Peer Teaching in Mathematics Classrooms: a Case Study

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As teachers we often make a distinction between superficial learning and deep or meaningful learning, as it is usually called. [1] Motivated by various teaching incidents, such as the low performance of students in various comprehensive tests or theory application tests, we usually say to them that it is one thing to simply read something and another to understand it in depth

Most of the students in the higher grades, owing to lack of time, or for their convenience, simply read the textbook as they read a novel without having an active attitude toward the mathematical content So although they probably find out that the reasoning steps are correct, they usually don't realize the motives and the purpose of these various steps.

The phenomenon is corroborated by the fact that many times the textbooks' exposition is excellent in showing each particular point, but fails to show the main line of the argument. But learning with understanding demands not only the knowledge of "how" but mainly of "why". [Polya, 1973: p 207] Also the overanalytic models so often presented to children in their textbooks on maths are often a far cry from the messy weekday activity of the real-life mathematician and don't help the children to understand how the "authority" arrived at the end result

If we accept the opinion that mathematics "is not something that one knows about but is, rather, something one knows how to" [Bruner, 1974: p. 124], we can easily perceive why the reading of books without active participation leads to surface learning

Teachers are obliged to warn their students about the danger of superficial learning which comes from their passive attitude toward the various learning sources. Learning by passive listening or observing creates false impressions and hardly has any value. Learning with understanding, meaningful learning, demands active participation and critical thinking. So the teacher shouldn't miss the chance to prove this truth, thereby creating the proper conditions in the classroom.

Students are not educated just by words or instructions but through the development of practical attitudes [2] and the creation of a strong self-concept and self-confidence. If we accept this view, then it is the mathematics teacher's task to invent the proper ways each time which will help the students to realize the features of effective learning

Such a powerful learning tool is, for instance, students' writing in mathematics where they write not to show to the teacher what they have learned and to receive a grade but to develop valuable learning experiences for themselves. This writing can give them an opportunity to clarify their understanding, or organize their ideas and notions about a topic, since when we put down ideas on paper we are faced with them and are then obliged to recognize their eventual shortcomings. [Stempien & Borasi, 1985]

In this article we aim to present another learning tool that can help students, under certain conditions, understand the difference between surface and meaningful learning and, at the same time, be useful to the teacher as a fruitful feedback resource on the teaching act. This is "peer teaching" By this term we mean the presentation of a topic by the student in the classroom who in this case takes over the role of the teacher

The value of peer teaching in mathematics pedagogy

The elaboration of a topic by the student himself and then the presentation of this topic in the classroom may, under proper conditions, prove to be a very useful tool in the teaching process for both students and teacher

Advantages for the students

(1) During the elaboration stage and while dealing with the topic, the student is given a chance to investigate, study and comprehend in depth a mathematics unit by himself. This is an excellent motive that gives the student the opportunity to be initiated into the auto-learning process which is so essential for success in mathematics Mathematical knowledge, which has increased at an enormous rate, is available in applicable materialized form, in books, journals and machines. Hence the competence of people in the flexible handling of this knowledge as well as the reinforcement of the responsibility of the individual for the organization of this learning are important [Fischer, 1988]

(2) During the elaboration stage too, the critical attitude of the student towards the mathematical content is developed. The student is obliged to clarify the obscure points of the content by himself. This obligation reinforces his participatory attitude towards the learning procedure

(3) The difficulties a student faces in the presentation of the topic helps him to realize the several study levels of a mathematical text and finally the difference between surface-memorizing or rote learning and deep-meaningful-learning.

(4) Entrusting the organizing and presenting of any given subject contributes to the development of responsibility, self-confidence, self-control and self-concept. These are some of the typical features which characterize mathematically capable pupils [Aiken, 1973; Sherman, 1979]

(5) The experiences which a student will gain from the

peer teaching process will help him to develop a more positive attitude towards mathematics and the learning process in general

Advantages for the teacher

(1) Through peer teaching the teacher is given the opportunity to observe the positive and negative points of a teaching process and work up stimuli which will function as a sort of feedback for his future work

(2) Watching his students as they try to present some mathematical topic in the classroom, he discovers the communication problems created during the teaching process These problems may have been forgotten in the daily teaching routine Their realization resounds positively in the accomplishment of this task and contributes to the improvement of teacher-student communication.

(3) Peer teaching offers a chance for the development of better teacher-student relations in general During their mini cooperation, which is preceded by the presentation of a mathematical subject, a friendly climate is generated which helps them to get to know each other.

(4) With peer teaching the teacher is given a chance to break the everyday monotonous teaching routine. The whole process creates a pleasant atmosphere in the classroom and revives the interest of the pupils for the subject.

The peer teaching project (P.T.P.)

In the following we will try to describe such a program which we applied in grade B of the comprehensive senior secondary school of Patras in Greece (16-17 years old pupils), during the school years 1985-86, 1986-87, 1987-88 This grade is suitable for the following reasons: (a) These pupils have acquired a mathematical maturity from their earlier years and have been well initiated in mathematical reasoning [3] (b) In this grade students are not busy with general entrance examinations for colleges or universities This fact favours the cultivation of an atmosphere in which the students will have the opportunity to develop interests and be occupied with activities beyond the formal curriculum [4]

26 pupils participated in this project (17 boys and 9 girls) taking the teacher's place out of 197 pupils divided into 8 classes At present the program is repeated in the same grade by other colleagues who had attended some of these peer teaching sessions. The mathematical units that the students presented were the following:

ALGEBRA
Unit 1: Sum of terms of arithmetic progression; applications
Unit 2: Sum of terms of geometric progression; applications
Unit 3: Properties of logarithmic function; applications
GEOMETRY
Unit 1: Ratio of areas of triangles (theorems and corollaries)
Unit 2: Metrical relationships in regular polygons

All the units were selected according to the order in which they are presented in the textbooks Basic criteria for their selection were: their simplicity, the exactness of the content, and the clear way in which they are presented in the textbook. In the following we will describe the basic ingredients of this project.

Organization of the project

1) During the last term when the students are more familiar with the mathematical subject, the teacher proposes that one of the next lessons be presented by a student. In the discussion that follows the teacher explains the benefits which everybody may gain from peer teaching. Also in order to motivate their interest he explains that those who decide to participate in this program should try to present the lesson avoiding the drawbacks and the weak points of their teacher They should also take into consideration their needs as students and apply a methodology which would, in their opinion, satisfy these needs as well: a methodology which they would like the teacher to apply in the classroom. In the beginning the students, of course, are reluctant but once the start is made the involvement is great.

2) The students who participate in the P.T.P as teachers are not chosen in advance; each time a peer teaching lesson is finished and after the criticism has taken place, as we shall see below, some students willingly ask to undertake the next peer teaching task. The prospect that they will do better than their schoolmates did is, it has proved, a strong motive for participation in this program.

3) Peer teaching is not adopted in successive lessons. This is necessary because after the student's presentation the teacher has to go over and correct any mistakes or emphasize other points during the next hour so that there isn't any danger of misunderstanding the topic that the student presented.

4) The student prepares for the subject that he will present for about a week and, in between, he consults the teacher In this cooperation the student's questions are solved and the difficult points of the mathematical unit are analyzed. Generally the student consults the teacher about the problems that the development of the unit presents.

5) In the scheduled teaching hour the student presents the subject to his peers. During the presentation, the teacher stays at the back of the classroom without participating or intervening in the teaching process. The only thing that he does is to take notes about the teaching, to be ready for the next, the evaluation stage.

Evaluation

The evaluation process comprises three levels.

(1) *Criticism by the students exposed to the teaching*

For the students' convenience, the teacher at the end of the teaching hour gives them a response-sheet on which they are asked to note their observations by answering five simple questions.

Response-sheet A (for the student)

1) To what extent had your schoolmate mastered the teaching unit?
2) Were the difficult points in the unit well illustrated?
3) Was there enough clarity in the exposition?
4) Did the rest of the class participate in the learning process during the teaching?
5) Write down your observations and impressions of this peer teaching

2) *Self-assessment by the student-teacher*
At the end of the teaching hour the teacher gave the student-teacher a response-sheet for the student to write down his impressions about teaching, replying to two questions.

Response-sheet B (for the student-teacher)

1) Write down the most important impressions you had from this teaching experience
2) Write down any other comments you may have on the peer teaching project.

3) *Teacher's criticism*
The teacher devotes the following teaching hour to criticizing the previous teaching and explaining certain points which in his opinion need greater emphasis. The teacher expresses his opinion about the peer teaching according to the notes he has taken, and he introduces a debate in the classroom. The students participate by expressing orally what they have noted in the response-sheets A and B so that the whole class benefits

Analysis of results

The students' answers on both response-sheets were open-ended. Each could give his own characterization and formulate his answer as he liked. The only demand was that the answers should be as clear as possible, exact, and brief.

Students' answers on response-sheet A

1) To the first question about 70% of the students replied that although the student-teacher knew the mathematical subject, he could not transmit it to the others. 10% replied that the student-teacher had mastered the subject sufficiently.
2) To the second question about 64% of the students replied that they would like their schoolmate-teacher to emphasize more some difficult points that they had raised. 23% replied that they were satisfied by the teaching and the rest (13%) that they were not.
3) To the third question, about 82% of the students replied that their schoolmate-teacher was not clear and exact and could not express with clarity what he had in mind. 12% replied that their schoolmate-teacher was good enough and about 6% that they were not satisfied. The large percentage of the student-teachers who didn't present the subject clearly corroborates empirical studies according to which clarity is one of the most important factors characterizing the effective teacher [Rosenshine & Furst, 1971] It is natural that the inexperienced students were inferior at this point.
4) In the fourth question about 79% of the students replied that they didn't participate at all in the learning process and that they merely watched their schoolmate teacher who tried to explain to them the new concepts with a lecture. The remaining 21% replied that their schoolmate-teacher tried some times to inspire them by asking them some questions but without any great success.
5) Maybe the most interesting answers came from the fifth part of the response-sheet, where they made comments and expressed their impressions about P.T.P. Below we will quote the most representative answers, which appeared in order of frequency. Each time we present the best formulated answer (according to our opinion) which represents a class of answers with the same content [5].

> "I was impressed with the fact that an excellent student can be such a bad teacher. I thought that in order for one to teach something all he had to do was to know his subject well. I can't believe how such a good student as Stathis, who had prepared for this lesson for about a week, faced so many problems while teaching. Finally there must be a vast difference between knowing mathematics for yourself and knowing mathematics in order to teach someone else. All the years I have studied mathematics I never thought about something like that. I must show greater understanding of my teacher." (Nikos)

This answer shows the false impression that students have about a good teacher. The image of the good teacher for them coincides with the image of a teacher who ignores entirely the pedagogical dimensions of the teaching act. The training of prospective mathematics teachers is based on this very view, since scientific qualifications are considered more important than their pedagogical education [Krygowska, 1979; Otte, 1979]. The realization on the part of the learners of the multidimensional character of the teacher's personality reinforces the students' regard and respect for the teacher and contributes to the creation of a climate of mutual understanding and mental communication in the classroom.

> " . I observed that as long as no questions were asked, everything was fine. Problems started when some student asked something related to the subject but not directly based on it. The moment, e.g., that Kostas asked Demetris [teacher-student] if a certain formula was valid as well, Demetris lost control and almost abandoned the teaching. At that moment I realized that claiming you know something well is abstract. There must be many degrees of learning. I wonder if I am using the correct process. I read the theory, learn some rules by heart, I do some exercises and that is all. Do I really understand mathematics in depth? I am afraid not, I am just simply accepting it." (Maria)

In this quotation one can see the students' doubt about what mathematics learning means. The inability of their schoolmate teachers (who had studied the subject very well) to answer spontaneous questions during the peer teaching is a very good challenge for them, as it was proved, in order to realize the existence of several learning levels and the basic difference between surface and deep learning. This ascertainment may help in a future change of the students' attitude towards mathematics learning and in a possible modification of their behaviour as autonomous learners.

> "Many of the things that Alexia told us were very easy. I will also be able to present them without special preparation. I think that she confused the proof of the sentence that the sum of the terms of an arithmetic progression which are equidistant from the extreme terms is constant. So I did not understand it as she described it. At home I found an easier proof, different from that of the textbook. [The proof is presented] .. That is how I will write it if it comes up in the exam . ." (Pavlos)

Here indirectly, but clearly, the problem of the students'

involvement in the teaching process is pointed out. The student likes to participate in the conquest of knowledge utilizing his own potential and his own particularities. He prefers to discover something by himself, in his own style, instead of being made to listen and accept someone else's reasoning. Being made to follow someone else's thought creates a sort of intellectual compulsion and enslavement; this is against contemporary pedagogy, which suggests that the teacher's purpose is to help the students become "knowledge-makers" [Lovell, 1978]. Pavlos chose his own proof, therefore corroborating the common teaching principle "never do for the student what a student can do himself"

> " During the first ten minutes I couldn't understand what Stavros was doing He drew a circle, a regular polygon inscribed in it, its apothem, and after that he applied the pythagorean theorem on a right-angled triangle During all of this time I tried to guess what he had in mind His presentation was very boring and it seemed as if it came out of the blue. Only after he had proved the formula which connects l_ν, a_ν and R did I understand what was going on. It would have been better if he had told us his purpose from the beginning " (Sofia)

Many students like Sofia pointed out the issue of knowing the learning objectives in advance. First of all the teacher has to challenge students and to give some sense of the goals that he wants to present. Then all together they must decide how they will reach the target To know the goal in advance is the first step in the participating learning process [Bouvier, 1985]

> " George entirely ignored the fact that we were hearing these things for the first time and therefore we needed some time to reflect on it He should have gone more slowly. I can't understand why he was in such a hurry If he was a real teacher it wouldn't bother me. They do not usually understand our needs. But since he is a student and he knows the problems that we face with mathematics, why did he behave like a real teacher? If I take on such a peer-teaching task the first thing that I will take care of is to speak slowly and ask my students constantly if they understand " (Natasa)

As one can easily understand, the problem of fast pace emerges mainly in expository teaching. Most student-teachers, being inexperienced, taught with the lecture form, as we noticed above Therefore they taught what they had studied without taking in consideration the intellectual level of their schoolmates' comprehension of mathematical concepts

We would like to state that when Natasa became a student-teacher, she kept her promise: she was one of the few students who was interested in her schoolmates' participation in the learning process Can one assume from this incident that teaching is in greater part an art which can't be taught and therefore some teachers are more gifted, i.e artistic, than others? [6]

Self-criticism of the student-teachers

1) Most student-teachers (85%) considered that they benefited mainly from the preparation stage of the teaching unit which they were to present For the first time they had a unique chance to organize and initiate by themselves in the auto-learning experience This responsibility functioned as a strong motive for personal study and as an effort in understanding mathematical concepts without the teacher's help This was perhaps their first effort to obtain independence and self-power in the learning conquest via auto-instruction and individualized learning.

> "It never crossed my mind that I could study new mathematical concepts by myself and even to understand them well enough to present them to my schoolmates When I began to study, I realized that it isn't so difficult I understood most of them immediately. The difficult part was to repeat the proofs. Then I realized that I needed a different kind of study, one in black and white." (John)

> "When I studied the proof in the textbook I understood it almost immediately. But within a few days I forgot the reasoning steps in the book. Then I realized that I had to study more carefully I forgot the steps because I had not thought of them by myself Now I understood how right you were [meaning the teacher] when you told us: "In mathematics, a path is meaningless if one has not made it by oneself", or "Knowing mathematics means knowing how to do it" . You can't read mathematics as you read a newspaper." (Demetris)

> "I observed that many of the points that are characterized in the book as obvious are not obvious at all Because I feared that some schoolmates might ask me a question and put me on the spot, I was obliged to think out myself how they are explained. I confess that I found difficulties and in one case I needed your help [meaning the teacher's help]. I realized that subject matter understanding for teaching is different from understanding it for yourself' (Spiros)

Another issue mentioned by student-teachers in their self-criticism was the change of their attitudes towards mathematics Many of them mentioned that through the peer teaching process they reinforced their self-concept and self-confidence regarding their mathematical abilities and their possibility of conquering mathematical knowledge

What gave them more security and self-confidence was the corroboration of their ability for auto-learning and understanding the mathematical concepts by presenting them to their schoolmates This functioned as a sort of self-rewarding of their mathematical abilities since an indispensable priority for one to become a teacher is to "know" mathematics.

> ". Mathematics is the most difficult subject. I never felt secure with it. There are pitfalls everywhere. The unexpected always exists. There will always be something that you do not know. You can never be sure about something The peer teaching experience helped me to see things more clearly. The fact that I managed to take the place of the teacher reinforced my self-confidence and raised my spirits. Mathematics demands a correct approach. It is not terrible ." (Angela)

> "I find it important that I could teach despite all my errors . It is not such a terrible thing to be a mathematics teacher after all This experience helped me to gain a more realistic view about this subject. I am not sure, of course, but I think I can handle mathematics from now on All I need is to pay a little attention." (Alekos)

> " I think that most of us have misunderstood mathematics: it helps us to hide behind excuses . . This experience makes me believe that if I take it seriously I will succeed " (Andreas)

The third important issue that puzzled the student-teachers was the conception of the teacher's role in the learning process. The unique experience they had in taking the teacher's place, even for only fifty minutes, gave them the possibility of evaluating by themselves the difficulties that everyday teaching presents with its unexpected events and liveliness. Some of them referred to the communication problems between students and teachers and tried to analyze the causes that don't permit the teacher to understand his student's perceptive abilities and so teach them in a language adapted to their intellectual level.

The realization of these teaching problems helps in the student's change of attitude towards the teacher since it makes the teaching act more down to earth. The distance which separates the two basic factors of the teaching process is shortened and the teacher's image of authority begins to fade in the student's conscience. This is one of the most important conditions for the future improvement of the mathematics teaching process.

"... When I was preparing to present the properties of the logarithmic function I found it very difficult to understand why a to the power $\log_a x$ equals x (although in reality it is very simple) So I thought that my schoolmates would have the same problem as I did and so, during the teaching period, I persisted with it very much and I think that all of them understood. But if I was a real teacher everything would seem easy to me. I would not suspect that this particular point would puzzle my students. So I wouldn't give it any special attention and I would overlook it. That is why our teachers don't understand us sometimes. They don't do it intentionally" (Gerasimos)

"... The textbook in order to prove the formula which connects l_ν, α_ν and R in regular polygons doesn't mention anywhere how one can make a regular polygon with ν sides inscribed in a given circle So when Vasilis suddenly asked me how can I be sure that such a polygon can be made, I got confused. This question almost blew up the teaching process before I even started. If the teacher hadn't helped out I wouldn't have been able to continue. These are the unexpected The teacher must know much more than what he has to teach. Now I understand much better what it means to be a teacher." (Nikos)

"... If I ever become a teacher I will make it a habit to ask my students every moment: Did you understand it? Did you understand it? Did you understand it? I will never forget that I was a student once." (Kostas)

This last quotation proves how the student through the peer teaching process begins to realize the basic teaching principle, that understanding mathematics in order to teach it means being able to think pedagogically about the subject, and that teachers must view mathematics through the eyes of their pupils [Dewey, 1916/1964]

2) The student-teacher's comments regarding P.T.P. were complimentary and expressed an instinctive enthusiasm for this rare experience. Even those who did not do well and were rated badly, felt enthusiastic with the whole P.T.P. process. They managed to discern the positive points and gained from the useful experience. Also in some cases they also suggested their own ideas for its improvement, or they expressed some other views which could revive mathematics teaching

> "I think that the student-teacher should be given two chances to teach I believe that the second time he will do better. I, for example, would correct many of my mistakes the second time ..." (Vagelis)
>
> I know that I do not have a mathematical mind. It has never appealed to me. I cannot find a motive to deal with mathematics ... In the beginning I thought of participating as a teacher just for the fun of it But now I see things from a different point of view ... I think that the only thing I will remember about mathematics after years will be this experience" (Thanasis)
>
> "... Why should the mathematics always be carried out in a conventional way? (Some exercises are solved on the blackboard, some theorems are proved, some applications are done, and the daily task is completed) We can find some ways to make it more interesting for all of us Peer-teaching is one of these ways. But there are others too, as for example: a) Have the class analyze a topic b) Write essays about those who discovered the various theorems in order to understand their reasoning c) Divide the class in groups and have each group work on a mathematical topic which will be presented to the rest of the class at the end of the school year d) Publish a small magazine where we will present our discoveries, etc." (Spiros)
>
> "... After this experience I believe it is most beneficial for us to prepare the next lesson at home each time so as to spot its difficult points So when the teacher presents the lesson in the classroom we will pay more attention to these points and understand them better In the beginning, of course, we will come across difficulties and we will have many questions But later I believe that we will get used to it and eventually this process will seem easier So we will learn to study by ourselves ... We were brought up to believe that we can't learn anything without a teacher's help So, sometimes when we get sick and we try to study by ourselves it seems impossible But it isn't possible to have a teacher with us for ever." (George)

In the last quotation the student indirectly refers to the issue of autonomous learning through self-instruction In learning mathematics all the active aspects of using mathematics as a tool are of paramount importance. "They who have acquired competence in the technique of reading mathematics gain a powerful means therein for widening the scope of their mathematical understanding ... Developing in students the habit of learning from books or from oral exposition is an important component of mathematical education." [Servais & Varga, 1971, p. 28]

Concluding remarks

The P.T.P. was an informal procedure designed as a mechanism for the realization of some important aspects of learning and teaching of mathematics in the classroom for the benefit of those participating. It was a mechanism aiming at reviving the teacher's pedagogical background as well as stimulating the students' interest in mathematics, the improvement of student-teacher communication, and finally the facilitation of students' reflection on meaningful learning We believe that the accomplishment of the aims of this

program and the enthusiasm with which students welcomed it, gives us the right to recommend it to other colleagues as a powerful pedagogical tool and a technique which enriches the mathematics teaching process with the motives it offers

Also the simplicity and the practical application of this program makes it a very useful pedagogical tool. From our experience as mathematics teachers we know that despite the great number of research reports appearing in special journals, the mathematics teacher remains helpless in his daily struggle with the problems which appear in a common mathematics classroom [7] What he needs are the simple, realistic ideas springing from the classroom; the specific, simple techniques and procedures which can be used without special instruments or special preparations and sophisticated plans Long experience has taught us to distinguish the tangible and realistic But realistic is what stems from raw, daily, school routine

P T P relies on the live act of teaching and the experiences of the daily adventure which is called mathematics teaching in the classroom Although in the beginning it started as a means for the students to understand the difference between surface and meaningful learning, its application in practice revealed a more powerful procedure which covered a broad spectrum of important aspects of mathematics teaching.

The criticism and self-criticism of students gave rise to serious issues which concerned all four basic teaching factors: the content, the teacher, the student, and the mathematics learning phenomenon. [8] We believe that this idea could be valuable for longitudinal research, with the systematic planning, recording and analysis of activities which would be based on peer teaching.

Notes

[1] This characterization is used in order to show the distinction between deep and surface approaches to learning [N Entwistle, 1986: ch 10] From the late 1950s conceptual approaches to mathematics education made increasing inroads on the former traditional mechanical approaches to learning. This teaching movement coincided with the period of curriculum reevaluation of the 1960s and supported the "modern" mathematics reform, developing new teaching methods to make mathematics learning "meaningful" [Resnick & Ford, 1984: ch 5]

[2] Methods of change or modification of "attitudes" as well as their learning conditions are complicated issues for which more research is needed For the value the possibilities, and learning conditions, of "attitudes" see McGruire [1969], Triandis [1971]

[3] These students have been taught mathematics as a compulsory subject during the past four years of secondary education, for an average of 4½ hours per week.

[4] The negative role of examinations in the teaching process has been repeatedly indicated We can say that the examination system exerts a major influence upon the mathematics curriculum and effectively controls it. It is often asserted that this system is more suitable for "sieving" and ordering pupils than evaluating their abilities as creative mathematicians About the influence of examinations on mathematics education and how they destroy creative mathematics teaching, see Christiansen, Howson and Otte [1986: p. 19].

[5] The students' answers on the response-sheets A and B are in Greek The English translations are ours.

[6] For a criticism of this view see Cooney [1976]

[7] It is common knowledge that those occupied with mathematics education at all levels (administrators, curriculum developers, teachers, etc) complain about the poor guidance that research gives to their decisions The reality of mathematics instruction, and mathematics learning particularly, has been influenced by research outcomes on a very small scale only [Bauersfeld, 1979]. The situations that a teacher deals with everyday are complex and therefore the focus of research must be neither the laboratory nor the armchair; it must be the classroom So the educator is impatient with fancy theories that try to explain everything and end up saying nothing, and he is skeptical of advice based on someone else's experience if that experience does not seem to correspond to his own

[8] Many of these issues are noted in recent reviews of mathematics education as, for instance, Bauersfeld [1980], Freudenthal [1981].

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