

That Was Not the Intention! Communication in Mathematics Education [1]

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Mathematical cognition and competence are developed through communication. However, it is difficult to demonstrate a direct, causal connection between communication and learning. Educational communication takes place here and now, whereas learning is a phenomenon without time limits - happening to the individual student over a period of time and not always in connection with the educational situation. For that reason, the concept of learning is misinterpreted if learning effects are only searched for in here-and-now investigations.

The claim of our thesis also carries with it a certain attitude towards mathematics and mathematics education. If mathematics education means that the teacher is supposed to transmit mathematical knowledge to the students, then dialogue becomes less important. The teacher provides the information to be learned and decides what is true and what is false. As a consequence, structuralist-inspired mathematics education has not been very interested in teaching-learning processes from the point of view of dialogue.

If, on the other hand, one believes that the development and clarification of mathematical concepts, and perhaps their application in particular, is achieved alongside the development of everyday knowledge of the students, then dialogue becomes central in the educational process. This attitude entails that mathematical knowledge is negotiated and created through teacher-student interaction. Of course, the teacher with his or her educational background is supposed to take the lead, but the teacher does not necessarily have the only correct interpretation or solution or suggestion. It is possible for both parties to learn, and it becomes the job of teachers to maintain the dynamic between their own intended meaning and the meanings and intentions of the students.

It is not possible to connect communication in education directly to the learning of the individual student. However, how communication functions in the classroom and which consequences it has for the processes going on here and now can be discussed. Furthermore, what consequences different communication patterns can have for learning in the long view can also be discussed, but that would reach beyond direct communication analysis.

What happens to the communication when the aim of the educational process is not made obvious to the students?

In what follows, we have chosen a somewhat negative perspective. Our thesis is that communication in mathematics education assumes a particular character when the overall

aim of the teaching is not made clear to the students. To illustrate this thesis, we refer to the teaching unit 'How much do newspapers fill?', based on a video recording made in a Danish Folkeskole (primary and lower secondary school). [2]

When a new teaching unit starts, both teacher and students may feel that a discussion of aims and intentions is needed. But if the proper terms and tools for speaking about aims and intentions are not present, the result will be an amputated discussion of purpose, rather like what follows.

The teacher introduces the unit, raises some questions and employs terms from the field, but the students cannot quite understand what the unit is about. The students try to enter a conversation with the teacher and try to answer his questions, but their interest is centered around the questions 'How are we going to take part in this?' and 'What is the purpose of what we are starting?' They seek to become integrated, and they search for an answer to the intention of the teaching by answering the questions of the teacher. But their interest is at a meta-level in relation to the teacher's terminology, whereas the teacher for her part has been at a meta-level in the preparation for the class. The students can be part of the educational process on different premises - either by understanding and accepting what is going to take place in the classroom or (even though the intentions never come to be drawn out clearly) by accepting that the only possibility is to follow the teacher blindly.

Meta-knowledge, meta-learning and meta-communication

There is a complexity of different language games in the situation we are looking at. In gross terms, it is true that the language we use when talking about mathematical matters - e.g. that a fraction does not change its value when you multiply numerator and denominator by the same number - is not the language that we use in talking about the *intention* behind learning how, with fractions, you can multiply the numerator and denominator by the same number. The language for talking about aims and intentions must (so to speak) include part of the mathematical language as its object. It must be possible to refer to mathematical matters. Still, it is problematic if the relationship between the mentioned language games is viewed as a relation between a superior and an inferior language.

The metaphor we prefer to use is that concepts from one language are reflected in another language. Talk about intentions and ideas behind learning to do fractions reflects a 'fractions language', but at the same time the fractions lan-

guage gives flesh and blood to a discussion of intentions. With these added remarks, we will allow ourselves to use the prefix 'meta-'. A language which reflects concepts from another language makes up a meta-language. But at the same time concepts from the meta-language can be reflected in the original language.

Reflections about and discussion of mathematical knowledge are necessary if the students (and the teacher) are to acquire meta-knowledge – i.e. knowledge about the knowledge they acquire or have acquired. In the same way, it is possible to talk about meta-learning as an expression of what the students learn about their own learning. Meta-learning produces certain learning strategies for the students. For instance, the student will act according to a certain expectation that the teaching in a certain subject with a given teacher will take place in a certain way (Mellin-Olsen, 1989, p. 99ff). Meta-knowledge and the content of the meta-learning thus help to form the learning dispositions of the students. If the students repeatedly do not understand what the intention behind building a certain competence is, they will achieve a meta-learning which can cause a number of learning problems.

Meta-knowledge and meta-learning can be expressed and made to be shared by everybody through communication in education. Furthermore, a discussion of knowledge and learning will cause the students to develop further nuances in meta-knowledge and meta-learning, and result in the teacher and the students obtaining a mutually deeper insight into the way each side perceives matters. All this happens through their communication.

But what can be done when the communication is not clear, and when misunderstandings occur? To recognise a misunderstanding is the first step to mutual understanding. To communicate is the next one. Thus, meta-communication – communication about communication – becomes a key concept and an important tool for mutual understanding.

What does it mean to 'fill'? [3]

Our teaching example starts with the following question: 'How much do newspapers fill?' According to the earlier so-dominant structuralism, it is essential that complicated mathematical concepts are decomposed into simpler elementary units, and that these units are learned piece by piece, so that the composite understanding is achieved by the students in the course of their schooling. With this in mind, the newspaper topic can easily be interpreted as confusing and without any educational idea. However, the textbook MATEMA [4], from which the topic has been taken, is marked by a process-oriented idea, which involves the students developing mathematics through a working process rather than through instruction. MATEMA seeks to create opportunities for teacher and student to enter an educational mathematical interaction. It does not have to be controlled; and it can be productive, because the different proposals from the students can be used educationally.

It is consequently essential to place the students in situations where mathematics enters gradually, since no ready-made mathematical concepts are introduced to help the students handle the situation. Clarification does not take place in the shape of a message from the textbook or the

teacher. It will become clear that such a process orientation is not so simple to establish. One way of doing it is to create a situation which on the one hand is determined by certain structures and premises, but on the other partly leaves the generation of ideas to the students.

MATEMA 13 does not declare any overall mathematical intention with the unit 'How much do newspapers fill?' All that appears from the teacher's manual is that the topic contains 'lots of mathematics'. It is suggested that work can be done with items such as weight, height and breadth, but also with the comparison of different newspapers with respect to content and size. The textbook thus leaves it to the teacher to choose an angle. The students' book for MATEMA 13 launches the question, 'How much do newspapers fill?' Judging from pictures of laid-out newspapers and matrices for marking the height and breadth of newspapers, it is reasonable for the teacher to assume that the topic is meant to throw light on the concept of 'area'.

The concept of 'filling', however, is not precise: 'You fill too much of that chair', 'It fills my consciousness', 'The pie fills my stomach', 'The picture fills most of the wall', 'The tabloids fill people with gossip', 'Reading the newspaper fills all of my mother's morning'. Nevertheless, it is a clear educational idea to start mathematics education with such an open concept. But 'fill' is a many-valued term in the question, 'How much does a newspaper fill?' It could refer to time, volume or area. 'Fill' demands specification.

How much do newspapers fill?

First lesson on Monday morning. The teacher enters the 3rd form. He carries a plastic bag and has a smiling, although perhaps somewhat stiff smile on his face as he surveys the class. He has some difficulty catching the attention of the students, which is focused on the cameraman. This is the first lesson that is to be videotaped. The light in the room has been measured several times, and as everything seems to be working the lesson can start.

The situation takes place in a classroom which can be characterised by many positive terms. The students have great confidence in the teacher. The teacher communicates openly with the students. The students seem interested in mathematics. Generally, they are active. In this energetic atmosphere, we meet the teacher and the students when the teacher introduces a new topic.

At first, it is unclear to the students what it is all about. Naturally it is not easy for the teacher to clarify the situation with a direct statement of the whole idea, i.e. that "it is about 'area' and 'volume', but that is something we will have to find out as we go along". The difficulty is that 'How much do newspapers fill?' is not integrated in a situation that provides some context for answering the question. The unit has no frame of reference available for the students. Consequently, they have to follow the lead of the teacher. They have to find an answer to how much newspapers fill without understanding why they need such an answer.

It is possible to talk about *vantage points*, which are established through the *scene-setting* of the class. [5] The scene-setting consists of the class being related to a frame and an overall idea, such as e.g. 'collecting newspapers'. This defines a number of tasks, and if these make sense to

the students and can be discussed and described by them, vantage points are established. In this sense, scene-setting and vantage points can give content to a discussion about the intentions of a teaching unit.

It is from such vantage points that students and teacher can prepare for the direction of the teaching-learning process. It is characteristic of the teaching unit 'How much do newspapers fill?' that such a vantage point is not established. And this becomes a basic premise for interaction in the classroom. It means that the teacher cannot include the students in the unit in any other way than by arousing their curiosity and gaining their confidence: by letting them play their way into the problem area.

A vantage point can be sketched in the following way. A collection of newspapers is to be made. Is it enough to use a Volkswagen to collect old newspapers in Mill Road? That depends on how many people give newspapers, doesn't it? It also depends on how often collections are made. What if we collect once every month? What is the price of old newspapers, by the way? Could there be a thousand Kroner's worth piled in one corner of the classroom? These questions cannot, of course, cover the multifaceted meanings of 'to fill'. But it is clear that if such a vantage point is developed, a language develops in which the students can talk about the problem area: how much does a newspaper fill?

The ideas could also take another direction. The teacher could start by piling up papers: how many are needed to reach the ceiling? But they will fall over before they reach the ceiling! Especially if they are folded! But do they have to be folded when we pile, for us to be able to find out how much they fill? How many papers are needed to fill a bathtub? Do they fill more or less when they are wet? Or how about the idea of filling newspapers into milk cartons? That would also give us a unit of measurement. How many litres does a newspaper fill? The local paper hardly fills more than half a litre. And what if we are going to paint the ceiling? How many papers are needed to cover the floor. Of course, students from the third form will not be painting the ceiling, but then they can examine, for instance, how many papers are needed to cover the floor of the classroom when they are doing finger painting next week. It is probably best that the papers overlap. Let us try to calculate precisely: how much does a newspaper fill? Here the story can begin.

Openness from the start

One overall purpose with the question 'How much do newspapers fill?' is that the students learn something about 'area'; but how does the teacher lead the students on the journey?

Teacher: Now, the topic we are going to work on this week is right here in this bag [putting a pile of newspapers on the desk]. What do you say to this?

Several students: Are we going to read?

Teacher: Are we going to read? ...What?

Peter: No, we are going to play. Yes, play.

The teacher starts by introducing the newspaper as the topic the class is going to work with. The students are not quite sure where the teacher is going with his question and ask whether they are going to read or play. [6] The teacher main-

tains his openness about the subject by asking whether the students have newspapers in their homes, and if so, which newspaper, and what is the difference between a newspaper and a local commercial paper, and if any of the students can spell the word 'commercial'. In this indirect way, the teacher reaches the question which is to be the turning point of the class: 'How much do newspapers fill?'

Teacher: Newspapers like these, do they fill much? Do they *fill* much?

Marie: [Shaking her head.]

Teacher: They do. They fill an awful lot, look here. Look here, how thick it is. If you have papers from, say, a fortnight, you would have a pile like this all of a sudden [measures with his hands]. I have a basket in my place, you know, where I put my papers. A basket ... that looks something like this [illustrates a box with his hands]. Then I put my papers in that when I have read them, and sometimes I take them out if I want to read them again. Then they sit like this [makes a downward movement with a newspaper]. Then all of a sudden it is quite full, and it *pours* out with papers, and then I get annoyed with them. Then all of a sudden I just take them and collect them in a pile, and then I take them down to the bin, you know. They *fill* a lot, newspapers. [...]. We are going to try to measure newspapers. We will measure them in various ways. I have three different papers for you. I have the county paper and I had ... have this one called 'Politiken', and then another one called 'Aktuelt', it looks like this. It looks a little different from the other two, as you can see. Now look at it, you can see there is a difference, can't you?

Tim: It is smaller [about *Aktuelt*].

Teacher: Yes, it is smaller. Yet maybe there is no less in it.

Tim: Right.

Teacher: It may be that there is as much text in it, it depends on the number of pictures. By the way, you can see the weather forecast here on the back. Can you see? You are going to measure in the groups you are in at the moment. ...

As the teacher is the only one who knows the purpose of the lessons, the students are obliged to feel confident in the direction the teacher is taking them. If you take his words at face value, it appears as if 'to fill' is to be related to something about volume: the papers are filled into a basket or a bin. The students are told that they are going to measure, but what are they really going to measure: how many papers a bin can hold? Which paper is the thickest? Or is it only the weather forecast that is going to be measured? ("By the way, you can see the weather forecast here on the back. Can you see? You are going to measure in the groups you are in at the moment ...") The message is obscure, and the students will necessarily be confused.

Why do the students not step out? Most likely because they already know that if they follow the teacher all will be well. And then it is a matter of finding out where he is

going. And when you are not told by asking, then you have to make guesses. Through this the students express an intention of taking part in the class. Another possibility is that the students know the teacher and his way of communicating so well that they are able to make sense of a message which to outsiders sounds absurd.

The openness could nevertheless also mean that the students might lose their interest in taking part in the class altogether. That they would stop trying to make sense of the teaching and just follow instructions that seem unavoidable. When the students in 'How much do newspapers fill?' persist with their attempts to define a purpose it can thus be interpreted as a vote of confidence for the teacher.

Yet perhaps there can be too much openness. Openness can result in a 'global' confusion if there is no frame from which a 'purpose for the teaching' can be construed. Contrary to this, a precise scene-setting can ensure that a 'local' confusion, e.g. about the meaning of a concept like 'to fill', becomes constructive.

Guessing what the teacher has in mind

Some students have a formidable faculty for guessing what the teacher is getting at and an elegant way of appropriating his or her thoughts. That does not mean, however, that any learning is going on (Voigt, 1984, p. 232). Rather, it may be a technique that the student has learned to be able to get on in class. It can also be an expression of the student's wish quite simply to be part of what is going on in class.

The guessing strategy we are going to study from 'How much do newspapers fill?' is of a different kind, because the students are trying to work themselves into the teaching topic. Their interest is not instrumental, but rather directed towards content.

Teacher: There are two things you have to measure at any rate. That is: how much does a newspaper fill, and then you have to [incomprehensible talk] then you have to find out which paper fills most. And how do we go about finding out how much a newspaper fills? How much does a newspaper fill? And then you can try and study the picture of the student there, who is measuring ... you know that student there [pointing at the overhead projector] is measuring how much the newspaper fills. And what does the student do ... Camilla?

Camilla: She opens it.

Teacher: She opens it. Because what does it mean to fill? What does it mean to fill? That a newspaper fills something - what does that mean? What does it mean, what does a newspaper fill? What ... what ... what? [John raises his hand.]

John: It can fill a room.

Teacher: How?

John: If you ... [Teacher interrupts.]

Teacher: What do you have to do then?

John: If you take a lot of newspapers.

Teacher: Yes, if you ... well, you are only allowed to use one.

John: Then you can spread it out.

Teacher: You can spread it out. You can take the pages apart and spread them out and then find out how much it fills.

John: Yeah ...

Teacher: But can't it ... can a newspaper not fill in other ways? [John is looking away.]

Jackie: You can weigh it.

Teacher: That's true, but that's something different. That has not got to do with filling, has it? That is different, that will appear later on this [pointing at the overhead]. But first you have to find out: which newspaper fills the most? And you have to use these three for that. Which one fills the most?

John's idea that newspapers can fill a room seems straightforward - it can be understood as analogous to filling a basket or a bucket. The suggestion is both logical and viable, but it is not what the teacher was getting at, so John has to make another guess. This time he hits the target, but apparently he loses all confidence when the teacher asks if a newspaper can fill in other ways. After all, this is what John just suggested and had turned down. He pulls out and Jackie suggests that you could weigh newspapers to find out how much they fill. But again the teacher turns the student down by noting that weighing is different, and that it is not on the agenda till later.

We will never know where John was going with his idea: it can fill a room. Was he approaching a suggestion about measuring the volume of newspapers? If this is the case, then John's suggestion could be an accurate step in clarifying the concept of 'filling'. Only if it is presupposed that 'filling' is to be interpreted in terms of area can John's idea be turned down along with Jackie's about weighing newspapers. But the rejection means that the educational idea of introducing the open concept of 'filling' gets partly lost.

The communication shows that the teacher has one idea or one intention that he pursues, while the students have to guess their way. This results in two different ways of communicating. The teacher has already prepared the topic and has a certain understanding which he wants to realise. The students are only trying to clarify what the unit is all about. Thus, the progress-directed communication of the teacher is confronted with the students' circularly-searching communication. They are talking at cross-purposes, not the most fortunate circumstances for negotiation of meaning.

The ambiguity of area and volume is productive if seen as an opportunity to make the students clarify concepts. It is an important insight to reach that you must have a common goal to be able to talk consistently about how much newspapers fill. However, the significance of this clarification is lost as the effort to find out how much newspapers fill is replaced by an attempt to find out how much they weigh.

The unit 'How much do newspapers fill?' has then entered the second dimension of the concept 'to fill', i.e. the idea of volume, but still without this having been made clear to the students, and neither in the workbook nor in the discussion in class is it revealed what could be the purpose of comparing the weight of the newspaper with what it fills. In the sequence below, the teacher intention is still the same - the students have to find 'the right answer'. The difference in relation to the previous example is that the teacher here initially turns down the student's suggestion, but he changes his mind, accepts the suggestion and carries it through. Here the teacher accepts the premises of the students. [7]

Teacher: What do we need in this one over here, Michael?
 Michael: Another newspaper.
 Teacher: [Promptly] No! [3 seconds] Well, it would be possible. We will do that then ... There Michael, I did that now, then. And what did I find out by doing that, Michael? ...
 Michael: That it was heavier.
 Teacher: That is right. It is possible to do what Michael suggests. That can make us find out which one is heavier ... Politiken is heavier than Aktuelt ... The two newspapers there do not weigh the same.

So far so good. But the students have still not guessed the teacher's answer, so he continues as follows.

Teacher: *But* what can we also find out, if we put something different in that thing there, John? [Pointing at the scale.]
 John: Put a weight in it.
 Teacher: We can put a weight in it. Here they are, both in plastic and brass. What can we find out then, John?
 John: Then if we know how much they [the weights] weigh, then we can find out what they [the newspapers] weigh.
 Teacher: What can I find out? What am I finding out now, John? [Puts a weight in the bucket, which tips.]
 John: Now you are finding out that the weight is too heavy.
 Teacher: Right, I maybe need to take off some weight, but now I will find out how much the newspaper weighs, and the figure ... now I get a figure ... and I can write down that this newspaper here, for instance, weighs 100 g.

John guesses the right answer, i.e. that newspapers can be weighed on the scale by means of weights. But that is not enough - the teacher also wants John to say what the teacher will find out by that. That John knows the strategy can be seen from his way of phrasing his answer: "Now you are finding out that ...", but still he does not quite hit the target by focusing on the weight itself. The teacher wants him to say something about the weight of the newspaper, but in principle they are talking about the same thing - only from different perspectives. And why would the newspaper perspective be more correct than the weight perspective? The result is that teacher and student do not meet in their communication. The meaning of the example is not negotiated, and no mutual understanding is established on the basis of the different perspectives of the two parties.

If we regard the two examples from a more general angle, we can say something about the nature and purpose of such a question-and-answer strategy. The teacher poses a question, and the students readily give examples. But their answers do not correspond to the hidden expectations of the teacher, and consequently he turns down the examples (Voigt, 1989, p. 30ff). This shows a contradiction in the communication of the teacher. Apparently, the teacher is posing an open question, but at the same time a closed question is being communicated, requesting a particular answer. Most likely, the result in the first instance will be that the students get confused about their answers being turned down.

What is the purpose of asking for examples, if you are not really interested in listening to them? After this, they can get annoyed at not having guessed his answer. Or they can become resentful because of the teacher's premises and because they have not been listened to.

If the teacher is thinking of a certain way to the right answer, and if this way is hidden to the students, it will block the way for a mutual understanding. On the surface, it looks like an open dialogue, but really the teacher is not listening to the students' suggestions. If this form of communication shows itself as a repeated pattern with the teacher, the students will come to a meta-learning, like: the teacher is always right and whatever we believe, it is the teacher who sets the agenda and decides what is right and wrong, important and not important. Consciously or unconsciously, the students will react on the basis of this knowledge, and consequently they will hold it futile to oppose. This can lead to a sadly repetitive pattern. Perhaps their meta-learning could also assume the following shape: in mathematics, there is always one and only one correct answer. The students' 'philosophy of mathematics' can thus be a reflection of the way the teacher communicates.

If the communication is locked into a certain pattern, meta-communication can be the solvent that makes it possible to move on in a different manner. If, for instance, the students were able to make it clear how they perceived the situation and in this way clarify the contradictions in the teacher's communication, the teacher's double meaning would become clear. And so a basis for changing the pattern would have been formed. Questions like "What do you really mean by that?" and "Why can't newspapers fill a room?" could be the starting point for communication about the learning itself. From being "Whatever we believe, it is the teacher who sets the agenda", the students' meta-learning could become "It is worth giving my opinion; the teacher is open to what I'm saying, I can help change things."

The students take over

After the introduction in the classroom, the students are now to go into groups and start measuring. One group is working in the corridor, measuring how much newspapers fill when they are spread out. This is where a problem of finding a unit of measurement arises.

Teacher: ... well you see, what fills is that which you can walk upon, touch ... But are there other ways? You can also measure how much they fill with a metre rule, that would be a possibility. You decide how you want to tell the others how much this newspaper fills when it is spread out.

The exercise consists of two things - first to find a unit of measurement: "you decide", and secondly to measure the newspapers: "how much this newspaper fills when it is spread out". The teacher does not reveal how or with what purpose they are to measure the newspapers, but suggests that the students can use a metre rule. However, the group decides to count the pages of the newspaper (in consultation with the teacher), having first turned down measuring in steps as an unreliable way of measuring. But then another problem arises. The pages of the different papers are not the same size.

Teacher: But look at this, here is another newspaper. You remember that small one, but look at this page, a page like this in this paper here, in fact it fits, as you can see, it fits right here [puts a double page from *Aktuelt* on top of a single page from *Amt-savisen*]. And that means that when you spread out that newspaper like this, when you tear it apart, then that is a page, then you have to count how many pages there are of this kind here, not of that kind there, that is a double page, then, isn't it? So now you are to count how many there are of that kind, so just count how many there were and write down how many.

The group is then left to its own devices, and it turns out that it is not so simple to decide what a page in a newspaper is. One problem is that the teacher has communicated a double message. On the one hand, the students have been told to decide for themselves what a page is; on the other, he in fact gives his own definition. This duplicity is repeated at a later stage when the students fetch the teacher, because they disagree and want him to decide who is right.

The double message of the teacher is reflected in their internal communication and disagreement. Camilla takes the teacher's offer of deciding about the size of a newspaper for themselves on face value: "Then we can just say, this is a page of a newspaper", but Malene refers to the definition provided by the teacher.

Malene: No, no, that is no good. No, if we spread out the paper like this ...

Camilla: Then we can just say, this is a page of a newspaper.

Malene: No [very determined/irritated tone of voice].

Camilla: There is one.

Malene: Well, no, that is two pages. There, now look at this [takes the newspaper from Camilla]. There are two pages, one, two [counting and pointing at the newspaper page].

Camilla: [Making objections, which cannot be heard clearly on the tape.]

Malene: There *are* two pages, do you not remember what the teacher said, that then we could ... [incomprehensible talk] ... well, it *is* two pages.

Thomas: I can't see whether this one is *Aktuelt*.

Malene: *Aktuelt*? Oh, but it says so on the front, doesn't it? [taking the newspaper from Thomas and beginning to study it].

In this group work, we see a disagreement between Camilla and Malene. Whereas Camilla makes her proposition in a timid voice, Malene has a determined and evidently irritated tone of voice in her rejection and explanation. At the same time, she is very much engaged with solving the exercise. Malene presents her argument with a certain authority by referring to what the teacher has said. The authority is strengthened further as she is capable of rejecting, arguing, explaining and directing at the same time. Not only is she able to argue against Camilla, she also directs Thomas, who is trying to find out where the different newspaper pages belong. It looks as if Malene is taking control [8], and this tendency is strengthened in the following sequence.

Camilla: Is this not the way to count: One, two? [Pointing at the pages.]

Malene: No. One, two. [Irritated tone of voice and pointing in a different way.]

Camilla: One, two. [Sticking to her suggestion.]

Malene: No! That stupid one or the one we were measuring yesterday, right?

Thomas: Now, look at this.

Malene: There were only these small pages, so he says: a page like this, this is a page.

Thomas: No.

Malene: That's what he said.

Camilla: No.

Malene: Yes.

Thomas: He said, one like this was one.

Malene: No, listen: In this paper ... which one was it we had a moment ago? What was it called?

Thomas: *Aktuelt*.

Malene: *Aktu...* *Aktuelt*, O.K. There were only these small pages, what can we do?

Camilla: Can't we try to do like we did?

Malene: No, please listen, will you? There were only those small pages, they cannot make up a big page.

Camilla: That is one, two [sticking to her earlier proposition].

Malene: Well, but the one we measured before, right?

Thomas: Yes. [They all laugh at the situation.]

and Camilla:

Malene: It was a page like this, not one like that one and that one, was it?

Thomas: [Shaking their heads.]

and Camilla:

Malene: That is why I say: in that one it has to be ... it has to be twice the size, because it is not a page like this [Camilla laughs], listen will you, now then it can't be the same, because there were twelve pages, and it was only small pages like this.

Thomas: It was not.

Malene: Then fetch the newspaper.

Camilla: I will go and ask the teacher.

Malene: But he would not know, would he?

Camilla: Yes he will.

Malene: If there are two papers like that, well, let us see ... I will go and get that paper, and then let us see, let us see if it was a page like this [Malene gets the newspaper], can't you see? It is not at all like this, can't you see?

Thomas: Right.

Malene: Then we cannot measure it, can we, because they are not the same.

It is noteworthy here that Malene in fact steps in to assume a traditional teacher role. She argues and explains, but she also gives out orders: "Then fetch the newspaper"; she disciplines: "listen, will you"; she asks questions, which she (partly) knows the answers to: "Which one was it we had a moment ago? What was it called?" When the others question her authority, she refers to the teacher's "He said so". The teacher is invisibly present in the communication, and Malene is his advocate. In the example, however, the rest

of the group oppose Malene as an authority and they fetch the real one to settle the disagreement.

Malene's taking control can be understood as a reaction to the missing teacher control. A possible student logic can be that when the teacher will not decide, then somebody else has to do it. It may express the student's attempt to create clear guidelines for the direction of the education. The situation can also be seen as an example of a group dynamic 'law', which takes effect when the official authority is absent. There is often a battle about who gets to decide.

Apart from the power struggle, it is noteworthy how engaged and serious the students are in their work with the problem, even though they are working in the corridor and without the presence of the teacher. The absence of the teacher has the immediate advantage that more students can have a say, while at the same time the discussion is on their premises. Thus, the organisation of the teaching gives the students the possibility to explore the topic themselves and discuss the problems among themselves - in principle, without the teacher having to interfere. In other words, there is both 'learning by doing' and 'learning by talking'. This last concept we use to focus on the ability to communicate what one experiences and does. Being able to explain opinions, methods, solutions, etc. is part of a communicative process and can be regarded as an example of how mathematical cognition is developed through communication.

'Zooming in' as a student activity

The students work actively although they do not know the general aim of the educational process. Therefore, we interpret the two student strategies 'guessing what the teacher has in mind' and 'the students take over' as their active attempts at 'zooming in' on the whole purpose. We use this metaphor to describe the process of focusing on certain selected elements of a setting. The students' 'zooming' strategy made them able to participate in the classroom activities.

'Zooming in' is an activity performed by students in which they try to accommodate their 'learning concerns' to the situation. We can imagine that the students and the teacher get to an understanding. The students come to see what it is all about. However, 'zooming in' which is not successful in this sense can also be observed. It was, in fact, unsuccessful but persistent 'zooming' which drew our attention to the phenomenon itself.

'Zooming in' has both positive and negative connotations. When you 'zoom in', you are able to see clearly some parts of a situation, but at the same time you (may) pay less attention to other important aspects or totally miss the general idea. We have indicated the negative consequences of missing the general idea if the students have to guess their way into the teaching-learning process. This led us to the conclusion that the focus of the classroom communication is related to the social participation in the educational setting, drawing attention away from the mathematical subject.

But 'zooming in' also has positive characteristics which are essential to learning. One of our main points is that the students are in fact concerned about coming to grasp the intentions of the educational process. 'Zooming in' can be seen as their way of trying to do it. Thus, the term becomes an indicator for something the students do with a special

effort, an intended action. Furthermore, it can be interpreted as a process, where the students seek for a vantage point. In this process, reflection is needed. Activity, searching, reflection and action then become central parameters of the strategy of 'zooming in', and these parameters are also essential to the concept of learning.

Besides something being out of focus, 'zooming' indicates that the classroom practice has not been made into a routine. New elements are available. Next, it indicates that the students are concerned about what is going to happen. They do not behave like a squadron of soldiers marching in a certain direction until a new command is given.

In many situations, we have observed classroom practices in which it is not possible to identify a 'zooming activity'. 'Zooming in' is not a common phenomenon. We can think of two factors which tend to eliminate 'zooming'. First, the classroom can be organised in a way that makes it absolutely clear what all tasks are about. Second, the students might not care about what they are doing because they are submersed into a well-defined instrumentalism. These two factors can be combined in the appearance of the traditional mathematical classroom organised in accordance with what can be called 'the exercise paradigm': the teacher explains the new topic and tells the students which exercises to do. The students do the exercises, and the teacher 'ticks' the results. In a classroom practice of the exercise paradigm there is not much need for doing any 'zooming'.

One point of the textbook MATEMA is that the mathematical topics are presented in a way that calls for 'zooming in'. This can be seen in several arrangements of what we called 'local confusion', which can be an educational invitation for 'zooming' activity on the part of the students.

Learning as action

How students learn cannot be decoded from educational communication patterns. But they influence the learning. Consequently, it is interesting to study what happens when the purpose of the education is invisible to the students. Here, we shall describe a terminology which can relate the students' 'zooming' strategy to a theory about learning.

The observations we have made could hardly have been done in a more rigid classroom governed by the exercise paradigm. Naturally, in such a classroom, students can also be acting and can decide to participate in things. However, in an open situation the students' concern for bringing themselves into the process becomes more directly revealed.

We see both 'communication' and 'learning' as closely connected to the concept of 'action'. For that reason, we want to draw attention to some important characteristics of action, which may help us to see less conspicuous but in our opinion important features of communication in mathematics education.

The concept of action can be associated with terms like 'aim', 'decision', 'plan', 'purpose' and 'intention'. An individual is 'involved' in his or her action. This means that we will try to separate the concept of action from 'biologically determined behaviour'. In fact, we are all doing a lot of things that could only in a very loose definition be classified as actions, such as e.g. nervously scratching one's scalp while writing an article. In order to be classified as action in the present sense, a certain intentionality is required.

A second requirement for being able to act is that one is not forced into a situation where no alternatives exist. If one is behind barred windows without a possibility of escape, then being imprisoned is not in itself an expression of an action – whereas planning to escape naturally is. It is not possible to act in an overdetermined situation.

Action, in other words, presupposes a certain degree of personal involvement and the presence of certain possibilities to choose. One can have an intention to do something, and the carrying out of the action means that the intention is followed up. Intention and action are interconnected, because an intention points to the actions that can fulfil the intention. Human action that is not rooted in an intention will normally not be referred to as action. [9] There is, however, no innate connection between intention and action in the sense that one intention points to one specific action.

We want to interpret learning in the same way as action. Viewing learning as action is on a par with constructivism as it has emerged more and more clearly in the discussion about mathematics education (see, e.g., von Glasersfeld, 1991). This also involves a complete departure from an 'empty vessels' pedagogy. What does it mean, then, to interpret learning as action? For action to be constituted like that, it has to be interconnected with an intention, just as learning is connected with an intention. The learning individual is an acting individual and has to be respected as such.

We view the students as 'involved' in their learning. For that reason, the framework around and conditions for learning can be discussed in terms corresponding to those of an action. Learning grows from a connection with an intention, and learning belongs in situations where there is a choice. This does not mean, however, that we connect all learning to intention in a uniform concept of learning. It is quite possible to imagine forms of learning where, for instance, a habit is assimilated, and where the learner has had no clear intention of adopting that habit.

People get influenced by the culture of wherever they live their lives. They learn the culture from within, perhaps with no clear intention of wanting to learn something. Thus, we do not claim that our way of viewing learning covers all forms of learning. We concentrate on analysing forms of communication in mathematics education, and here it is possible to spot the students' intentions about learning. In such analyses, we believe that the perspective of 'learning as action' is useful.

We have characterised the introduction to the unit 'How much do newspapers fill?' as openness from the beginning. This openness has both a good and a bad side to it. The students gain the possibility of finding out what purposes the unit might have. To the extent that they are able to find the intentions, and to identify with them, they can be joint owners of their education. [10] As learners, they can also be acting. On the other hand, the openness can lead to confusion, which makes it difficult for the students to see the intentions and thus also makes it difficult for them to take part in a joint effort.

We believe that the interpretation of the situations 'guessing what the teacher has in mind' and 'the student takes control' becomes more nuanced if learning is seen as action. In the guessing sequence, the students search for a purpose

for their education. They try to establish a basis for action – for learning. When the student takes control, the teaching unit will be given new energy, although it may lead the students off on the wrong track if the idea behind the unit remains hidden. We have characterised these actions as 'zooming in'. Therefore, we can see the students' 'zooming' strategies as observable tokens of their learning intentions.

Disposition, intention and learning

Intentions do not emerge out of the blue; rather, they are formed on the basis of experience, impressions, prejudice, preferences, etc. – what we refer to as pre-intentions or *dispositions*. A person's dispositions make up the raw material for her or his intentions. We see dispositions as individual, but at the same time as culturally determined characteristics for a person. It is not always possible to decide a person's dispositions on the basis of his or her actions, but they will somehow be embedded in these actions.

When we look at learning as action, we also use the concept of disposition to describe the motivation of the student to enter into a learning process. The student's dispositions for learning are thus indicative of the factual possibilities that the student has in a school system and for the student's interpretations of these possibilities. Correspondingly, the students' dispositions make up a heavily structured framing condition for intentions about learning. The students throw out such intentions in patterns and according to their notions about learning and going to school. The parts of the students' dispositions that are important for the shaping of intentions for learning correspond to what could be called 'the students' learning plan' (Lindenskov, 1992).

Intention and action are closely related, but not in the sense that the intention of doing something comes first, to be followed by a certain behaviour that is expected to fulfil the intention. The intention of the action is so to speak still present in the action itself. For that reason, we will talk about the intention of an action as characterising an action. The same goes for learning. The students do not first have an intention to learn before they let themselves be involved in learning. The intentions have to be present in the learning. The fact that the students here continue the intense discussion of what is meant by a newspaper page – even after they have left the classroom and the teacher's domain – we see as indicative of their wish to see the purpose of the exercise. This purpose has not been co-ordinated and prepared in an overall aim with the unit, but still the students have intentions in learning how to measure newspaper pages. This can be seen in their 'zooming' activities.

Intention and action are integrated via a decision. You have an intention and you choose to carry out a certain action in the hope of fulfilling your intention. An action has consequences, including consequences that are beyond the notions of the individual. This is because the intention cannot reasonably be said to describe the consequences of the action. Many consequences can be unforeseen and unexpected; the intention, at most, contains an inexact notion about some of the possible consequences. At the same time, the action and its consequences can end a circle (or open a spiral), because a significant restructuring of a person's dispositions is conditioned by that person's actions

and the consequences that these actions contain. In acting, dispositions are continuously changed. And that goes for learning as well. Meta-communication and meta-learning thus have special functions, because the latter helps change the dispositions of the learner, whereas the former can ensure that this change is not (merely) ruled by coincidence.

We find that the phenomena of 'zooming' supports the interpretation of learning as action. 'Zooming' consists of an attempt to relate intentions to activity. It is a trial-and-error strategy for looking for meaning of a classroom activity. The importance of establishing educational situations in which it is possible for the students to carry out zooming and, at the same time, to establish a 'culture' in the classroom in which the students really want to try out 'zooming' also becomes clear to us. Both conditions are important if teaching is to be different from classical teaching guided by the exercise paradigm.

Learning and teaching

Intentions about learning are not the only kind of intentions that manifest themselves in the classroom. The teacher is also an acting individual. This claim is, of course, not particularly novel. In many descriptions of educational situations, education has been described as a process subjected to planning and structuring. Teaching has been described as a complicated action, at times in management terms, whereas the receiving end, the students, have often been described in terms that place them as objects for educational planning. In our terminology, education is characterised by two 'agents' meeting each other. One of the problems is thus to co-ordinate two different types of actions, i.e. learning and teaching. For that reason, it is of special interest to see how teacher and students meet about the educational content. One kind of meeting we have characterised as a guessing strategy. Other forms can obviously be imagined: instead of the open guessing strategy, the meeting can be closed through an authoritative use of a textbook. [11]

Intentions connected with learning are far from the only set of intentions that the student has. A student can have intentions about avoiding being noticed by the teacher, about sitting next to somebody, about joining the game in the next break, etc. Here we will talk about *underground intentions* which refer to the students' 'zooming out' of the official classroom activity. Such intentions also flourish in the classroom, and they are connected with learning as much as other intentions, partly setting the scene for what is actually going on in the classroom. It is difficult to spot underground intentions in the classroom. Apparently, the students are calculating studiously, but this is only when the teacher is in their close vicinity. When the students find out that they can once again make themselves invisible in the abyss of the classroom, their activities change.

If learning is interpreted as action, it means that communication in mathematics education often does not manifest itself as communication about, for example, the understanding of a certain mathematical topic. This is so even if the conversation is about mathematics. On a meta-level, the conversation is aimed at the students' interest in understanding the purpose of the education. That is exactly the presupposition for entering the learning process as acting

individuals and having an intention to learn. *We have noticed students struggling to 'zoom in' on the learning process, but being blocked on their way exactly because the difficulties of communicating a purpose with the education are too big.* As a consequence, the students have to return to their large set of underground intentions or to guessing strategies, as we saw in John's attempt to find out where the teacher was going.

We take an interest in guessing strategies mainly as a result of our interpretation of learning as action. Do the students get a chance to discover the teacher's intentions, do they get a chance to express their own intentions, do intentions become negotiable? That is essential if the students are to own the aims of the teaching unit to the same extent as the teacher. In order for the students to be able to get involved in the class, it is important that the possible directions of the class are confronted. This communication can naturally not take place in a mathematical terminology; rather, a commonly accessible frame is required from which vantage points can be established.

Another reason for the difficulty in creating meaningful education is that the teacher often interprets the students' questions about mathematics and the mathematical content as if they are about mathematics and the mathematical content. They are, in fact, not always so. At least, they can often be interpreted differently. Really they are about the educational intentions and aims. They are part of the students' attempts to form intentions on the basis of their dispositions and of bringing intentions into their learning.

Often the students' attempts to make sense of the educational process are unsuccessful. Yet, there are many indications of the students really wanting to enter a learning process. That is not so difficult to explain. To do well at school is in fact the most prestigious project for young people. That is why it becomes tragic when discussions supposed to make intentions clear are relegated to other categories.

When the students give up locating their own intentions in what is going on in the classroom, they will establish underground intentions. This can be observed from the resigned glances out of the window after their questions and comments have been reinterpreted by the teacher. When underground intentions become dominant, the educational activities of the students turn into mechanical behaviour: they calculate because they have to. In other words, the lack of a possibility to find an intention in their learning appears as a lack of mathematical competence. This can furthermore be reflected in the teacher's assessment of the students: a weak and unenergetic effort in mathematics can be expressed as "This student needs a lot of help" or "This student is weak at calculating", when what is at stake is really that the student has not had the opportunity to see through to the intentions behind the activities in the mathematics class.

In 'How much do newspapers fill', the communication appeared confused, but we can see the openness of the situation as an invitation in practice to bring the students into a discussion of the teaching intentions. Such discussions are all too seldom present in mathematics education.

When a teaching-learning process is structured in an open way, it is important that possibilities are created for teacher and students to negotiate about what might be the idea and

purpose of the process. [12] This we see as an important lesson of analysing this example.

Returning to the thesis

The interest in studying the communication when the educational aim is hidden to the students is closely connected with the thesis about a connection between mathematical cognition and communication. In our description of learning, the participation and 'zooming' engagement of the students make up defining elements. We say that the students must have an 'intention about learning' as a part of their learning. We also talk about learning as a student action, i.e. we presuppose that the students are acting individuals in a learning process. The driving force in the learning must come from the students.

In order to act in a learning process, the students have to have an idea of the aims and intentions of the teaching unit. Furthermore, the students must be able to grasp the idea behind a unit to be able to let themselves be involved in the unit and to become familiar with its aims. All this is opposed to the situation where the students play the role of blind passengers in the classroom. Becoming able to see and grasp the aims of the teaching is intertwined with a communicative process. Factual matters may well be stated, but intentions and aims have to be made clear, and if a student is to be given the chance to grasp the teacher's intentions, negotiations are needed. It is a complicated process for the students to make the aims their own. At the same time, it is a process which presupposes that the aims are negotiable and can be changed.

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Notes

- [1] A first version of this article was published in Danish in *Nomad* 1(2), 1993. This version has been subject to revision.
- [2] The observation material was made by a teacher team in Randers Kommune in Denmark. It consists of three videotapes containing observations from a 3rd and a 7th grade classroom. Bjarne Würtz Andersen and Ane Marie Krogshede Nielsen were the teachers. Jan Boserup taped and edited the video in co-operation with Ib Trankjær who also co-ordinated the project. We were not present at the lessons we describe. The lessons were videotaped as part of a project making examples from classroom practice available to the public. It was not the idea to produce 'perfect' lessons, but rather to provide material which makes it possible for outsiders to discuss what takes place in the classroom: students, teachers, colleagues and parents, as well as researchers. We have had the opportunity to discuss details of the video and our analysis with the persons involved.
- [3] Throughout this article, the word 'fill' has been consistently used (even in places where the most idiomatic English translation might be 'cover' or some other word again) in order to reflect the fact that there is a single, common, Danish word *fylde* which is the central focus of the authors. *Fylde* can be interpreted as if it has to do either with volume or area, and is also used metaphorically in a variety of ways. Mathematics entails, among other things, the specification of language; hence, *fylde* is a good starting point, as its meaning can be specified in significantly different ways.
- [4] MATEMA is a textbook series from 1st to 10th form, prepared and edited by Peter Bollerslev, Vagn Harbo, Viggo Hartz, Peter Olesen, Leif Ørsted Petersen and Ib Trankjær. MATEMA can be seen as a Danish interpretation of some of the intentions which have been promoted internationally by Hans Freudenthal's activity perspective on mathematics education.
- [5] See Skovsmose (1994) for a discussion of 'vantage point' and 'scene-setting'.
- [6] This can, of course, also be understood as joking comments, since they know well enough that it says mathematics on their schedules.
- [7] This sequence is not included in the edited version of the video.
- [8] According to the teacher, Malene is not normally the one who takes the lead in class.
- [9] For a discussion of 'intention' and 'action', see Searle (1983).
- [10] Stieg Mellin-Olsen often used the phrase that the pupils have to 'own' the aims of their education.
- [11] See Alrø and Skovsmose (1996a) for a discussion of the notion of 'bureaucratic absolutism'.
- [12] We have described such a process in terms of an 'inquiry cooperation model' in Alrø and Skovsmose (1996b).

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