

The Alchemy of Mathematical Experience: a Psychoanalysis of Student Writings

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When asked to fantasize about an experience of being intensely involved with a math problem, a woman in my Elementary Functions (pre-calculus algebra) class wrote:

I felt as though I was jumping rope on a razor blade, and with each jump blood trickled onto the blank paper below me

Studying math (and especially solving problems) can be much more than a cognitive experience, as this student's writing shows. Dorothy Buerk [1982, 1985, 1988] and others have elicited similar images of mathematics experience from students of various levels, and have found them useful pedagogically for both teacher and student. What I want to share here in this paper is a more *psychological* look at student images of mathematical learning and problem solving. But this is not *psychological* in the usual sense of cognitive-oriented educational theory, but *psychological* from a *psychoanalytic* orientation, based on the writings of such Jungian analysts as Edward Edinger [1985] and James Hillman [1975].

I see a need in mathematics education for a more holistic type of thinking, such as can be provided by Jungian psychoanalysis. Math as it is often seems so very *rational* that even when teachers speak of intuition it seems a very *rational* type of intuition, especially when compared with the tumultuous inner-workings of many adolescent students.

Trying to learn mathematics, struggling with a math problem—these can push a person into deep psychological waters. Most math teachers are probably familiar with some of this—student writings or classroom/office discussions on topics such as teacher-class relations, feelings about math, questions of the purpose of studying this or that topic, the purpose of studying math at all. Less frequently raised, but very present are ideas and images concerning the deeper meanings of success and failure, life and death, identify and suffering—to name just a few. I believe these explorations, these steps toward self-knowledge, can emerge and should become part of shared mathematical experience.

I have begun to do just that. In the second half of each semester I ask all the students in my classes (at the Armstrong County Campus of Indiana University of Pennsylvania) as an extra credit assignment to think of a recent math problem which challenged them and to find and write about fantasy images which capture the feelings they had. I suggest that they can describe one image alone (an example of which was at the beginning of this paper) or write a sequence of images, like a story or dream. Then I analyze

the material they submit: I try to see through and into the fantasies, looking at the images with an eye for timeless and universal themes. Whenever I sense the students might be interested, I invite them to discuss the fantasies with me and explore the personal meanings. I also encourage them to write more fantasies and continue the process.

The goal of my fantasy work with the students is to assist them in developing a deeper, more imaginative perspective from which to view their mathematics experience, and perhaps their lives. The goal of this paper, then, is to share some of their efforts and to look deeply into them, and thus to show how we as teachers can make a richer and more meaningful activity for our students and ourselves.

I must state clearly that the direction of this image work is not upward: the goal is not to somehow produce better math students, any more than the immediate goal of psychoanalysis is to cure symptoms. It is to look more closely at what we (teachers and students) do, see what is going on, play with images, issues and feelings traditionally ignored, devalued, forgotten. Let's not harness insight and fantasy into the service of growth and learning! For it cannot be done without killing the free spirit of fantasy. Sometimes what we do leads upward to sunny success and growth, resulting in glowing testimonials to the effectiveness of this or that method, textbook, or teacher. At least as often and of no less significance, in the gloom of the night we see, in spite of our achievements, our path leading downwards into dark failure, a sense of galling limitation and insignificance, a life wasted with death coming soon. We should honor both movements, upwards and downward, the forces of Life and Death in all their forms and meanings, in order that we might learn to use our imagination and creativity to the fullest.

Student response to my extra credit fantasy assignments has been good. In Fall and Spring semesters 1990-91, for example, 46 out of 202 students turned in material, and six students did it more than once. Most of the material seems psychologically genuine and shows a high degree of involvement.

Many common themes and images emerge from the material. Particularly intriguing, though, are the many thematic connections with the process of alchemy, as interpreted psychologically. These form the focus of this paper. The alchemists were pioneers in solving problems—their investigations contain a large amount of fantasy of great psychological value. When viewed from the perspective of alchemy the student's writings become luminous examples of alchemical processes.

It should be noted that alchemy is just one of the possible sources from our cultural heritage which we could draw upon to organize and amplify the fantasy symbols and motifs. With a different perspective we would probably get different patterns and somewhat different meanings.

Alchemy as it is to be used here deserves a bit of an introduction, because many people, especially science-oriented people, misunderstand its true purpose. They tend to see alchemy (and by this I mean classical, pre-seventeenth century alchemy) as merely a pseudoscientific ancestor of chemistry, full of baloney and obfuscation. They imagine the alchemists in their laboratories searching vainly and foolishly for the so-called *Philosopher's Stone*, that secret which would transform foul, lowly and degraded materials into pure, noble, shining gold.

Much scholarship by Carl Jung [1953-76] and others has found, however, a deeper and more significant purpose to alchemy—that of seeking to understand the process of psychological transformation, whereby the foul, lowly and degraded parts of the human psyche can be turned into psychological gold, i.e. wisdom. If this is so, then why the laboratories, the equipment—alembics, melting pots, furnaces, chemicals? The question is a good one. Basically, the alchemists projected their psychological states and processes onto matter, so that chemical transformations took on psychological significance. What is relevant here, however, is that we begin to follow in the alchemists' psychological footsteps whenever we seek to solve a mathematical problem—or any type of problem. The foul, lowly and degraded parts of the alchemist's psyche correspond to the ignorance we feel when we can't see which way to go. Our goal is to transform that, to create understanding and ultimately the *golden* solution. How is this possible?

If I were a modern alchemist, and had read James Hillman [1975], I might respond something like this: We enter into a difficult problem through a process of deliteralizing, *seeing through* the problem to the fantasies behind it. We play with the images brought up by the problem. We fantasize special cases, what would happen if we changed one little thing. The point is, we move from a fixed state of staring at the literal problem to a more fluid state of imagining. But I would also say that it is not easy to teach this, to ourselves or to others. A math problem may be small potatoes, but in our psyches it looms larger and larger the more time and energy we invest in it. The problem becomes our Ignorance, our efforts become a Quest for Wisdom and Truth. And it is not easy to induce change here—our psyches cling to the literal, resist seeing things, seemingly preferring to wallow in stupidity. To break down old structures, old ways of seeing things, is painful and leads us into psychopathology. Which explains why the processes of alchemy typically involve such strong themes as burning bodies, drowning, murder, dismemberment, and putrefaction. Students complain often that math problems drive them crazy. Math problems are this way because they so effectively constellate the psychopathologies which lie beneath our rigidities, slumbering in our souls.

Let us now turn to some of the fantasies which were written by my students, as viewed from an alchemical per-

spective. We return to the example given at the beginning of this paper:

I felt as though I was jumping rope on a razor blade, and with each jump blood trickled onto the blank page below me

For this student working on a math problem would seem to involve a playfulness (jumping rope) which is also a wounding. Blood flows, and we can see that blood as her life force, her creativity. I had the idea that the drops of blood take the form of the solution, because this student was very good at math and never turned in blank pages—but we don't know whether she intended this meaning. Pain is not mentioned, interestingly. Perhaps she has learned a perspective, an attitude, with which she accepts pain and is not overwhelmed by it.

Having the right attitude is important in any creative activity—we have to be unconditionally willing to undergo wounding.

In alchemy there is much written about the proper attitude for the work, the *opus*. It is sacred, this *opus*, this search for the secret of transformation, and without a proper attitude success is impossible. What qualities are necessary? Patience, courage, a willingness to live with anxiety. As one alchemist wrote:

Anyone who gives himself up to this search must therefore expect to meet with much vexation of spirit. He will frequently have to change his course in consequence of new discoveries which he makes [...] The devil will do his utmost to frustrate your search by one or the other of the three stumbling blocks, namely, haste, despair, or deception [...] [quoted in Edinger 1985, p 5]

Most students have not found the proper attitude, and there is much suffering. It need not be pointless—suffering can be a good teacher. We can help our students develop a proper attitude by hearing their complaints and helping them find images for their suffering.

When we attempt to learn something difficult, or to solve a problem, what we experience as we begin work on it can be likened to what the alchemists called *prima materia*. By this the alchemists meant the primitive, undeveloped, "primal" condition of matter, worthless and despised yet full of potential. It is an image of the absolutely ordinary, leading down into bottomless depth and darkness. For our use we can perhaps say, by analogy, that the *prima materia* consists of the mathematical concept or problem with all the ideas that come to mind about it, plus all the personal issues and feelings which well up in us. The combination often is chaos, seemingly hopeless confusion and frustration—a despised state, yet full of potential. From the student writings.

When working on one of these problems I feel mass confusion, my mind feels exhausted. My mind is cluttered with thoughts trying anyway possible to come up with an answer.

Also:

I was so frustrated that I saw myself grappling at ropes over a gorge, and everytime I reached the ropes would get further and further away ...

Confusion, frustration and danger too:

Working on a math problem is like a time bomb ticking down to its last seconds and if I don't disconnect the precise wires, me and my whole world will be blown to pieces

Here we have three different images of *prima materia*. The first student struggles with confusion to the point of exhaustion. This student is desperate, will try "any way possible" to get an answer. Creativity is down; perhaps there is even the possibility of a criminal act, maybe a willful violation of the rules of mathematical thought, just to be able to have "an answer."

The second student, grappling at ropes over a gorge, is clearly in danger. There is the possibility of a terrible fall, of being shattered (which in fact happens later on in this fantasy) The large vertical drop emphasizes the huge gap between expectation and personal reality. Furthermore, there is a trickiness about those ropes, in that they move away each time the student reaches for them. It would seem to be a cruel game.

The third piece, about disconnecting the time bomb, pictures math as a matter of great precision, right ways of doing, with death being the outcome of error or indecision. The problem is inherently dangerous, a bomb waiting to go off. One wonders what this student's former math teachers were like. Perhaps, too, the student may be responding in part to me as a teacher: Whether I intend it or not, the images in my soul become part of my teaching. Math problems in graduate school sometimes felt to me like bombs! Sometimes students respond to the *prima materia* by seeing themselves as fierce warriors:

Like a Spanish Conquistador, I turned the page to face yet another battle. With the precision of a surgeon, and the sheerness of his scalpel, I cut through a vast array of problems. Nothing on this page could even slow me down. My confidence was booming, my work was speaking for itself, I could conquer!

Also:

I felt as if the problem had risen off the page and become larger than me. It was like it had taken over and I couldn't go on until I defeated it.

A warrior mentality works for some problems. Certainly we as a society use it frequently. Witness: the War on Poverty, Vietnam, the War on Drugs, Desert Storm. We see a problem, we feel threatened—Boom! we declare war on it!

A severe limitation of the battle metaphor is that the "it" we may rush to battle is too often a literally interpreted "it." We think we see the problem, but we don't really. There are other ways of seeing, other foregrounds and backgrounds, other groupings of causes and effects. The enemy then proves elusive, and the war may end in despair, as in Vietnam.

Some students, when working on a math problem, find themselves going deeply into their own psyches, re-experiencing painful childhood memories and/or exploring deeply personal inner worlds. A math problem can thus begin a journey into Soul.

A woman in her late 20's describes how working on a math problem put her in touch with memories:

When I'm concentrating on a tough math problem and run

into difficulty solving the problem, I frequently feel deeply frustrated. I tend to let my mind wander and instantly I find myself floating back in time, when I was a young child suffering the same math frustrations... [I am] inside my third grade classroom, where I'm being tutored by Mrs. Reed, the remedial math expert. I can still clearly see Mrs. Reed's face, as she sternly scolds me for being unable to solve the math problem in front of me. Her face is mean and distorted, like a wicked witch who continually barks commands angrily at me because I just don't understand the math problem. I huddle down further in my seat and fidget restlessly as her mounting rage continues. I feel deep terror and a suffocating fear... I wish the numbers would disappear. I wish there was no such thing as math.

Another student found math problems took her down into her depths, an underworld:

The world was full of people who were sick with incurable diseases, people that kept on the run from the law, they were in all sorts of trouble. I felt so sorry for these people. I wanted to help them. I tried to tell them I was sorry I got so upset about a math problem, but they said it's too late—you are here, you have to watch us and wonder how to help us.

By working on a math problem this student discovers the dark side of herself, a world of sick and criminal inner figures that represent undeveloped, repressed sides of herself. This is *prima materia* of an unusually strong order, for a math problem and for a college freshman. In fact, the whole alchemical problem is articulated extremely well: How does one cure the incurable, redeem the unredeemable? Or, in other words, if cure and redemption and solving problems are upward movements, then how is transformation possible? The stage is set for a Quest: This could be the problem of her life.

According to alchemy, after the *prima materia* has been obtained (not necessarily an easy task, and one which can mirror the entire alchemical *opus*) a series of procedures or processes has to be undertaken to transform it. There is some agreement among authors as to the identity of the processes, but as to the overall course of the operation, the ordering of the process, there is little agreement. All the procedures overlap and interconnect in various symbolic ways—not surprising if we see them from a psychological perspective.

We will now look at some student fantasies which contain images of alchemical processes. We begin with *calcinatio*. (I will follow Edinger in using the Latin terms for these processes to emphasize that they are *psychological*, not chemical, processes.) This is the intense heating or burning of the *prima materia* in order to drive off water and impurities.

A variety of *calcinatio* images occur in the fantasies. One example is from an actual dream by a Probability & Statistics student:

[I dreamed that]... all the frustrated people on the Armstrong County Campus of IUP were having a ceremonial book burning. The top books in the pile were: *General statistics*, *Applied calculus*, *Precalculus functions*, *Basic algebra*,... They were burning books like there was no tomorrow. The smoke was a thick green, blue, and red mixture. The smoke was so intoxicating that I could hardly breathe, and couldn't see.

The *prima materia* to be cooked is here the student's relationship to math, symbolized by his textbook, the book on the top of the pile. The other math books, and the fact that there was a mob doing the burning, emphasize the frustration and rage felt by this student toward math in general.

As stated above, the goal of *calcinatio* is to burn off impurities. Here one way to see the multicolored smoke is that it contains the "impurities" (emotional conflicts, projections) in this student's relationship to math.

The fire in *calcinatio* can be seen as coming from frustrated desire, which in broadest terms is the internal blockage resulting from strongly wanting something and not being able to get it. This student had a strong desire for success, a hunger for good grades, which, however, was not paired with a willingness to work for them. He did little homework, missed important lectures, had a hard time admitting he needed extra help. Hence he did badly on tests and became severely frustrated.

The dream tells us that this student's frustration may be of value: it provides the heat for the *calcinatio*, and in this there is the promise of transformation. I believe it is important to help such a student see the value of "staying with" the frustration, whatever other advice we give him or her. It is an image of incubation.

On another level of meaning it is possible to see the burning as releasing some of the spirit hidden in the books, with the three different colors suggesting three different dimensions of this spirit. Interestingly, in alchemy there are four basic colors: red, symbolizing emotion; blue, symbolizing thought; green, symbolizing sensation; and yellow, symbolizing intuition. We note the absence of the fourth color, yellow, in the smoke from the burning books. We might tentatively suggest that the *calcinatio* was perhaps failing to release the "yellow smoke", his intuition. Perhaps with more "cooking" this too may sometime come.

To focus on the smoke as being of value instead of the residue takes us out of *calcinatio* and into another alchemical process, known as *sublimatio*. *Sublimatio* is an ascending process, a liberation of spirit, in which a broader, more objective viewpoint becomes possible. We rise up above our entanglements and see things clearly.

Another alchemical process is known as *olutio*, the dissolving of a partially-developed substance. There's an interesting word play here: a student seeks a "solution" to a math problem, but experiencing *olutio* may not seem at all linked to the solution of the problem. Instead it may seem to be an anguished abandonment of effort. What is happening is that there is a reductive loosening of structure. Of course in the long run this may be just what is needed to see the problem differently.

In alchemy *olutio* is often pictured as being performed on a king. Psychologically, the king refers to our conscious will, although it gains additional meaning if we think of him as also our conceptual approach to a problem—the way we see it, define it, carry it, limit it. The drowning of the king refers to the fruitlessness of our efforts. We are defeated, we dissolve back into the *prima materia*, the primal waters where we began.

It is also a theme of alchemy that from the waters the king may be born again—we start over on the problem. This is *olutio* less as an image of drowning than of baptism, a cleansing experience.

A student had the following fantasy about taking an Elementary Functions exam, which combines images of *calcinatio* and *olutio*.

I felt as though I knew everything, then the exam came. Everything changed, my mind went blank. I became real tense, the blood rushed to my head like it was a volcano getting ready to erupt. I suddenly became real hot. I felt like I was on fire, the smoke then began to pour out of my head like it was a chimney. The sweat began to flow out of me like I was a water faucet turned on full blast. Within minutes I was drenched and I felt as though I could swim in my pool of sweat, like you swim in the pool at the YMCA. Then things changed, I started to panic, the water was getting too deep. I couldn't move, I felt as though I was chained to the chair, the water kept rising, it wouldn't stop. It was up to my neck and getting higher and higher, faster and faster. The water was now over my head and I couldn't breathe. It was like someone was smothering me. I couldn't do anything about it, no matter how hard I tried there was nothing I could do. Finally it was all over, I was at peace again. I thought I knew everything once more, but little did I really know because I was dead.

This fantasy begins with a cascade of images, including a volcano (recall the time bomb fantasy?) and a *calcinatio*. Three things seem to stand out here: (1) Here the student *himself* is the bomb in the form of a volcano—not the math problem, he *himself* is on fire and not a textbook. Psychologically this difference suggests that this student cannot separate from his feelings. (2) The student drowns in his own sweat. The sweat is his excessive amount of feeling and mis-directed effort. He is trying too hard, huffing and puffing instead of thinking. His ego is inflated, swollen, just like the imagery at the beginning of the fantasy. (3) The sense of peace the student feels at the end is interesting. Death is sometimes seen as a blissful *olutio*. It is highly significant that he says he "knows everything once more." In the surrender there is the hint of a return to Self, perhaps a rebirth. A *olutio* experience can be a baptism, a cleansing of the world (flood symbolism). The authentic parts of the psyche survive the flood.

Since this fantasy dealt with death, let us now look at the stage of alchemy known as *mortificatio*. Here there is darkness, defeat, the anguish of dying. Usually *mortificatio* reduces a substance back to the *prima materia*, but sometimes the *prima materia* itself is reduced by *mortificatio* back to a simpler, more elemental form. For us, we can see *mortificatio* as the death of our effort to solve a problem or understand a concept. Such a defeat for the ego, however, may be necessary in the long run. *Mortificatio* can be the eventual outcome of any other stage of alchemy, although it doesn't have to be. For example, in the previous example we saw a *olutio* that apparently ended in death. As we saw, it is possible to see it instead as more of a baptism. Many fantasies and dreams contain the threat of death, but the crisis is resolved without the ego, the "I", dying. Other times this extreme image seems to be necessary.

For an example of *mortificatio* we rejoin the fantasy of the student who we left grappling at ropes over a gorge:

... I continued on and tried different solutions and as I tried the last possible one, not solving the problem, I fell through the air and felt my body shatter as I hit the boulders and water beneath me.

His body shatters. His body is his ego, his self concept, his investment in the problem. Note the rigidity and even inflation in his statement that he has tried the *last possible* method of solution. Is there ever a *last possible* method of solution? Rigidity invites shattering, inflation invites deflation: by overstating the case, by acting too high and mighty when he probably felt the opposite, he insures his fall into failure and ruin.

One way to help students who are caught in a death fantasy is to encourage them to accept the experience in an imaginal, nonliteral way. Help them to see that *something* needs to be transformed—but that the ego doesn't need to identify with it, to literally "die".

In alchemy *mortificatio* is sometimes pictured as a king being murdered. As before, the king represents one's conscious will, the ego. The game of chess is built around the idea of *shahmat*, death to the shah (checkmate, to us).

In a few of the student fantasies there is an explicit separation between the ego and the object of *mortificatio*. Consider this example:

When I have trouble with a math problem I first skip it. When I go back to it later if I still can't figure it out I get really frustrated. At this point is where my fantasy begins. I just picture myself kicking my math book out my front door onto Rt. 85 and just watching the Mack trucks running over it. After imagining my book in shreds I feel a little less tense and I try it again.

The *problem* undergoes *mortificatio* here—it had become inflated, carrying too much meaning: it had turned into a monster. Deflation follows: The down-to-earth no-nonsense trucks of everyday life kill the monster, cut the problem back down to size. There is a rebirth of effort. This seems to me a very healthy-minded fantasy.

A good number of the student fantasies contain a resolution of the original dilemma: The problem was finally solved. Consider:

The final try at the problem brought me out of the clouds onto a fixed place and the equation with the correct solution was in my grasp.

This motif of coming "out of the clouds onto a fixed place" was used by several students to represent finding a solution. The confusion and lack of clear vision of the "clouds" yields to something hard, fixed, substantial.

The clouds can be said to represent the *prima materia*, and the alchemical process which turns it into substance is called *coagulatio*. It is a creation experience; from clouds of gas are born stars and worlds.

It is interesting, then, that the words "finding a solution" refer to *solutio* and not *coagulatio*. Perhaps finding a solution to a math problem is a coagulating of intellectual efforts but at the same time a dissolving of emotional energies and tensions. As the concepts come together and solidify, we relax free again.

Only in the easiest problems would *coagulatio* seem to represent final solutions, by the way. To achieve *coagulatio* a problem must be taken literally on some level, its meaning fixed. This limits it, binds it in time. The solution today may not hold tomorrow—either a flaw will be discovered or an extension will suggest itself. *Solutio*, *mortificatio* or another process may follow.

There was one student's fantasy which was most remarkable in the depth of its imagery relating to the discovery of a solution. This remedial algebra student had taken a quiz which contained three factoring problems. He came to my office a few weeks later to turn in a fantasy writing. He was excited, "the fantasy had written itself," he said. He explained that it had just flowed out of him, and claimed nothing like this had ever happened to him before. Here is the final part of it:

I felt like I was staring down a well, looking for something, but not quite sure what that something was. My mind was playing tricks on me. It was making me more and more confused. As I stared down in that well, I could see everything, except for the very bottom, which was covered by a white cloud. The well was about 40 feet deep, built out of big stones. It looked like it was built sometime in the early 1900's. As I stared at the bottom of the well, I became more and more curious at what could be laying at the bottom of it. I knew that the only way I could find out what was laying at the bottom of the well was to crawl down the side of it and check it out. My mind was struggling harder than ever before, when finally three Indians appeared around me. The Indians knew I wanted to find out what was at the bottom of the well, and they grabbed my hands and made a circle. They started to softly whisper a prayer of some sort. I'm not quite sure what they were saying, but somehow I knew what they were saying while I joined in on the prayer. Instantly, out of nowhere, I could see what was at the bottom of the well, and the answers to the three factoring problems came to me. I couldn't believe it. The answers just came to me. I really felt strange.

Several themes stand out here. (1) There is the symbol of the well. Where symbolic underground waters come to the surface—whether natural spring or man-made well—can be a place of miracles, of transformation. There is a vertical drop here, as with the fantasy of grappling at ropes over a gorge. But psychologically they are very different images. That was of heights, this is of depth. (2) The three "Indians" are interesting. Aside from what they symbolize in themselves for this student (which we don't know), the fact that there are *three* of them stresses the connection with fate and destiny (according to Jungian psychology). There is also the implicit message that the ego alone cannot do the job—help is needed. (3) As to the connection between the vanishing of the cloud and the answers—there are two ways to see this. One is an image of the alchemical process *coagulatio*. Generally *coagulatio* is a downward movement, as the insubstantial cloud descends into materiality. In this fantasy, however, there is a twist—the cloud is at the bottom of well, below the student. He thinks at first he will have to *descend* to find out what is lying below it. But thanks to the "Indians" the cloud vanishes and he can now see *downward below* the cloud.

