Geometry and Spatial Learning: some Lessons from a Jamaican Experience

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There is now overwhelming evidence that students in the developing, predominantly agricultural countries of the tropics have a much lower level of spatial ability than students in the more industrialised nations in the temperate regions [Deregowski 1980, Mitchelmore 1975]. While psychologists argue over the causes of the difference, educators are rightly more concerned with its effects in the classroom. The major research areas have been in picture production and perception, where some teachers have devised successful means of helping students overcome their initial difficulties [Davies 1976; McCrae 1973; Nicholson, Seddon and Wornsop 1977].

However, spatial ability tests measure only one aspect of a person's knowledge of space, namely the ability to predict the results of various transformations of depicted objects, using only such everyday words as draw, turn, fold, look like, longer than and nearer to. Much of what one knows about space requires a more technical vocabulary, including such words as line, square, angle, area, parallel and congruent, and is expressed in statements about the relations between the corresponding concepts and in the application of these relations to given configurations. This second aspect is what one usually calls geometrical knowledge. The two aspects are clearly not independent; all geometrical concepts are entirely visual in origin and owe their importance to their invariance under some common transformation, and many spatial ability tasks can be made significantly easier by the application of a little geometrical knowledge.

In Western studies of secondary school children, it has been found that geometrical knowledge is only moderately correlated with spatial ability, with verbal ability and logical reasoning often proving to be more strongly correlated [Werdelin, 1961]. In younger or less sophisticated students who are still in the process of forming the basic concepts of geometry, a much stronger relation might be expected. One would therefore predict that students in developing countries would show a deficit in their geometrical knowledge similar to their deficit in spatial ability. Such a relation was in fact found in two recent studies comparing samples of students from urban areas of Jamaica and West Germany. In one study [Mitchelmore, 1982a], the geometrical knowledge of Jamaican Grade 9 students in non-selective secondary schools was found to be less than that of Grade 5 students in German comprehensive schools. In another study [Mitchelmore, 1982b] it was estimated that the spatial ability of Jamaican Grade 5 students was equivalent to that of German students 8.6 years younger.

The difference in geometrical knowledge and spatial ability which these studies show is more than half the length of time which students usually spend in school. Since Jamaica is one of the richer developing countries and is highly accepting of Western culture, one might expect other developing countries to show even bigger differences, to the point where they exceed the normal length of schooling. Obviously geometry teaching in developing countries has to be qualitatively different from geometry teaching in Europe and North America, at least for the great majority of students.

However, the mere finding of a correlation between geometrical knowledge and spatial ability does not help us to understand what aspects of spatial understanding are important in early geometry learning, and so gives us no clue as to how geometry teaching in developing countries should be designed. We need answers to the following questions:

1. In what ways does low spatial ability affect geometry learning in developing countries? Or, to put it another way, what specifically spatial problems do students encounter in learning geometry?
2. Can students of low spatial ability learn geometry, and if so, how should it be taught?

The purpose of this paper is to describe the results of a research project carried out in rural Jamaica which throws some light on these questions [Mitchelmore 1982c]. Although the urban areas of Jamaica are similar in many ways to small cities in fully industrialised countries, the rural areas remain largely dependent on small-scale farming and are probably not very much different from rural areas all over the tropical world.

Background

At the time the project took place (1980-81), students in Jamaican teachers' colleges were placed in schools as interns in their third year and as part of their work were each required to make an empirical study of an educational problem in their school. Fifteen interns from two colleges, all of whom were teaching in rural primary schools, agreed to study the teaching of various topics in elementary geometry. The topics, procedures and results were discussed and coordinated at a number of seminars held under the direction of the author, but the studies were the responsibility of the individual interns working under the supervision of a college tutor. The quality of the study reports varies considerably, but one can have a fair degree of confidence in the many observations which appear in slightly different form in several reports, and it is these we shall rely on.

In rural primary schools in Jamaica, the principal may be the only trained teacher. Classes are often large and frequently crowded into one large room, the students sitting three to a desk designed for two and the classes separated by no more than a standing blackboard. The typical child comes from a large one-parent family living in a two-room house, frequently comes to school without breakfast, and tends to attend irregularly [Figeroa 1971]. There has been no formal study of what mathematics is taught in rural schools in Jamaica, but anecdotal reports suggest that teaching tends to be authoritarian and little influenced by the textbooks and curriculum guides distributed by the Ministry of Education. Anyone who has visited a rural primary school in a developing country in the tropics will recognise the situation [see, for example, Roberts and Kada 1979].

The studies

The fifteen studies fell into six categories: two on the recognition of basic shapes, three on fitting shapes together, four on parallels and perpendiculars, four on mirror symmetry, one on angles, and one on solids. By force of circumstances, most followed a one-group, pretest-teach-posttest design in the intern's regular class or an experimental group selected from it. To indicate the flavour of the results, here are summaries of the three studies which dealt with fitting shapes together.

Study 1: Making patterns (Grade 3)

In the pretest, a circle was correctly named by all the 25 students in the class, and a square, a rectangle and a triangle by 62-64%. However, knowledge of patterns was very weak: only one student could point to a pattern in the classroom or name one outside, all students made rather poor efforts at continuing given patterns, and only three could create their own patterns. Figure 1 illustrates students' difficulties in continuing patterns.

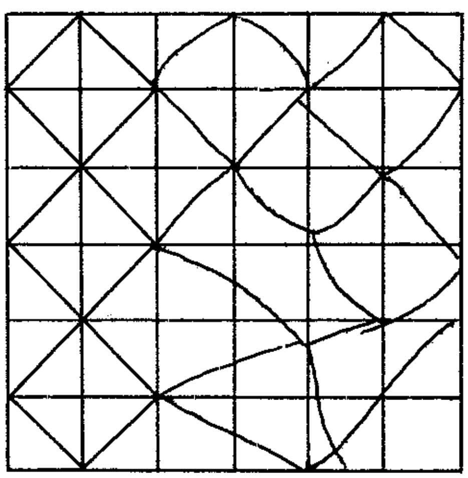

Figure 1
A Grade 3 student's attempt to continue the pattern given in the two left-hand columns. From the pretest in Study 1

The teaching unit consisted of 23 lessons over four weeks – ten from mathematics, twelve from art and craft, and one from language arts periods. The art lessons consisted of identifying patterns in the classroom and during a nature walk, and then making patterns using scrap material, string pulling, paper folding, drawing, ink blobbing, paint blowing, block printing, tie-dying and weaving. The emphasis here was on the regular repetition of a basic shape to create a pleasing pattern. (Naturally, many of the patterns were rectangular.) The mathematics lessons dealt with different types of triangles, the difference between a square and a rectangle, finding perimeters, making tessellations of squares and rectangles, and finding rectangular areas by counting squares. Students also wrote a composition on "Patterns".

The intern reports that, after an anxious start, children became very eager and worked carefully trying to create "better" patterns than their friends. They brought materials and patterns from home and talked freely about their discoveries; no stigma seemed to be attached to using scrap materials. It was noted, however, that children preferred to copy patterns freehand rather than use tracing paper. Also, some children had difficulty dividing rectangles into squares in order to find their areas.

On the posttest, all the children named all four basic shapes correctly and many identified patterns in their environment. There was still difficulty in continuing given patterns, but "after much effort" about 80% produced adequate completions; by contrast, they showed much enthusiasm and creativity in constructing their own patterns. The comparison with the pretest shows that students had made enormous progress. The intern also reports that students seemed to have become more alert, observant, creative and willing to "experiment and research".

Study 2: Tessellations (Grade 6)

The 18 students in this class were given a written and an oral pretest. On the written test, children were asked to draw various figures on squared paper, to say whether given shapes could form a tessellation, to draw tessellations using a given rectangle and square, and to find the area of four rectilinear shapes drawn on a square grid. The mean score was 8%. On the oral test, students were shown cards with diagrams of several shapes and asked to point to all the examples of various named shapes. Performance here was surprisingly poor, the best-known shapes being the right-angled triangle (55% correct), the square (33%) and the rectangle (33%). There was frequent confusion between the square and the rectangle; 33% called a rectangle a square. Similar frequencies were found when students were asked to name the shape forming a given tessellation.

Teaching consisted of 15 one-hour lessons spread over four weeks and fell into three sections. In the first, designed to reinforce concepts of basic shapes (the various types of triangles, square, rectangle, pentagon and hexagon), children outlined shapes on squared paper, drew shapes to given dimensions, and made shapes on geoboards. The second section dealt with tessellations of these shapes, the students firstly sticking card squares or triangles on to squared paper and then drawing given tessellations on squared paper and creating original ones; they were also encouraged to produce coulour patterns by using card shapes of different colours and by crayoning their tessellations In the final lesson, children learned how to find the area of a rectangle by dividing it into unit squares and counting the squares.

Children's work collected during the teaching period showed evidence of considerable individual differences and much learning Elementary errors included pencil lines not following the grid lines or made of two distinct segments; vertices not on a grid vertex; sides of the wrong length; square cards not quite fitting on the grid lines, so that adjacent squares did not fit together; card shapes rotated so that they could not possibly fit together (see Figure 2); and drawn shapes fitting together but changing shape from one side of the paper to the other. These errors gradually disappeared as students did further work on each topic The major remaining difficulty appeared to lie in drawing isosceles triangles on squared paper; there was no discussion of symmetry in this unit

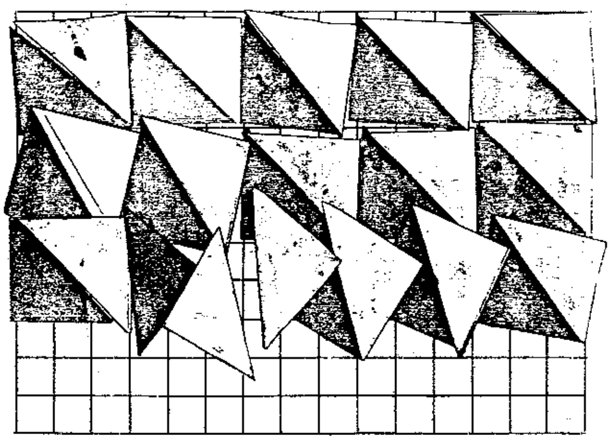

Figure 2
A Grade 6 student's attempt to stick right-angled triangles on to a square grid to form a tessellation From the second week of the teaching unit in Study 2.

The posttests were identical to the pretests On the written test, the mean score was 59%. On the oral test, only one student (who had not been in regular attendance) could not identify all the shapes on the cards; the shapes in triangular, rectangular and square tessellations were identified by 83%, 89% and 100% of the students respectively. Students had obviously learned a great deal not only about the basic geometrical shapes but also about shape in general.

Study 3: Area (Grade 6)

This study, done with a group of eight selected students, was intended to fall into a measurement category, but it was in fact largely concerned with teaching children to appreciate how certain shapes can fit together.

On the written pretest, half the students gave the correct values for the areas of various squares and rectangles, but none could state how many unit squares would fit into each shape. A group of ten Grade 7 children was also tested; they all found the correct areas but again none could state the number of unit squares The intern concluded that some students had learned the "length × breadth" formula but that none had any concept of area measurement

The four-week teaching unit started with exercises on covering regions with congruent shapes in a rectangular pattern; books were covered with bottle caps, desk-tops with books, and the classroom with desks (!) The rectangular pattern was highlighted by looking at squared paper and by pulled-thread activities with crocus-bag material (hessian) It took some time for students to appreciate that the covering pattern is formed by intersecting, regularly-spaced perpendicular lines; and they counted squares one by one instead of using multiplication. The general idea of area measurement was reinforced in art and craft lessons; for example, children made shapes representing people, houses, etc. on squared paper and counted the number of squares covered; or they covered a sheet of squared paper in a tessellation, coloured it in a pattern, and counted the number of unit shapes of each colour Areas of triangles and circles were found by counting squares, and students also learnt to estimate the areas of objects in the classroom.

Students nevertheless had difficulty in proceeding to the semi-concrete case; they did not find it easy to draw the squares in a rectangle of a given size, and they still counted the squares one by one. Towards the end of the teaching period, a few students noticed that there are the same number of squares in each row and the same number in each column when a rectangle is divided into squares.

Performance on the posttest was much improved, with about two-thirds of the students showing understanding of the items testing their concepts of area measurement The most frequent error was the use of linear units in place of area units.

Results of the other studies were similar. Students in Grades 2 and 4 learned to identify squares, rectangles and triangles successfully and learned their basic properties; students in Grades 4, 5 and 6 learned to identify parallel and perpendicular lines without much difficulty, and were able to investigate and use the corresponding properties of squares and rectangles; students in Grades 3 to 6 learned to recognise bilaterally symmetrical figures and to construct them using paper folding and sketching, but were unable to draw symmetrical figures accurately; Grade 6 students learned with some difficulty to measure angles to the nearest 45°; and a Grade 3 class learned the names of the commonest solid figures and their basic properties. All units, typically of 8-12 lessons over four weeks, followed a concrete-exploratory approach similar to that exemplified in Studies 1-3, in which shapes or relations were identified in the environment and then constructed for study in the classroom using geoboards, squared paper, art materials and whatever scrap materials could be gleaned by the intern and the children.

Conclusions

The results of the fifteen project studies answer our second question conclusively: young children of low spatial ability can learn geometry. On 8 of the 15 studies which included an identical pretest and posttest, the mean gain was 53%; similar gains seem to have occurred in the other studies. Even though no study included a retention test to find if the improvements were lasting, these are certainly impressive gains. Students certainly met difficulties, but the important thing is that most children tried and succeeded in overcoming many of them. Of course, many problems still remained, but you cannot expect to teach everything in one unit. Were students exposed to a sequence of units such as those which were taught in this project, one could reasonably expect that the elementary difficulties would be progressively eliminated and that students would be ready to proceed to a more abstract and analytical study of geometry by the end of primary school.

The teaching method which produced these gains was based on structured concrete explorations. The structure was imposed by the teacher in order to ensure that the target concepts were encountered; the teacher was also directive in presenting the geometrical terms and their definitions, but only after the need for them became apparent. The approach was concrete in that it dealt with real objects embodying the concepts under study, and it is interesting to notice that many of the spatial difficulties which children could not overcome so easily seemed to result from the transition to a semi-concrete representation (e.g. from covering a rectangular object with square cards to dividing a rectangle into squares by drawing). Finally, the method was exploratory in that, within the structure supplied by the teacher, students were frequently set simple problems to solve cooperatively and openended construction tasks leaving scope for individual creativity.

It should be clear that by following this method the interns taught an understanding of space which went far deeper than mere knowledge of geometrical jargon. This is evident both from the improvement in the technical standard of students' creative work and from gains on test items which were not dependent on verbal knowledge (e.g. the pattern production items in Study 1 above). No standardised spatial ability tests were given, and indeed it is doubtful whether the gain resulting from any one unit would have been measurable by such means; but it does seem likely that a sequence of units studied over a period of time would produce significant increases in measured spatial ability as well as geometrical knowledge.

We now turn to consider our first question by looking in more detail at the types of spatial difficulties which the students appeared to face. Three concepts turned up over and over again in our seminar discussions of interns' observations and results: congruence, pattern and direction.

The concept which seemed to give most difficulty in many different contexts was that of *congruence*. This concept appears to be fundamental to our understanding of space, and to be formed intuitively long before it is given a name and a verbal definition. Several studies showed the concept developing in response to situations where two or more figures needed to be "exactly the same". Young children frequently did not see any need for the figures to be congruent, or if they did they did not know how to achieve congruence. This should not be surprising; a great deal of cognitive development may be characterised as learning what "the same" means in different contexts. The teaching units presented situations where congruence was seen to occur because two shapes could be fitted on top of each other (using identical squares of card, making symmetrical figures by paper folding, and so on) and where congruence was seen to be important in order to produce a desired effect (pattern continuation, filling a square grid, copying a shape on a geoboard, making a symmetrical figure). Through these activities, students become aware of the need in certain circumstances for one shape to be an exact copy of another shape, meaning that one shape would if moved fit exactly on top of the other. Some activities also provided students with methods of copying shapes exactly in order to achieve congruence (using tracing paper, a square grid, or measurement). Several studies showed that children learned to copy shapes fairly well when it was necessary, although they did not even try before the unit was begun.

In examining examples of children's work included with the study reports, it would be easy to attribute their errors in copying to carelessness or lack of manipulative skill. There was certainly some element of the latter, but it cannot explain the extent of the errors which were made (see for example, Figures 1 and 2). Also, the activities seemed to be universally interesting and pleasing, and it is unlikely that in such circumstances children would have done anything less than the best that they knew. It appears much more reasonable to assume that errors arose either because children did not realise that they should try to draw a congruent figure or because they did not know how to do so.

A second concept which turned up in several studies as a source of difficulties was that of *covering a plane with a regular pattern*. The basic idea, of repeating a given shape in some regular fashion, was soon grasped and led to much enjoyable activity of an aesthetic nature. At first, children did not fully appreciate the necessity of repeating the shape exactly, but, as we have seen, the experience probably helped the development of their concept of congruence. The next step was the tessellation, a special kind of pattern in which the repeated copies leave no gaps between them. Making tessellations also proved to be an enjoyable aesthetic experience; in addition to providing further opportunities to experience congruence, it also drew attention to the necessity to orient copies appropriately (see Figure 2). The most important tessellation is no doubt the square grid, which embodies several critical properties of squares and rectangles and of parallel and perpendicular lines and is basic to the ideas of rectangular coordinates and area measurement. Studies 2 and 3 showed that the idea of a square grid does not come at all naturally and needs to be supported by many practical exercises in making tessellations and experience of using squared paper in other geometrical activities.

A third critical concept was that of *direction*, which is closely related to the concept of an angle. Children did not seem to have any difficulties learning the ideas of parallel and perpendicular lines and of a right angle, all of which are rather static. However, we have just commented on the difficulty which young children had in keeping copies of shapes in their correct orientation; and despite copious real-life examples and even a specially-simplified protractor, Grade 6 children could not master the idea of an angle as an amount of rotation. (Other studies done in Jamaican secondary schools have identified angle measurement as a particularly difficult topic. [Mitchelmore, 1982c, p 44]) This seems to be another example of the relative difficulty of ideas which require a semi-concrete representation; Jamaican schoolchildren obviously need many more concrete experiences involving corners and turning before they can deal successfully with the semi-concrete angles which occur in diagrams.

There were doubtless many other spatial concepts which caused students difficulties; whatever they were, they were overshadowed by congruence, pattern and direction and we did not notice them. We did notice several difficulties of a linguistic, logical and social nature, but these were minor in comparison to those of spatial origin.

Implications

The question "Why should we teach geometry?" has recurred with monotonous regularity this century with recent discussions giving increasing attention to the role of geometry in the primary school [Henderson 1973, Steiner and Winkelmann 1981]. All answers have been in terms of the characteristics and needs of students in the developed, industrialised countries. What of students in the developing world?

The results of the project just described confirm the predictions from ability and achievement testing that geometry learning in developing countries is quite different from what it is in Europe and North America; so geometry teaching should also be different. Criticisms such as those of Fielker [1979] of "the various pretences to be concerned with "geometry of the environment" " on the grounds that they "are only involved in perceiving shape, and perhaps how shapes fit together" [p.85], however valid they may be of geometry teaching in England, cannot be applied to Jamaica. In fact, it would seem that the very opposite statement should be made: primary school children in developing countries need to study "geometry of the environment" *because* it involves perceiving shape and how shapes fit together. By environmental geometry, we understand the many and various practical activities both inside and outside the classroom and the mathematics lesson in which children can make, handle, copy, use, examine and learn about the common geometrical shapes and develop understanding of such spatial concepts as congruence, pattern and direction – in other words, spatial education (In German, *Raumlehre* as opposed to *Geometrie*). Students in developing countries have low geometrical knowledge and spatial ability because they rarely engage in such activities either at home or at play, so if they are to develop the critical concepts it is the school which will have to provide them.

A study of environmental geometry in primary school is likely to be helpful to students both in other school work and in out-of-school actitivies which involve any sort of spatial ideas, and is essential if they are to make sense of geometry in secondary school. There is no doubt that geometrical activities of the sort we have described can give children a great deal of pleasure and also teach them some elements of critical thinking. To twist Fielker's words again [1979, p 87], the case for teaching environmental geometry to young children in developing countries therefore rests on its utilitarian value, on its place as preparation for later geometrical studies, *and* on its intrinsic value to the educational development of children at the time.

The objectives of environmental geometry, as we have envisaged it, are largely long-term. The spatial understandings arise from many different units organised around geometrical concepts such as symmetry, tessellations and angle, and are rarely the subject of separate units. For example, an understanding of congruence does not come from a unit on congruence (although such a unit might appear late in the sequence), but from many "fitting" activities in which the concept is not made explicit. This means that the readily-identified learning outcomes of individual units (e.g. "state the name of the given shape" or "calculate the area of a rectangle of a given size") tend to be trivial and to distract attention from the long-term objectives – which are in any case extremely difficult, if not impossible, to tie down in words [Tahta 1980]. As Fielker [1979, p.85] points out, the situation is in strong contrast to the position in arithmetic teaching, with its emphasis on skills and techniques.

The clash between the identifiable short-term objectives and the elusive but more valuable long-term objectives of environmental geometry brings many difficulties both for teachers and their advisors. The advisor can soon convince teachers through hands-on workshops that practical activities can be instructive, for the teachers themselves will often have difficulty completing them accurately. Teachers may even feel that the activities are inherently interesting and agree that the development of the concepts involved should be considered part of everyone's general education. But the teacher still has to battle with parents and principals. There is a long history of opposition to "play" in schools in developing countries; and since this term seems to mean anything enjoyable with no readily-identified short-term learning objectives, environmental geometry is likely to be included in that category. Also, in auxilliary investigations carried out by several of the interns in this project, a large number of primary school teachers said that they did not teach geometry because of lack of instruments, which suggests that they perceive geometry mainly in terms of euclidean constructions – a practical activity indeed, and one with well-defined short-term objectives, but certainly not appropriate in the circumstances.

Even if teachers can be persuaded and are able to include some geometry teaching in their yearly plans, there is still a great danger that it is the trivial short-term objectives which will be emphasised, since they are easier to teach and to test. The effect of this sort of geometry teaching is already apparent in Jamaica; despite the acknowledged difficulties in learning to measure angles in secondary schools, items in public examinations at Grade 9 and Grade 11 on the sum of angles on a straight line or in a triangle have relatively high facilities [Mitchelmore 1982c, p.42]. These and similar highly specific objectives can in fact be taught more efficiently without the time-consuming practical activities which can be more valuable in the long run. There is, however, another danger: a teacher, recognising the poverty of short-term objectives, might swing to the opposite extreme, arrange a great variety of practical experiences and never ask what children are learning from them. The difficulty for the teacher advisor is to ensure that teachers are sufficiently aware of the long-term objectives that they can not only arrange suitable activities but can also recognise when the objectives are being achieved.

Teachers who themselves have problems dealing with basic geometrical ideas are likely to need a great deal of help in the form of curriculum guides, but the writer of such materials is faced with a further problem: how to specify the activity and the expected products clearly without destroying its exploratory nature and without restricting the creativity of both students and teacher. A most valuable help to the teacher would be diagnostic tests similar to the Nuffield Check-ups [Nuffield Mathematics Project, 1972] which could be used to assess achievement of the long-term objectives and advise the teacher on what types of activities are desirable and which are unnecessary.

Of course, these are not the only problems facing a teacher in a developing country who wants to teach environmental geometry. A class of 60 students cannot easily carry out a practical investigation together; they must be divided into smaller groups. But then each group must be given simple but clear instructions which they can follow independently and each group will probably have to discuss its own results and reach some sort of conclusion which they can then report to the rest of the class. This is, however, entirely different from the usual mode of teaching; it requires a very confident teacher to give up her authority in this way and very brave students to break free of it. Fortunately, it seems that practical geometrical investigations are highly motivating throughout the primary grades, so they provide an ideal means of starting to promote student self-reliance.

A further problem concerns material and equipment. Although we have argued that sophisticated materials are not necessary, there is a certain minimum requirement of paper and card, scissors and paste without which nothing can be done. Even these might be outside the resources of schools which have annual grants measured in cents per student. Squared paper, which seems to be so very helpful in so many ways, would be a luxury. In these circumstances, it requires a keen teacher to beg and collect sufficient scrap and commercial materials to supply a large class for any length of time.

A final problem we shall mention is perhaps the most serious of all. How can teachers organise large classes to conduct independent exploratory investigations and develop students' spatial and geometrical concepts when they themselves are products of an authoritarian system from which they learned very little geometry? The amount of inservice education needed to develop all primary school teachers' concepts of congruence, pattern and direction to the point where they can design and supervise appropriate student activities is so enormous as to defy contemplation. The only hope, it seems, lies with the next generation of teachers. Perhaps the most encouraging aspect of the project described above is that it constitutes an existence proof that teachers' colleges in developing countries can produce teachers who can teach elementary geometry successfully and enjoy doing it.

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