"STOPPERS"

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In October 1979 I invited three secondary school teachers to join me in reflecting on the teaching/learning process that was taking place in their classrooms Classroom research was seen as an exploration and our aim at that time was to examine our own practice critically and systematically I think it is important to say that we recognised difficulties would be encountered with the word "systematic" – what is systematic for one is not necessarily so for another The pathways of each are different and an understanding of the systematisation process of another requires toleration of the other's space In June 1980 "Teacher-based Enquiry into Mathematical Learning" [1] was produced. This contained accounts of lessons by the teachers involved, together with their own and the group's reflections on the lessons. We then – metaphorically – stood still and reflected on the experiences of the year.

One aspect of our experience that had grown in importance, in that it had engaged and captured our interest, was what one member of the group termed a "stopper". Now in Piaget's view, once interesting intellectual phenomena are discovered, the cognitive processes underlying them need to be identified and described [2]. The interesting intellectual phenomenon that had manifested itself for us was the "stopper" and before endeavouring to probe the cognitive processes underlying it, we attempted to crystallise our own thinking about the nature of a stopper. The group defined a stopper as the moment when a pupil is no longer able to "cope" with the work in which he is engaged, that is, there is an observable breakdown. This may be manifested in a variety of ways, one of which is as a "mistake".

When a stopper occurs we are left with the decision of how we, as teachers, may enter into the pupil's mathematical world This moment is delicately balanced The clumsiest action is to presume that the reason for the stopper is apparent. There is a need to probe the nature of the pupil's mathematical thinking so as to gain some insight into how the stopper has occurred. This calls for judicious questioning on the part of the teacher. An awareness that a pupil may be unable to verbalise his thinking or that his poor command over language may cause him to offer an explanation that is liable to incorrect interpretation is of importance. Our thinking at this juncture finds some resonances in H Ginsburg's article [2] where he says

> "Verbalisation can be misleading since the child may not have direct access to his cognitive processes or may have poor command over language."

Wason [3] has also referred to the pupil who strives to justify his solution rather than describe how he achieved it. This pupil may be considered as amongst those who do not have direct access to their cognitive processes.

Stoppers seemed to us to be embedded in both the cognitive and affective domain of teacher and pupil We also acknowledged the teachers' inability sometimes to recognise a pupil's stopper and pondered on how frequently hidden assumptions were the cause of this lack of recognition. The place of words, both verbal and written, in the creation of stoppers was not underestimated. However we felt the most difficult situation occurred when the pupil experienced a stopper that the teacher did not know he had (It is probably pertinent to add here that the teacher knowing does not necessarily prevent a pupil stopping!)

We were also aware of moments when a word or question to a pupil had allowed him to continue – I think all teachers can bring to mind these "Ah, I see" moments But can teachers probe these moments sufficiently to be able to know for themselves how their action created them?

The words "know for themselves" are important at this juncture for these words acknowledge that a teacher's knowing is interwoven with his interpreting of what he thinks happened.

In the course of these reflections the group felt the need to focus on a specific area of mathematics. Each member of the group compiled a list of stoppers that had occurred during their lessons. The one on which it was decided to focus related to number and place value This problem may appear at the outset as a comparatively simple problem – so much the better! It was given to a first year middle of the ability range group (11 years). What follows is the account of the teacher concerned.

"*Question:* Which is the biggest number?
2.19, .888, 1 699, 2.2, 1.8989

Out of a class of 30,

14 pupils gave the answer 2.19
9 pupils gave the answer 2 2
7 pupils gave the answer 1 8989

Approximately three weeks later, after the Christmas holidays, I gave them the same written question except that the order was:

2.2, .888, 1.6999, 2 19, 1 8989

The reasoning for this went as follows:

(i) After the long holiday, most, if not all of the pupils would have forgotten the question.

(ii) I altered the order of the crucial choices to see what, if any, effect the order had on the answers. I thought that there may have been a small effect – some children picking out the 2 19 in the first question because it came first.

The actual answers were: 2 19 (12), 2.2 (11), 1 8989 (7)

"The next step was to investigate this marginal improvement. However, an interesting point arose which was hidden in these figures. Two people switched answers between the 2.2 and 1 8989.

"Those answering 1.8989 all gave the same reason for this response – "It's the largest". When asked "Why is it the largest?", all the children found it difficult to put into words. I found it very hard not to put words into the pupil's mouths. I had the mental picture that they were putting down 1.8989 because it had more figures than the others but if I had said so, I suspect that the children would have seized on this as their justification.

"I asked the girl who had changed to 1.8989 why that was the biggest. She told me "Because it has more numbers in it." I then told her that she had put down 2.2 about three weeks ago, so why wasn't it that one? This obviously caused her some discomfiture but she stuck to her answer of 1.8989.

"I asked several of the children who answered 2.19 why it was the biggest. The favourite answer to this was – because it was more than the others.

"Which is the next biggest?"
The answer is "2 2"
"Why is this smaller than 2.19?"
"Because 2 is smaller than 19."
(This particular pupil had even said throughout "Two point *nineteen*")
"Two weeks further on I tried the same example (hidden amongst others) with substantially the same group. However, this time I substituted 2 20 for 2 2. The results were as follows: 22 pupils gave 2 20 as the biggest
7 pupils gave 2.19 as the biggest
1 pupil gave 1 699 as the biggest

"I asked two of the pupils separately why the answer was 2 19 but they found it difficult to explain it to me."

This work resonated with all members of the group and generated several lines of thought:

1. Is the stopper, in this case, due to:
   (a) the inability of pupils to appreciate the significance of digits placed after the decimal point?
   (b) words used, e.g. pupils tending to say "two point nineteen" rather than "two point one nine"? Is this in any way related to our everyday terminology for the use of money when, for example, we speak of two pounds nineteen pence?
2. Why are pupils naming the one with the greatest number of digits as the largest number?
3. The problem that generated the stopper is one that has become prevalent in post 1960 textbooks. Prior to 1960 this type of problem is rarely found in textbooks; consequently recent practice could be highlighting this particular stopper.
4. Would the use of calculators as a check help pupils in this particular instance?
5. How convinced is a pupil regarding his answer – do we have to affirm or undo his conviction? Is his conviction emanating from relational understanding [4] or has it entered the pupil's "skill bank" whereby he "knows" how to operate when confronted with this type of problem? The difficulty arises when the pupil's "knowing" or "conviction" is not in line with accepted convention.

With these thoughts in mind work was embarked upon by members of the group and the following methodology was agreed:

i. Before lesson(s): write down the intended approach;
ii. After lesson(s): write an account of what happened, highlighting anything considered to be a stopper;
iii. Write an account of conversations with some pupils about the stoppers they experienced.

What follows are excerpts from work with pupils by members of the group who focused on this particular issue.

TEACHER A

2.19, 888, 1.69, 2 2, 1 8989

At the beginning of the year I noted that my first year (11 years) were particularly weak at decimals and place value. Perhaps eight individuals at most would have ordered the above set of numbers correctly. During the year I have spent more time than usual with first years doing "decimal and place-value" activities. However, there are still a number of pupils who say 2.19 is bigger than 2.2 and this leads me to two main concerns:

(a) Why is 2.19 bigger than 2.2 for these pupils?
(b) How can we remedy this? At the last meeting we mentioned briefly that the conviction has to come from the pupils themselves.

At present there are about five or six girls in the class who consistently make this mistake with place value. For my data I set the following homework.

> Look at these sets of numbers. Decide which is the biggest and write down why. Then put the numbers in order.
> (A) 505, 555, 560, 559, 509
> (B) 3.2, 2 8989, 3 19, 888, 2 92
> (C) 7, 6 55, 7.4, 6.99, 7 29
> (D) £ 4.20, £ 2.89, £ 4 19, £ 4.09, £ 4 90

Set A and set D caused no problems. In fact the money aspect of (D) made things easier. The girls' answers were more fluent, even from weak pupils.

"I would much rather have 4 pounds and 90 pence rather than 4 pounds and only 9 pence." "The 90 pence is 81 percent more."

Set B is very similar to the original data, but here I found a new problem.
Two girls wrote:
"2.8989 is the biggest as it has the most numbers."
And one girl
"2.92 is the biggest because it has more tenths."

These three homeworks I found surprising as I would have expected the three girls to have ordered set B correctly. Besides this problem, there were seven girls who gave the following order:

3.19, 3 2, 2 92, 2 8989, 888

Unfortunately, these seven girls not being the brightest pupils did not express their reasons very well.

"3 19 is bigger than .888 because 3.19 has 3 whole ones." Only one girl tried to explain why she thought 3.19 was bigger than 3 2.

"I chose this as the biggest as it is point nineteen and my second is only point two."

Quite a popular reason! However, I do feel that the 9 is significant: would 3.15 and 3.2 give a different result?

This is how one of my third years ordered five decimals:

SMALLEST				BIGGEST
6 12	6 2	6 4	6 39	6 5

Set C produced another "surprising" result: six girls gave 7 as the biggest number in this group, and this included two girls who had ordered A, B and D correctly. The general reason was:

"7 is seven whole ones so it's biggest."

This is rather vague and I tried to probe it further with individuals but really got no further.

Overall, I was disappointed with their reasons given in this work. *Those with the stoppers had difficulty in expressing themselves fully* and I decided, therefore, just to try pairs of numbers a couple of weeks later.

I wrote up five pairs of numbers on the board and also handed out the calculators. I gave the instruction:

"Without using the calculators yet, decide for each pair, which is the biggest number and draw a ring round it."

(1) £ 4 80   £ 4 79
(2) 2 2   2.19
(3) 3 15   3 2
(4) 7   7 1
(5) 6.8989   6 9

Michelle had written (7) 7 1

Teacher "Can you tell me why you chose this?"
Michelle "Well, this is 7 whole ones and this is 7 1"
Teacher "What does 7 point 1 mean?"
Michelle "Oh – I'm wrong – I know what I was thinking – 7 whole ones and you knock off point 1"

We talked then about what the point 1 is and also Michelle went through the routine of 7 – 7 1 and 7.1 – 7 on the calculator; however, I felt she was uneasy – almost as if she didn't believe the little minus sign. How do I convince her? How does she convince herself? I also get her to draw lines of length 7 cm and 7.1 cm but I'm not sure this is enough.

Nicola

(1) £ (4.80)   £ 4.79
(2) 2.2   (2.19)
(3) 3 15   (3.2)
(4) 7   (7.1)
(5) (6.8989)   6 9

Nicola's answer sheet was interesting. Pair (3) is answered correctly but not (2) or (5). I try to ask her in a suitably unbiased tone why she chose 3 2 as bigger.

Nicola "I know that one's correct but these are wrong. I'm just going to change them."
Teacher "Yes – but why is 3.2 bigger?" I persist.
Nicola "I know why I've got them wrong. It's all right."

Her friend Jackie butts in "You have to look at the whole numbers and then the first number after the point with these ones, Nicola."

I know that Nicola was not convinced but her main concern at the time was to have all her answers correct.

With some girls I think the calculators certainly helped to convince. However, with a third year group (13 years) the following conversation was interesting. I had asked them to order 2.19, .888, 1 69, 2 2, 1.8989 and then to use the calculators to check their order.

Debbie's original order was: 2 19, 2 2, 1 8989, 1 69, 888

Debbie "I've done two point nineteen and two point two on the calculator and I see it's minus, but I still don't see why Point nineteen must be bigger than point two."

Before I can say anything her neighbour Alison had interrupted.

Alison "You do it my way and put in 0's – as many as you like." She adds a nought to Debbie's 2 2 so it became 2.20
Debbie "Oh yes, I see!!" exclaimed with almost a sigh of relief.

At the time I knew there was nothing else I could add. Debbie was convinced by Alison but will she do it correctly another time?

TEACHER B

Lesson with 1B – a mixed ability eleven year age group. Intention: To compare understanding of decimals without and with a calculator.

I asked 1B to space out and gave them a piece of rough paper each. I requested that they work on their own. On the blackboard I wrote: –

2.19,   888,   1.699,   2 2,   1 8989

I gave the instruction:

On one side of the paper I'd like you to put these numbers in order of size, from smallest to biggest and explain very clearly how you did it. They had ten minutes . most only needed five minutes.

The pupils were asked to report their answers, which I recorded on the blackboard.

Results

		2 19	888	1 699	2 2	1.8989
Smallest	1		26			
	2			21	1	3
	3			4		23
	4	17			9	
Biggest	5	9			16	

Although I knew there were some discrepancies from changed minds the purpose of this was to set up a visible reminder of different possibilities so that an individual would have to be secure and convinced to retain his or her position.

The ones where there were discrepancies were pointed out. I gave out calculators to those who needed them and asked them to turn over their pieces of paper. They could now use a calculator to try to convince themselves, if necessary, and could talk.

2.2 and 2 19 changed round after the use of the calculator since a positive remainder was left.... Nevertheless there were four stalwarts who were still convinced that 2 19 is greater than 2.2 and my sixth form helper went to see them. The rest of us shared verbal experiences and convictions, with quite a lot of interesting ones emerging. *Then* .

Write down £ 2.05 (said "two pounds and five pence")
£ 2.50 (said "two pounds fifty pence")
£ 2.50½ (said "two pounds fifty and a half-pence")

and (1) tell me what's the same, if anything ...
(2) if you get stuck with decimals do you think about money?

There were some who wrote £ 2.5 for £ 2.05 but mainly they were all right ... decimal points were commented on and the fact that the amounts were in order of size ...
... but a categorical NO for money/decimal image links ... interesting ...

With this group I have not specifically talked about decimals in any set way although they have cropped up with the use of calculators and machines. They had not done this sort of exercise before where they had to list a set of decimals in order of size.

My own thoughts were stimulated in the direction of "When do I know something in mathematics?" "When do I know that another individual knows something in mathematics?"

Examples of the work of five pupils out of the class of twenty-six follow. Although it is in type, the pupils' way of writing has been preserved in so far as numerals, words and symbols have been used where they used them.

Pupil 1

(Side 1)

888, 1.699, 1.8989, 2.2, 2.19
.888 is 0.888 so that is the smallest number because it hasn't got any whole numbers.
1.699 came next because it has less digits than 1.8989 and so is smaller, then 2.2 because that has less digits than 2.19

(Side 2)

I now think that 2.2 is 'larger' than 2.19 because if you take away 2.2 from 2.19 you are left with – 0.01 so 2.19 has to be smaller than 2.2 otherwise it wouldn't be – something.
£ 2.05p £ 2.50p £ 2.50½
1. They all have got a decimal point in them and they all go in sequence from lowest to highest.
2. No

Pupil 2

(Side 1)

888	I chose this as the smallest as it has'nt got any whole number before it
1.699	I chose this one now as the 2nd number is smaller than any other of the numbers which as a 1 before.
1.8989	I chose this one now as there are'nt any other numbers with 1 before
2.19	I chose this one now as it is the smallest of the numbers with 2 before.
2.2	I chose this one last as if I added a 0 on to 2.2 it would become 2.20 so it is bigger than 2.19

I am convinced that 2.2 is bigger than 2.19 because if you had a 0 on to 2.2 it makes 2.20 and 20 is bigger than 2.19.

(Side 2)

All the numbers I have written I am convinced that they are right.
£ 2.05 £ 2.50 £ 2.50½
What if anything has what we just talked about got to do with decimals? They all have a decimal . in They go from the lowest amount to the highest. If we are ever stuck about decimals do we ever think about money? Yes sometimes

Pupil 3

(Side 1)

888	.888 can be written as 0.888 you can not go lower than 0
1.8989	Because 1 comes after 0 and the larger the number the smaller it is
1.699	the number is getting smaller and so is the value.
2.19	we are now down to 2 number which is very small.
2.2	but I think this is the biggest

(Side 2)

I can tell the difference by looking at the size in number the one with the least numbers is the biggest
£ 2.05
£ 2.50
£ 2.50½

In writing numbers as decimals and money they both have a decimal point.

Pupil 4

(Side 1)

.888
1.8989
1.699
2.2
2.19
These numbers are in this order because .888 has not got 1 whole number than 1.8989 because its got a one and .8989 is smaller than 699 than 2.2. because 2 is smaller than 2.19

(Side 2)

.888
1.8989
1.699
2.19
2.2

£ 2.05
£ 2.5
£ 2.50

Pupil 5

(Side 1)

.888, 1 6999 1.8989, 2.2, 2 19
1 2 3 4 5

I have chosen these like this because .888 has not got a 1 or 2 in front of it, it has got 0 so I put that first, then 1.699/or 1 8989. I put 1.699 first because 6 is smaller than 8 than I put 2 2 as 2 is smaller than 19 then lastly I put 2.19 so the highest has 2 whole ones the smallest has no (0) whole ones

(Side 2)

I think 2.2 is larger because if you take away your left with – 0 01 2 19 has to be smaller.

2.19
2.2
– 0.01

2 05	£ 2.05
2 50	£ 2 50
2.50½	£ 2 50½

1 They all have decimals what has this in them
2. When you get stuck do you ever think about money
No.

The following comments relate to the work of these pupils.

Pupil 1 After dealing with whole numbers, the pupil's rule for deciding smallest to greatest hinges on the number of digits. The use of the calculator enabled the pupil to correct the ordering in the case of 2 2 and 2.19 but did not lead the pupil to think about 1.699 and 1.8989 Why?

Pupil 2 The reason given for how the pupil is convinced is interesting. Can it be assumed that the pupil is aware that zeros can be added until there are the same number of digits in both cases after the decimal point?

Pupil 3 This particular pupil would in no way move from this position. How does one deal with this stopper?

Pupil 4 Intriguing that the pupil stated 8989 is smaller than 699 but 2 is smaller than 2 19. This pupil would not appear to be using the number of digits as criterion. The use of the calculator enabled the pupil to correct the ordering of 2 2 and 2 19 but as for Pupil 1 it did not apparently help with 1 699 and 1 8989

Pupil 5 The use of the calculator enabled the pupil to order 2.19 and 2.2 correctly but it is interesting to note that she wrote
2 19
2.2
– 0.01
How convinced is she?

Having read through this article the reader can now make his own judgements as to how far we have answered some of the questions that we asked ourselves at the beginning.

How has the reader resonated with what is written? How many assumptions do we make in our work with pupils despite efforts to free ourselves of them? How far has our own psychology of learning infiltrated our assumptions as to how our pupils learn?

Finally we can certainly say that "samenesses" were identified in that the surface errors were the same, but probing beneath the surface revealed substantial differences. It is the implications of this that are of prime importance in our teaching.

Acknowledgement

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References

[1] University of Bristol, England Teacher-Based Enquiry into Mathematical Learning Working Paper of a Teachers' Research and Development Group 1979-1980
[2] H.P Ginsburg, The Clinical Interview in Psychological Research into Mathematical Thinking: Aims, Rationales, Techniques *For the Learning of Mathematics*, Vol 1, No. 3
[3] P G. Wason Dual Processing in Reasoning *Cognition*, Vol. 3
[4] R Skemp. Relational Understanding and Instrumental Understanding *Mathematics Teaching*, No 77