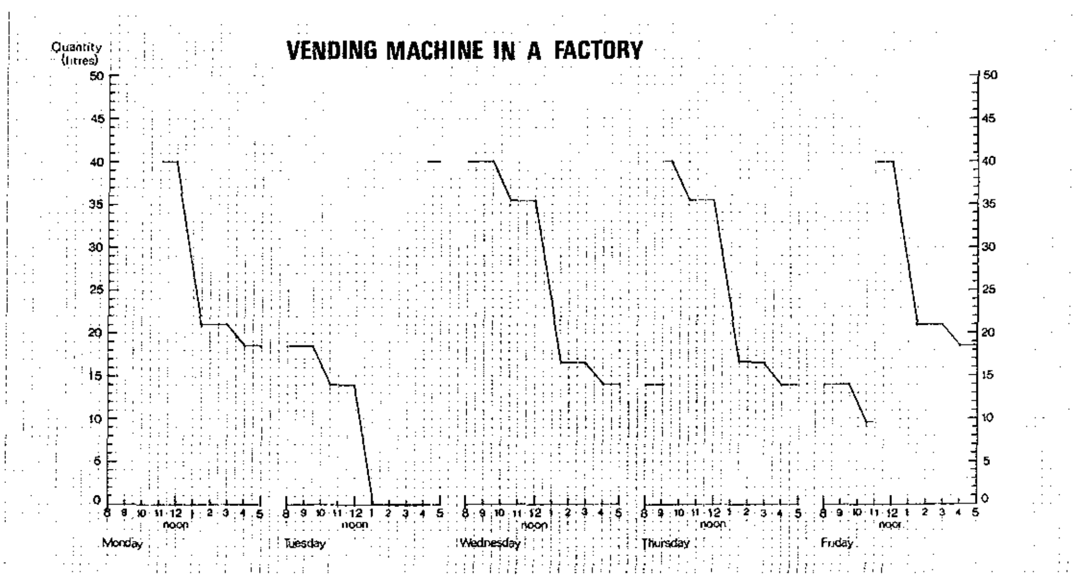
(2) non-systematic, relative

CRA *"It drops quite a lot from 12 till 1.30 But in the morning it does not drop so much . . "*

(3) semi-systematic, absolute

JAM: *There is 40 litres in the morning and there is 14 at the end." (The rest goes on with absolute values.)*

Other responses simply read off values of the amount of liquid on each hour ignoring the amounts between. Another difficulty lay in the fact that drinking, thought of as a positive action, had to be associated with a fall in the graph. Several responses were of the type, "the quantity went up from 14 to 40". The discontinuity on Friday created several problems. By several pupils it was initially seen as a *gap* where the curve suddenly disappeared and this was accounted for by such statements as "It ran out", "It's empty" or "It broke down" Even when the gap was seen to be a jump rather than simply a disappearance, it still proved hard to associate this with a refill of the machine. The comparisons required by the questions whether more liquid was drunk on one day than another and in the afternoon than in the morning, are most directly solved by moving the segment of the graph by eye and comparing it, and a number of pupils learned this skill in the course of answering the nine questions through which they were taken, but in many cases their earlier responses showed the phenomenon of attraction to high values which has been mentioned above Here it took the form of assuming that the graph



which began higher was the one associated with the greater consumption (i e. Friday afternoon as against Thursday).

We see here again clearly how *graphical interpretation is a progressive integration of the various pieces of graphical information with the situational background* The associations drop=drink, plateau=not drinking, zero plateau=empty, jump=refill all have to be formed into a usable language, and then combined to describe the week's events. Their formation and use are strongly facilitated by the ability to translate segments bodily, and to employ direct grid-reading of differences parallel to both axes.

Then there is a progressive interaction of graph and situation in which the former steadily grows in richness of meaning This is typically *not* achieved by a systematic working through the graph, point by point or section by section, but by a *scanning process* in which some parts acquire meaning first and are then used to help understand other parts

A comparative teaching experiment

In view of the above analysis we decided to attempt a teaching experiment to evaluate the effectiveness of a *language approach* to the learning of graphical interpretation We devised a rich setting in which the pupils would be encouraged to speak meaningfully the graphical language rather than to write a systematic series of lessons which would "grammatically" present a set of identified skills. To "speak meaningfully" means to use (and learn) a language in relation to a rich environment in which the links between the language and the situational facts are diversified and numerous while both get mutually richer and more complex as a result of the ongoing feedback process involved in learning any language.

A language approach is in essence indirect and natural The best analogy we can use to characterise it is that of a child learning his mother tongue No lists of words are referred to by his mother, no catalogues of grammatical forms. But the contact with his environment "supervised" by his mother "makes it happen. "

Having in mind to use this approach, we organised, with a science teacher, a series of experiments to be carried out in a science laboratory and during which the pupils would have to use graphs meaningfully (in relation with the experimental facts). For instance, in one experiment they were asked to heat up water and take the temperature every thirty seconds. This gave a graph for each group of pupils (2 or 3 pupils) The *shape* of the graph was then discussed, the various gradients were related to the "hotness" of each flame Also, the "graphical consequences of not stirring, of reading too late the thermometer, of adjusting the flame ...", were equally discussed In short, in comparing all the graphs, the teacher and the pupil "spoke" the graphical language in relation with the experimental facts.

A second group carried out the same series of experiments They recorded their results as tables rather than graphs but discussed them in the same way with their teacher and classmates.

Sample

The classes were two first year secondary classes, aged about 12 years, normally taught by the same teacher, who also conducted the teaching for the experiment. They covered a wide range of ability

Criterion test

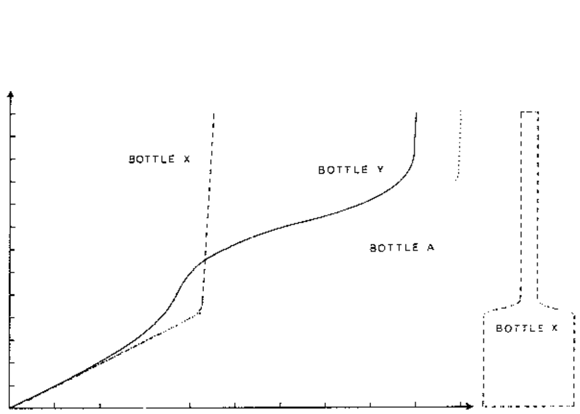
Conclusions were to be mainly drawn from observation of the teaching, but pre- and post-tests were used, consisting of two tasks (six marked items) similar to those discussed above Further tasks, both written and in interview form, were used as an additional post-test

Teaching material and procedure

During the nine 55-minute lessons, four experiments were carried out by the pupils: Heating Up Water (1 lesson), Cooling Boiling Water (1½ lessons), Stretching Rubber Bands (2½ lessons) and Filling Bottles (2 lessons). The first has been briefly described above. Pupils were subdivided into groups of 2 or 3, and each group had to plot their data on an already scaled transparency Class discussions organised at the end of the lessons were made more profitable as the transparencies could be compared by superposition. Comparisons between the curves of three different groups were then immediate. Small steps forming a "staircase" were drawn on these graphs to facilitate study of the changes in gradient.

Differences in steepness between graphs were discussed and attributed to the different flames of the burners; kinks were similarly traced to late readings, taking the thermometer out of the water, or failure to stir The other experiments were dealt with similarly In the case of Stretching Rubber Bands, the lengths with different weights hung on were recorded, short/long and thin/thick bands were distinguished by their consequent graphical features; also noted was the permanent extension caused by over-stretching. The Bottles task consisted of recording the increase of height when water flowed at a constant rate into bottles of varying shape.

Despite the interest of nearly all pupils, it soon became clear that the discussion at the end of each lesson although meant to be a synthesis for everyone, was not the occasion for a personal reflection on each question raised. A series of "synthesis exercises" was then designed with the intention of bringing about some fruitful reasoning or the experiments One example of such exercises for each group (graph and table) is shown below The exercise about filling curves turned out to be very difficult for the pupils.



During the experiment: HEATING UP WATER' the two following groups did not quite follow the instructions.

(1) group C skipped a few readings;

(2) group D read late a few times.

Can you recognise their tables?

TABLE 1 Time (min)	Temp (°C)	Difference	TABLE 2 Time (min)	Temp (°C)	Difference	TABLE 3 Time (min)	Temp (°C)	Difference
0	20	10	0	17.5	8.5	0	20	12
½	30	11	½	26	16	½	32	4
1	41	3	1	42	8	1	36	[illegible]
1½	44	6	1½	50	8.5	1½	[illegible]	
2			2	[illegible]	5.5	2		
			2'			2'		

Results and comments

Possible differences on the pre/post tests, between the "graphs" and the "tables" groups, were looked for, particularly under the following headings.

Reading skills
Here we noted no real difference between the two groups. This suggests that the reading skills already mastered by the pupils of both groups have been successfully extended to deal with the more complex cases of the tests

Interpolation
The fact that the table group did better suggests that the notion of completeness requires more than situational familiarity to develop Even though interpolation is simply a graphical artefact, it requires some thinking to be triggered and, apparently, this thinking is not fostered by the language approach, even extended over nine lessons. Since the table group improved more, we are tempted to put forward the idea that "numerical awareness is central in interpolation"

Fastest rate of change
On this topic, the "graph" group on the whole made a slightly better performance than the table group, although this was not statistically significant. The mean score of the graph group on the *Flower* problem improved because a significant number of pupils no longer confused "grow fast" and "be tall"

Graphical awareness
The *Racing Car* problem has shown that pupils of the graph group were more aware of the symbolic value of the graph in refusing to select the "distractor tracks" We are led to conclude that the "graph approach" is better at developing "graphical awareness", as expected.

Associating gradient with a situational feature
The notion of steadiness was developed significantly more by the graph approach than by the table one The association of the gradient with the "hotness" of the flame appears to have been handled more efficiently by the graph group. On other items, easy questions showed no differences but on more difficult items above average pupils of the graph group did better

Teaching observations

The teaching method used proved to be astonishingly successful most of all in that the *personalised* results plotted on transparencies were displayable in front of all and appropriate for direct comparisons. But, as *il ne faut pas abuser des bonnes choses*, after a month or so its effect appeared to be less striking, even though it enabled the teacher to keep the pupils more attentive than conventional methods do.

As mentioned earlier, the need for worksheets of individual exercises was felt, right at the first lesson. Despite their genuine interest in the discussions, only a minority of the pupils were really making the effort necessary to spell out in graphical (or tabular) terms their results and those of the other teams.

The positive features of the "table approach" did not take long to stand out, even though it must be said that the table group always followed the graph group. Consequently, the teacher was well guided in his synthesis and study of the tables using a pattern of analysis, so to speak, built up during the previous graph lesson But, on the whole, it was observed that the series of changes are more noticeable on tables, since the pupils have a greater familiarity with numbers It appeared clearly that a table of data with all its differences is very appealing to pupils and does not require as many "explanations" as the staircase does Let us quote the log-book

> *"A firm conviction today that the tables have provided a better source of discussion that the graphs But I am not sure that it is intrinsically true since groups are different, the teacher looks at the tables graph-wise and altogether he gets along better with the second group"*

But doubts seem to have gradually faded out:

> *"A table is a nice way to encapsulate a set of results and pupils like to look at it I think that in a real teaching scheme both tables and graphs should be used"*

As mentioned before, the use of tables proved a powerful tool to study "how variables change" The results conclusively show that the table approach certainly spelled out many ideas to the extent of making possible transfers from tables to graphs. Consequently, results suggest that the use of tables should be included in our graph teaching scheme.

The results suggest even more It seems evident that more analytical ingredients should be injected in the "graph language approach" which stresses synthetic elements We believe that this should mainly come in mathematics rather than science lessons For instance, graph-reading techniques could be more seriously developed in mathematics. Also, we think that *complex graphs should be introduced and analysed in graphical terms without reference to situations*. Such activities (learning or teaching) would represent a sort of backing grammatical support to the language approach. For example, the language of growth would be this way also developed at a more analytical level, completing the synthetic approach.