The Role of Research in the Practice of Mathematics Education*

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There is a growing body of research on teacher education, focusing mainly on attitudes and beliefs of student teachers [e g Thompson, 1984; Lerman, 1986; Ernest, 1989], but little that focuses on teacher educators and our practice. This may be because of the tradition that research is generally carried out by an outsider, as a consequence of a perceived division between practitioners and researchers It is this division in relation to teachers that is examined here, and through that, some reflections on change in my practice as a teacher educator

Research in mathematics teaching is generally carried out at institutions of higher education, or specialized research centres, by people who are either attached to the institution for that research, or are permanently employed there, usually in teacher education. The results are occasionally descriptive, normally prescriptive theories and information, which are intended to improve and develop the teaching of mathematics in schools. It is by no means clear however, in what manner the research reaches teachers, nor whether teachers find the research relevant. There have been a few important articles in the literature [e.g. Scott-Hodgetts, 1988] but in general the "teachers as researchers" movement has had little impact upon the world of mathematics education research, taking the proceedings of the International Group for the Psychology of Mathematics Education as an example. Indeed I have heard the opinion expressed, by experienced mathematics educators, that the presence of teachers at research conferences lowers the level of debate

This separation between practitioners and researchers cannot be productive, and this article attempts to indicate the essential connections between the two, and to suggest fundamental misunderstandings of the nature of the theory/practice interface by both groups. The separation also does not reflect what is actually going on, in that there is a considerable amount of small-scale research being carried out by teachers in schools which is clearly not receiving recognition in the mathematics education research community.

A brief overview of some of the influences that led to the rise of the "teachers as researchers" movement will first be given, although it is not the intention of this article to give a complete account of the movement's development, nor of some of the debates taking place today [see e g Weiner, 1989], but merely to indicate some of the factors that have led to research by teachers in their own classrooms This

will be followed by a description of some work by student teachers engaging in classroom research, which will highlight the reflective function of research, as well as its function of the generation of knowledge. As mentioned above, a major focus of this article is the recognition that teacher education is practice too, and reflective practice on the part of mathematics educators involves research in teacher education in an analogous mode to research on the practice of teachers, and children's learning of mathematics

The "teachers as researchers" movement

The stimulus for the "teachers as researchers" movement has come from a number of directions, including the following:

(i)   Top-down curriculum reform brings the necessity for teacher education through in-service courses to the fore For instance, [Elliott and Adelman, 1976]:

> The fundamental problem for curriculum reform and the implementation of change was the clash between the theories of the reformers and those implicit, often unconsciously, in the practice of teachers

In the case of mathematics curriculum reform, the "New Maths" changes in school texts were not matched by investment in courses for teachers However teachers themselves began to recognize the mismatch, and to instigate programmes of investigation of the changes required, and of the results of the curriculum developments

(ii)   The desire on the part of teachers to improve their own practice has resulted in small-scale research, by individual teachers and in groups, in mutual support A recent publication that illustrates this development is that by the Association of Teachers of Mathematics in Britain [ATM, 1987] Similarly, curriculum reform from grass roots level brings about research in the classroom, as shown by some of the studies of the individualized learning projects in Britain [e.g. see copies of the journal *Investigator,* published by SMILE].

(iii)   One consequence of the traditional model of the researcher being an outsider coming into the classroom has been the disaffection of teachers with this model. For example, a survey of teachers who had found themselves in this situation revealed the following comments [McCutcheon, 1981]:

1) Outsiders often investigate questions of little concern to teachers

2) Teachers realize that it is they who must act, whereas outsiders recommend how others should act

3) Teachers or their pupils are studied not studiers and have little control over the research and its subsequent dissemination.

4) When the teacher has only this limited control, "eaves-dropping" can be very unsettling.

5) The researchers are often seen to make only sporadic visits to the school/classroom which may give them a false picture.

6) Reports are often written in difficult-to-understand jargon and published in obscure journals

7) Research is often thought to be done to promote the self-interest of the outsider

8) The tone of the research is often condescending to teachers.

iv) The alternative model of teachers as researchers has been characterized by some curriculum theorists as potentially emancipatory. Bell [1987] writes:

> Teacher (or action) research stood in opposition to a dominant tradition of positivistic social science in which the study of causal relationships was pursued by testing hypotheses, controlling variables through sampling procedures, and interpreting the concept of evidence as the statistical processing of data.

Hopkins [1984] suggests that this model:

> . leads to an image of an individual liberated from normative values and free to be experimental, radical and vulnerable in his/her work . Teacher research is a pedagogy for emancipation.

Finally, Weiner [1989] writes:

> Stenhouse regarded teacher research as potentially emancipatory in that it could liberate teachers and pupils from a system of education which denied individual dignity and was predicated upon external authority and control. It could also enable teachers rather than academics or external researchers to develop educational theory grounded in classroom practice

(v) A major influence in the "teachers as researchers" movement has been Schön's notion of the Reflective Practitioner. Schön, who has written widely on the false separation between experts and practitioners in various fields, particularly teaching, describes situations, such as reacting to novel interventions from students, as "the *indeterminate zone of practice*" [Schön, 1984, p. 6] He describes the action of the teacher or practitioner in this situation as follows:

> In these zones, competence takes on a new meaning. There is a demand for reflection, thought turning to the surprising phenomena and, at the same time, back on itself to the spontaneous knowing-in-action that triggered surprise. It is as though the practitioner asked himself, "What *is* this?" and at the same time, "How have I been *thinking* about this?" . Such reflection

must be at least in some degree conscious . It has a *critical* function . a *restructuring* function . gives rise to *experimenting* . to *test* new understandings to *explore* new phenomena . [p. 6-7]

Schön's work emphasizes the theory-in-practice nature of the actions of the Reflective Practitioner. To make a separation between those who practice, and those who develop theories for the practitioners, is not an adequate characterization of the business of good teaching

From the perspective of a teacher educator, it seems inconceivable to consider teacher education without continually drawing upon current research. In an attempt to make that research accessible to student teachers, and consequently to themselves as practising teachers in a few years time, an experiment has begun at South Bank Polytechnic, London, to investigate the notion of student teachers becoming active researchers as part of their course. At the same time, this change in practice of teacher education becomes the focus of a reflective process to evaluate that experiment and its implications.

Students using research

The students described below are mature students, mostly women, who have been looking after their own children, and have enrolled on the course, intending to become primary school teachers (In Britain, primary schools cover the age range of 5 to 11 years.) Under new regulations of the Department of Education, students must study one subject in depth for a large part of their course, in addition to the general preparation and training in all subjects of the curriculum. These particular students have chosen mathematics even though almost all have bad memories of their earlier experiences in school mathematics. There is no doubt that most do so because a qualification in mathematics is a distinct advantage for employment chances, but also many choose mathematics because they feel that as they are to be responsible for children's early learning of mathematical concepts, they must overcome their particular fears and lack of confidence. The students are well-motivated and demanding. They will make it clear if they are unhappy or do not understand. At the same time they are fearful about succeeding in learning mathematics. In another article their attitudes to mathematics have been described in some detail [Blundell, Scott-Hodgetts & Lerman, 1989].

Before their first teaching practice, a number of small-scale research studies were described to the students, appropriate to the age-range 5 to 11, and they were invited to choose one or more to investigate further. This being their first extended contact with their particular school and pupils, as well as their first experience of research, it was suggested that they should attempt to replicate previous studies, and then modify the research if they wished. The students worked in groups of those who had chosen the same piece of research, discussing how they might implement the research, preparing materials, etc. As their tutor, I was available for consultation and to facilitate discussion. Students were asked to take notes while at school on the implementation of their chosen study, including in particular their reflections on the work, to be written up later on their

return to the college. On this occasion it was not feasible for me, as their tutor, to visit the schools and assist them, but it is hoped that this will be possible on future periods of teaching practice.

Some found that the stress of teaching practice in a London inner city school was such that they could not carry out the research. Others were strongly discouraged by the teachers with whom they were working. The majority, however, were able to try out this work, and what follows are a few extracts from their written descriptions. For the purpose of this article, what is most important are the students' comments on the experience of carrying out research, and what they learnt from it, not the results of their work. The extracts mention the content of the particular case studies described here, there being two students' comments on one study and three on a second study.

Example 1 The role of language in class inclusion tasks [Liebeck 1987]

> "Despite the reports I have read I find it difficult to believe that the children can't distinguish between the group and a sub-class of the group *(written before the teaching practice)*
> All the evidence I collected supports Liebeck's work, but despite all that I still find it difficult to understand how they can pick up the right things and yet be unable to answer the questions
> The teacher with whom I was working . didn't want to know what the tests were, what they would lead to, and she didn't want to know the results
> I thoroughly enjoyed the research and would like to be in a position to do more of it . ."

> "I began this research work with some trepidation, mainly of the type — will I do this correctly? Will this make any sense?
> I did not receive a great deal of encouragement from . the class teacher . . in fact she was rather derisive about the whole thing.
> I have really enjoyed doing this work — the children enjoyed it too!
> It really does bring home how little I know of how children learn.
> With later reflection I must agree with her (Liebeck) suggestion that the choice of language seems to make a significant difference to a child's response."

Example 2 Visual mathematics [Kerslake 1979]

> "This was valuable research for my part because it blew up my preconceived ideas that all children of this age would know a triangle whichever way round it was shown. It opened up the idea of the way children discern differently . and taught me not to assume. The children gained from the research in that after it they all seemed to understand that a triangle was a triangle whichever way round it looked.
> The teacher was keen to see the research and interested in it, and very encouraging."

> "I am surprised at the wide variations even within that small sample. Not one child identified correctly every

shape. I think the class teacher would have been surprised if she had bothered to examine the results.
I think it was of value to me because I was able to offer them maths which they understood once I realized how much they did not understand of the language I was using, and the concepts which I had been told they understood
I would have liked to continue onto other children, and ideally repeat the research again after several weeks had passed."

"Rather surprisingly only one triangle was recognized by all the children.
I found the whole exercise very interesting. The children seemed to enjoy it too. I should like to be able to do something similar on another teaching practice
After looking at the results, the class teacher told me she was planning to do a follow-up. She was amazed at the number of children who seemed to want triangles, rectangles and squares to be sitting on a firm base . . She was very positive about the whole thing."

The tutor's reflections

(i) *What the students have learned*

After the writing up stage, the students were encouraged to share their experiences and thoughts. This enabled those who had been unable to do some research themselves to gain from the work of others, and the different benefits derived by the student-teachers to become the learning experiences of the whole group, to some extent.

The students appear to have gained a great deal from their research. Amongst other things, they recognized the mismatch between what they expect children to know, and how children learn, and what they observe of the process of children's knowledge construction. They recognize the significance of the language the teacher uses, in the mathematical understanding of children. Research of this kind "blows up preconceptions". They have learnt how to ask questions of children, and then sit and listen to their responses. Some of the class teachers were supportive and interested whereas others perhaps found the work a threat, or were too busy, or possibly considered this typical of tasks set by tutors in their colleges, not aware of the constraints in schools. One teacher, who later became very interested in the research work, initially commented "Do they think you have time for this?" This has helped them reflect on the different ways a class teacher can react to classroom situations. There are comments that suggest that the student-teachers are eager to continue this kind of work in future periods of teaching practice, and have their own ideas as to how to extend the research. They have also become aware that children actually learn from such research, as it reveals to the children too, what they have previously constructed from their experiences, and how it may need extending and modifying

(ii) *What I have learned*

As an educator of teachers, this work was an innovation in my practice, and required reflections of an analogous kind to those of the student-teachers, and thus what follows are conjectures, based on this work, of the way this has altered my conception of my role as tutor

Firstly, carrying out small-scale research has been far more effective in helping the students to listen to children than any other method that I have used before. I have tended previously to exhort students to listen to children when asking questions, but the concentration on one's own response and the other activities in the classroom perhaps precludes the possibility of giving full attention to listening to children. The research is focused on listening to children's responses and recording them, and hence facilitates this

These students have become aware that research exists, is accessible to them, and is relevant in an essential way, to their role as teachers They learned where research is written up, they learned to read research, and discuss it, they learned to design some research themselves, even where it was straightforward replication, and to think about follow-up studies, extensions, etc Most important, they have become aware, through reflection, of what they gain from such work, and what children gain too I have always brought snapshots of research into my work as tutor, but this task has achieved much more in terms of "ownership" of research than those sessions

Through the reactions of the teachers with whom they have been working, they appear to have seen the significance of "reflection-in-action" of which Schön talks. They were genuinely surprised by those teachers who were not interested in the research, even where they understood the reasons for that reaction It seemed clear to the student-teachers that the results ought to influence the actions of the teacher. Opportunities for analogous reflection on the part of a teacher educator are not common, but perhaps only because of a lack of recognition that the process is analogous, and just as essential

My concern at imposing too many expectations on student-teachers in their first teaching practice were justified in part by the problems faced by many of the students, but the success of the programme, as a first stage, perhaps outweighs that concern. It is intended that this will be a study that will extend to following the student-teachers through into their first posts as teachers, and that the continuing programme will also provide vital opportunity for my own "reflection-in-action".

Finally, to return to the issue of the theory/practice interface, the student-teachers' awareness of research and its potential role in their work as teachers, both as methodology for themselves as practising researchers-in-action and as

material developed by others and available for illuminating their practice, brings together these two aspects as part of the whole process of teaching. I have discussed the nature of theories and the implications for action elsewhere [e g. Lerman, 1989]. Further, what the study has clarified for me, is the meaning of research-in-action for the teacher educator, and brings together theories such as the nature of the research with the practice of teacher education

References

ATM [1987] *Teacher is researcher* Association of Teachers of Mathematics Derby, England

Bell, G. [1987] Action Research in the educational field: European contexts In Weiner, 1989, p 47

Blundell D , Scott-Hodgetts R , & Lerman S [1989] Mathematizing the hard stuff *For The Learning of Mathematics* Vol 9, No 2

Elliot J & Adelman C [1976] *Innovation at the classroom level. a case study of the Ford Teaching Project for Open University course E203 Curriculum design and development* . Open University Press, Milton Keynes, England

Ernest, P [1989] "The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: a Model ' *Journal of Education for Teaching* Vol 15, No 1

Hopkins, D [1984] *A teacher's guide to classroom research* Open University Press, Milton Keynes, England

Investigator, a newsletter published by the SMILE Centre, Middle Row, Kensal Rd , London, England

Kerslake, D [1979] Visual mathematics *Mathematics in School.* Vol 8, No 2

Lerman, S [1986] Alternative Views of the Nature of Mathematics and their Possible Influence on the Teaching of Mathematics Unpublished PhD Dissertation, King's College (KQC), London

Lerman, S [1989] Constructivism, mathematics and mathematics education *Educational Studies in Mathematics* Vol 20, No 2

Liebeck, P [1987] The role of language in class inclusion tasks In [ATM, 1987]

McCutcheon, G [1981] The impact of the insider In J Nixon (Ed) *A teacher's guide to action research.* Grant McIntyre, Edinburgh

Schön, Donald A [1984] Education for Reflection-in-Action Talk given at the Harvard Business School Anniversary Colloquium on Teaching by the Case Method

Scott-Hodgetts, R [1988] Why should teachers be interested in research in mathematics education? In D Pimm (Ed) *Mathematics. teachers and children* Hodder and Stoughton. London

Thompson, A.G. [1984] The relationship of teachers conceptions of mathematics and mathematics teaching to instructional practice *Educational Studies in Mathematics* Vol 15, No 2

Weiner, G [1989] Professional self-knowledge versus social justice: a critical analysis of the teacher-researcher movement *British Educational Research Journal* Vol 15 No 1