Democratic Competence and Reflective Knowing in Mathematics

OLE SKOVSMOSE

1 Introduction

In *Schooling for Democracy* Henry Giroux emphasises that school needs to be defended as a service that educates students to be critical citizens who can challenge and believe that their actions will make differences in society. Students therefore, have to be introduced to forms of knowledge "that provide them with the conviction and opportunity to fight for a quality of life in which all human beings benefit" [Giroux, 1989, p. 214]. As part of this enterprise the discussion of "literacy" has played a major role, especially provoked by the work of Paulo Freire, who developed the political dimension of education from this term

Antonio Gramsci has indicated that literacy is a double edged sword. [1] Literacy is a necessary condition in today's society for informing people about their obligations, and for people to be used in essential work processes. However, literacy can also be wielded for the purpose of empowerment because it can be a means to organize and reorganize interpretations of social institutions, traditions, and proposals for political reforms Literacy is not just a competence having to do with ability to read and write, an ability which can be simply tested and controlled, it possesses also a critical dimension. Giroux formulates it like this: " . literacy as a radical construct had to be rooted in a spirit of critique and a project of possibility that enabled people to participate in the understanding and transformation of their society. As both the mastery of specific skills and particular forms of knowledge, literacy had to become a precondition for social and cultural emancipation " [Giroux, 1989, p. 148] Later he underlines: "... literacy is not just related to the poor or to the inability of subordinated groups to read and write adequately; it is also fundamentally related to forms of political and ideological ignorance that function as a refusal to know the limits and political consequences of one's view of the world. .. What is important to recognize here is the need to reconstitute a radical view of literacy that revolves around the importance of naming and transforming those ideological and social conditions that undermine the possibility for forms of community and public life organized around the imperatives of a critical democracy." [Giroux, 1989, p. 151]

The question we shall discuss in this paper is: could "mathemacy" be substituted for "literacy" in the previous formulations? [2] By mathemacy we may initially understand an ability in calculating and using mathematical and formal techniques; later we may hope to provide the concept with a more differentiated content. Could mathemacy be conceived as a double-edged sword? Could mathemacy also be wielded for the purpose of empowerment? Could mathemacy help people to reorganize their views about social institutions, traditions and possibilities in political actions? If we reformulate another of Giroux's formulations it becomes: mathemacy as a radical construct has to be rooted in a spirit of critique and project of possibilities that enables people to participate in the understanding and transformation of their society and, therefore, mathemacy becomes a precondition for social and cultural emancipation Could this be more than a half empty assertion? What might be the meaning of mathemacy if it is to fit into this formulation? Freire developed literacy to include more than simple reading and writing abilities, but what sort of extension of mathemacy is needed? Is mathemacy also related to forms of political and ideological ignorance that function as a refusal to know the limits and political consequences of one's view of the world? Could mathemacy be involved in a project of naming and transforming those ideological and social conditions that undermine the possible forms of community and public life organized around the imperative of a radical democracy? [3]

It would naturally be too simple to assume as axiomatic that mathemacy has a similar role to play in society as literacy It may be the case, but differences and similarities have to be analysed The intention in what follows is not to "prove" that mathematics education has to be developed along some specific lines. The intention is more limited. It is to discuss the possibility of providing meaning for such developments. Giroux talks about critical democracy, several educators have reflected on this term, and I find that a more careful look at ideas and assumption related to democracy can indicate some answers to the problem raised. Although several other lines of analysis are relevant I shall, in this paper, concentrate on the concept of democracy and try to connect it with mathematics education

2 Democracy and education

There is a broad agreement that democracy is a most attractive feature of society, but at the same time widespread disagreement about what democracy might mean [4] The concept of democracy refers to a bouquet of different ideas, hopes and utopias Therefore, although it is impossible to pinpoint any simple definition of democracy, we could try to grasp the concept by outlining ideas which relate to democracy. [5]

Democracy refers to at least the following four aspects: (1) Formal procedures for electing a government and for the government to carry out its ruling (2) A fair

For the Learning of Mathematics 12, 2 (June 1992)
FLM Publishing Association, White Rock, British Columbia, Canada

distribution of social services and goods in society, such as welfare, education, hospitals, etc. Consequently, a substantial part of the theoretical analysis of democratic ideas is concerned with the types of goods and facilities which have to be distributed in an fair way. And what is the interpretation of "fair"? (3) Equal opportunities, rights, and obligations for every member of society. There must be no differences in opportunities based on differences in social background, sex, or colour of skin. Everybody must be treated equally according to the law, and similarly everybody must obey the law. But what does "equal opportunity" mean? According to the liberal and idealistic tradition, it means the unrestricted possibility of trying to do whatever a person wants (legally) to do; while the materialistic tradition has underlined that it is not enough to decrease the number of restrictions, society must actually provide the conditions for everybody to be able to pursue his or her interests. In this way every discussion about democracy becomes a discussion of freedom. (4) The possibility for and the ability of citizens to participate in discussion and evaluation of the conditions for and consequences of the governing which takes place: a "democratic life" is presupposed.

In other words: Democracy refers to *formal* conditions concerning algorithms for election, *material* conditions concerning distribution, *ethical* conditions concerning equality, and finally conditions concerning the *possibility for participation* and re-action.

It is important to discuss all these aspects in relation to education, and in fact the second and the third have been discussed in great detail. [6] A fair distribution of social services implies that in a democratic society every child and adolescent must have equal access to schooling and to learning. This naturally leads to a discussion about equality. What does equality in education mean? Obviously, children and students seem to receive very different sorts of education, even in the same society, and even in societies supposed to be democratic. How is this possible? Some find it tempting to maintain that only children with the same abilities can be treated equally. This may be convenient for some practical purposes: it seems easier to teach a group of children who are at about the same level. But in what sense is this in accord with democratic ideals? Several investigations have indicated that in some countries working class children receive less from schooling than other children. [7] It is also well-documented that differences in achievement correlate with gender, at least in some subjects (in some countries). School seems to serve a reproduction of social structures including the division of labour, the distribution of power between the individual and the state and between social groups, and finally it seems to reproduce traditional cultural values. What does that mean for our interpretation of education by democratic standards? To be in accordance with the ideals of democracy, schools have to react to the different ways in which society reproduces itself, and it must try to counterbalance some of these reproductive forces to provide an equal distribution of what schooling may offer, including opportunities in further education and vocational life.

Education has to do with the content as well as the distribution of acquired competences. That I am not going to pay attention to the distribution of competences does not mean that this is of minor importance. However, I find the questions relating to content and to aspect (4) above — democracy as involving a sort of participation — sufficiently complicated, and this brings us back to questions about the interpretation of mathemacy. What sort of competence, if any, important for participating in a democracy, can be supported by developing mathemacy? What is the nature of such a competence in a highly technological society? Can mathematics education be of value in providing a foundation for children's and adolescents' later participation in a democratic life as critical citizens? Does it in fact make sense to relate the discussion of the content of mathematics education to the discussion of the nature of democracy? [8] We must take a closer look at the idea of democracy.

3 Democratic competence

In *The social contract* published in 1762 Jean-Jacques Rousseau makes a classification of different types of government, and he writes: " ... the sovereign may put the government in the hands of the whole people, or of the greater part of the people, so that there are more citizen-magistrates that there are ordinary private citizens. This form of government is known as democracy." [Rousseau, 1968, p. 110] This definition closely follows the ancient conception of democracy stressing the importance of participation, but Rousseau takes a step further to express explicitly the idea of *direct democracy*. Everybody (or most of the people) should actually be able to participate in the ruling. If we conceive this to be a genuine definition of democracy, it is obvious that democracy is impossible in most modern societies. The concept of democracy gets a limited range of applications; only very small and homogeneous societies can be democratic.

If we give up the idea of direct democracy and try to find a more feasible interpretation, applicable to a wider range of societies, we face the problem of delegation of sovereignty: How is it possible to combine democracy with the necessity of selecting a small group of people to actually do the ruling? This question always follows *representative democracy*. The necessity for the delegation of sovereignty is also implied by the fact that ruling presupposes specific qualifications that are not of a common nature, particularly not if we have a complex modern society or organization in mind. The people in charge must have specific knowledge about the domain to be ruled. Perhaps specialized education is needed. The people in charge must possess a competence that includes information and knowledge. We shall accept the delegation of sovereignty as a necessity in what follows. But how is it possible to control the people in charge? This problem accompanies every attempt to extend democracy beyond exclusive direct democracy. [9]

We have to make a distinction between the competence which the people in charge must possess if they are to be able to take well-founded decisions and act in an appropriate way, and the competence which is presupposed if people are to be able to judge the results and consequences of the ruling. We shall distinguish between *competence in ruling* and a *democratic competence*. Democratic competence is to be ascribed to the majority, it must be presupposed in a functioning representative democracy. Democratic competence is the basis of knowledge and understanding which is necessary if the delegation of sovereignty is to be subjected to any sort of control. It is a condition for participation and reaction. Interpretations of this competence vary between two extremes. One sees democratic competence as a natural ability of human beings, the other sees it as an acquired ability. In what we could call the classical and idealistic interpretation of democracy the basic idea is: while the ruling-competence of the people in charge is of a special nature, the judging-competence is natural (but perhaps only as a potential capacity because only a certain attitude will stress the importance of a democratic way of social control, and a democratic attitude is not of a common nature).

It is my assumption that we cannot presuppose a democratic competence automatically exists. It has to be developed. However, I do not find the fundamental reason for this in the "limitation" of the (although limited) human nature but in the demands which the complexity of society makes on democratic competence. To participate in democratic life in a highly technological society seems a challenging task. But by which institutions in society has a democratic competence to be developed? It is a basic assumption that this question makes sense. Does this imply that we assume the existence of some sort of evaluative authority? But can it make sense to accept a delegation of sovereignty but not to accept that people are sovereign in their evaluation of the result of this delegation?

Before getting into that discussion I shall mention a particular way of "solving" the problem of the extension of democratic competence. The classical interpretations of democracy draw attention to the question: How to deal with ruling? The actual election of the people in charge has not been the important issue. Naturally this election has to be democratic, and formal rules for election have to be specified and obeyed. But election has been seen as subordinate to the real concern of democracy: the appropriate way of ruling. However it is possible to turn this upside down and rephrase democratic life as primarily concerned with election and not with ruling. What must be democratic, then, is the election of the government. This makes the formal condition important in a discussion of democracy. The other conditions for democracy become insignificant and no attention need be paid to the establishment of a democratic competence. That is a possible "solution" to our problem about the nature and development of democratic competence.

Such an interpretation of democracy is suggested by Joseph A. Schumpeter in *Capitalism, socialism and democracy,* first published in 1943. Schumpeter conceives the election of the government to be the primary concern of a democracy. He takes the view that the role of the people is to produce a government (or an intermediate body which could produce a government), and he defines: "The democratic method is that institutional arrangement for arriving at political decisions in which individuals acquire the power to decide by means of a competitive struggle for the people's vote." [Schumpeter, 1985, p. 269] Democracy then becomes a formal characteristic; it does not have to do with the actual questions to be dealt with in governing. In fact Schumpeter maintains that producing a government practically amounts to deciding "who the leading man shall be". Schumpeter's interpretation is provocatively simple, but let me point out one of its "advantages": It has great descriptive value. It explicates how the concept of democracy is normally used when countries describe themselves as democratic.

This interpretation neglects every concern about the non-formal conditions for democracy, such as a fair distribution of goods and equal opportunities. Even the basic idea that democracy should be a way of keeping power in the hands of the people is eliminated. Also it is obvious that if we take Schumpeter's interpretation literally we need not care any more about the existence and nature of democratic competence. Democracy does not presuppose participation and has nothing to do with decision making, or with criticism and the evaluation of decisions and proposals put forward by the government. Democracy has to do only with the production of a government. It is the procedures and algorithms for election which are democratic or not. That is the most simple and radical solution of the problem of delegation of sovereignty. The only input from the people to the democratic life is their votes. Beside that they have only to receive.

A corollary of Schumpeter's interpretation is that education need not pay any attention to democratic life. To do so just simply expresses a misunderstanding not only of education but also of the nature of democracy. Considerations about the potential of literacy as a support of a critical democracy become irrelevant. Several trends in (mathematics) education ignore the analysis of democratic demands as part of the guidelines for the curriculum — although without an explicit acceptance of Schumpeter's interpretation. However, such ignoring may express a Schumpeter-like interpretation of democracy.

The concept of democracy to which I subscribe does not have the great descriptive value of Schumpeter's. It is much closer to the classical interpretation which regards democracy as a characteristic of governing, although the actual production of a government should not be ignored. I shall not return to the utopia of direct democracy, so the delegation of sovereignty has to be dealt with. Therefore, democracy also characterises the ways of participating in discussions and criticism of the actual ruling. A democracy must make room for a critical citizenship, which is the actual performance of a critical competence.

So the problem of democratic competence is back on the agenda. I see the development of this competence as one fundamental condition for democratic life. We have, therefore, to characterize the possible content of democratic competence in relation to the main questions which concern the society in question. *The content of the* *democratic competence depends on the nature of the problems which face the society.* And my focus is on the highly technological society.

4 The problem of democracy in a highly technological society

Humanity is surrounded by technology. Society and technology are integrated and technology has become the dominant feature of civilization. The relationship between types of technologies and sciences varies greatly. Manual tools are developed independently of science. The development of steam engineering took place in parallel with the theoretical understanding of thermodynamics, while information technology is totally based on developments in mathematics. Mathematics is the logical underpinning of information processing, and mathematical thinking is the foundation for the actual applications of information technology as well. In fact every application of a computer can be seen as an application of a simple or complex mathematical model. Therefore, from a logical point of view, information technology need not be interpreted as a *new* way of formal manipulation; but it is an enormous extension of these manipulations. The effect of computers is that the applications of formal methods have colonized all areas of life. This is what characterises the information society. [10]

In a society based on the use of manual tools a classical interpretation of democratic competence remains plausible; no specific technological knowledge seems to be needed to evaluate the acts and decisions of the people in charge. Quite the contrary is the case in a highly technological society, where all sorts of social, economic, and political decisions also concern technology. On the face of it only a limited group of people is able to manage this complexity. In fact this competence seems to presuppose a certain amount of technological knowledge. How can anybody evaluate decisions which must take into consideration consequences of technological enterprises without a fair amount of technological knowledge? [11]

Democracy may be destroyed by a dictatorship which obstructs formal democratic procedures. That has been seen often and is normally conceived as *the* problem of democracy. It is what some countries accuse some other countries of not solving. But unless we accept Schumpeter's interpretation, the algorithm for the election of the people in charge is only one aspect of democracy. Democracy can be undermined in other ways than by just neglecting the rules of election. Democracy refers not only to formal but also to material and ethical conditions and to possibilities for participating and reaction. In particular democracy can be destroyed if a critical citizenship cannot be brought to life. When society changes rapidly the main principles in the mechanisms of the development of society become hidden and difficult to identify. How is it possible to evaluate decisions taken by the people in charge if neither the conditions for, nor the implications of, their decisions are visible? How can anyone other than experts control experts? Will the conditions for critical citizenship be eroded by social and technological development itself? This phenomenon we shall call the *problem of democracy in a highly technological society* (keeping in mind that this naturally is far from the only problem which faces a democracy).

In the information society the ability to collect, systematize, and use information seems to be the vehicle for social development and, simultaneously, it becomes a source of power. In the article "The Social Framework of the Information Society" Daniel Bell maintains that a knowledge elite has power within intellectual institutions but only influence in the larger world where policy is made. Bell finds that even if political questions become more and more mixed up with technical issues, the knowledge elites have no power to take decisions, although they can define problems, initiate questions, and provide the technical bases for answers. Bell concludes that the political power belongs, inevitably, to the politician rather than to the scientist or economist: "In this sense, the idea that the knowledge elite will become a new power elite seems to me to be exaggerated." [Bell, 1980, p 542] [12]

However, even if this argument may be sound it does not solve our problem about the existence of democratic life. Let us assume that Bell is right: the elite with knowledge may have power within intellectual institutions but only influence in politics. But the problem of democracy raised by the development of the highly technological society is not just that of the influence or the power of a knowledge elite placed outside the arena for political decisions. The problem is not just whether or not technological development reduces politicians to marionettes giving voice to the consequences of prefabricated technological calculations. The problem concerns the relationship between, on the one hand, the people in charge (the elected politicians) and the technological elite and, on the other hand, the people who are affected by the ruling. So even if the knowledge elite is only able to influence while the politicians remain with the power, the conditions and the argumentation for the decisions to be made may go beyond the reach of ordinary people.

Therefore if the condition for critical citizenship is important in a democracy we have to return to the problem that the grounds for the decisions taken by the authorities may be inaccessible to people other than the technician and the people in charge. Technological development may *erode* part of the non-formal conditions for democracy, leaving behind only an algorithm for election. That erosion is a real threat to democracy in a highly technological society. But is it possible to secure a critical citizenship in a highly technological society? To find a positive answer to this question is equivalent to conceiving democratic life as possible (also) in the future (without being forced to return to Schumpeter's definition). We shall not try to turn the clock back. We cannot propose abandoning technological environments. We cannot expect the conditions for critical citizenship to decline. The problem is to develop a general critical competence which can actually match the social and technological development. This constitutes the problem of democracy in a highly technological society. To analyse this further, I shall take a look at the position of mathematics in society. This formal science can be seen as a structuring part of "expertocracy"

5 The formatting power of mathematics

Technology today can be assumed to be characterised by the dominance of formal methods so we shall take a look at the role of mathematics in society. [13] The thesis which we shall discuss says that mathematics makes a real intervention in reality, not only in the sense that new insight may change interpretations, but also in the sense that mathematics colonizes part of reality and rearranges it. The thesis is that *mathematics is formatting our society*. This thesis of the formatting power of mathematics does not imply that mathematics itself cannot be seen as a social construct (which I conceive it to be) and interpreted as colonized by economical and cultural interests. What is emphasised by the thesis of formatting is that this social construct, although formal, is able to do something to reality, and I concentrate on this aspect because of my interest in the conditions for democracy

We shall distinguish between two different kinds of theoretical constructs, *thinking abstractions* and *realized abstractions*. [14] Thinking abstraction are used to facilitate reasoning, and this type of abstractions may be exemplified by mathematical concepts and mathematical modelling. Reasoning about economical development may be helped by using a concept like the National Product, defined as a mathematical function Thinking abstractions only exist as mental models or as images exist Their existence is similar to the existence of a character in a novel. However, realized abstractions have a different ontological status. They are taken for granted. We normally do not question whether what we are dealing with is a realized abstraction Perhaps we do not even have the possibility for identifying such abstractions We live with realized abstractions Ways of calculating taxes, child benefit, salaries, strategies of production, etc., are not just thinking models, they have a real influence on our lives. [15] Exchange values of goods in the form of money are real, they are not just models for expressing the degree of usefulness of some goods or for expressing the time needed for their production Money systems become real states of affairs, and even the National Product becomes real; it has reached a different status from being just a mathematical summary of calculations based on the values of some parameters. The National Product enters the political and economical discussion as an independent object and as a real figure

Every society and every culture have developed a realm of realized abstractions. But from what source? Where do they come from? They must be brought into existence by some creative act. Going back in history we may be able to trace some realized abstractions in ideological and religious structures and metaphysical systems; but habits becomes norms and standards and rules Realized abstractions have to be taken into consideration as part of our lives. They are not any longer just models for our thinking In that sense thinking abstractions become realized abstractions. Further, the main source for realized abstractions is different in today's highly technological societies. Mathematics and formal sciences have become a new bubbling source for the invention of rules and structures. Formal sciences not only create ways of describing and handling problems, they also become the main source for the reconstruction of reality.

We can formalize a language or a part of a language, and mathematics can be interpreted as such a type of formalization. But it is not only possible to formalize a language: actions and routines, i.e. way of behaving, can also be formalized In this case the result of a process of formalization is not a new language but new structures for management perhaps in the shape of "manuals", i.e. descriptions of how to behave in a prescribed algorithmic way. This phenomenon can be illustrated by "scientific management" as developed by F W Taylor. [16] The basic idea is that complex work processes have to be decomposed into their atomic components. Then each component must be investigated in order to find out the best way of handling the operations, and the "proper time" for its execution must be measured. Then the atomic components have to be sequenced to define the work processes for each worker, and the total of the worker's algorithmic behaviour will make up a new "mega-machine" Taylor described how he investigated specific work processes and, as an example, he gives the story of the engagement of the "perfect" worker named Schmidt, who never asked questions but just followed the prescribed algorithms Schmidt thus became the first Taylorized man

Formalization of language and formalization of actions are closely connected. The application of a formalized language in order to describe phenomena makes it easier to "see" formal structures, and by doing so, a first step in adapting reality to our image of reality is taken. A formal description highlights some aspects and ignore others If, for instance, the object of our description is work processes or economic transformations and our intention is to do further systematization and Taylorizing, a formal description will facilitate our steps. It makes it easier to work out new algorithms of behaviour We witness a transmutation of thinking abstraction into realized abstraction, caused by the transformation of a formalized language into a formalization of routines We create a semantics for our formal description by inventing algorithms and routines, i.e. sorts of behaviour referred to by the formal language. Mathematics intervenes in reality by creating a "second nature" around us, by giving not only descriptions of phenomena, but also by giving models for changed behaviour. We not only "see" according to mathematics, we also "do" according to mathematics. Mathematical structures come to play a role in social life in the same fundamental way that ideological structures organize reality Schmidt is no longer the only Taylorized man.

6 Reflective knowledge

I find that the problem of democracy in a highly technological society has to be seen in the perspective of the thesis of the formatting power of mathematics If mathematics intervenes in reality, one of the principles for organizing work-processes, economical management, etc., is created by a source hiding behind the scene of politics.

But if mathematics has a special role to play it becomes natural to suppose that mathematical education must be put into focus, and that brings our discussion back to the concept of mathemacy To specify the content of this concept some distinctions may be useful.

Let us as an example look at the problem of motoring: Too many (private) cars cause pollution, too much petrol is burned into the atmosphere, etc. This form of transport carries some serious risks (of an ecological nature, for instance) which we are going to face in a not too distant future. The way to confront these emerging problems is not to develop the drivers' driving skills, i.e. their abilities in manoeuvring a car in the traffic, nor is it to give drivers more information about mechanics — how the car is actually constructed, how the brakes works, how it could be repaired, and so on. Naturally, it is useful both to be able to repair a car and to drive it in a better way, but this is not a satisfactory answer to the problem of motoring. To face this problem and to react to it in an adequate way we have to develop a better understanding of "motoring", seen as the complex phenomenon of organising transport and traffic in general. What are the economic and ecological consequences of "motoring"? What social and political actions are needed and which seem to be possible? We have to learn *about* motoring to address such questions. Obviously, to learn about motoring is not itself a solution to the problems caused by motoring but it is the epistemological step to be taken to get hold of the problem itself. Knowledge at a metalevel must be developed if our actions are not to degenerate into measures of desperation. To stick at the improvement of driving skills would be to adopt an ostrich-like policy

This can be stated in a more precise way. Let us call the knowledge necessary for developing and using technology *technological knowledge* and, as an example, we may include both the knowledge necessary for driving a car and the know-how necessary for repairing and constructing it Driving skills are not of the same type as knowledge about motoring. The latter is an example of meta-knowledge, and we call it *reflective knowledge*. [17] The fundamental thesis relating technological and reflective knowledge is that technological knowledge itself is unable to predict and analyse the results of its own production; reflections are needed. The competence in constructing cars is not sufficient for the evaluation of the social consequences of car production. Improved road ability does not produce a better understanding of "motoring". Technological knowledge is born shortsighted. Reflective knowledge must be based on a wider horizon of interpretations and pre-understandings It has to grasp the situation in which technological knowledge is at work, but no simple steps lead from technological to reflective knowledge Technological and reflective knowledge constitute two different types of knowledge, but not two independent types. It may be important to master some technological insight to support reflections. To be able to understand and discuss the social implications of the pollution caused by motoring we have to know about the basic principles and conditions related to the constructions of cars; but it is not necessary to master all aspects of these constructions. If that were the case, democracy in a highly technological society would become impossible.

Reflective knowledge cannot be analysed into technological components. Even if we collect every bit of technological information we shall not be able to build up reflections from these parts alone. Technological knowledge does not address a self-critique, nor a specification of the alternative trends in technological development, so reflective knowledge does not have its epistemological basis in technological problems but in the technological way of handling such problems. While technological knowledge aims at solving such problems, the object for reflective knowledge is a suggested technological solution to some (technological) problems

It may be necessary to make a further distinction between technological and *mathematical knowledge*, which refers to the competence normally understood as mathematical skills, including competences in reproducing mathematical thoughts, theorems and proofs, as well as mastering a variety of algorithms. These competences differ from abilities in model building, i.e. the ability in applying mathematics in the pursuit of technological aims

Now we can reformulate (a part of) the problem of democracy in a highly technological society. The question is whether the development of reflective knowledge can take place without presupposing a complete development of mathematical and technological knowledge. If it is possible to show that the instruments for identifying and criticizing the use of formal methods in society do not push us completely into the formal sciences themselves, then we may hope to find a solution to the problem of democracy. In that case technological development need not necessarily erode the conditions for democratic life, leaving behind only Schumpeter's algorithm

Which institutions in society could take over the job of developing democratic competence? It cannot be assumed that it can be done in a straightforward way, but one answer given is that education must be in charge. I take in the assumption that education plays a specific role in developing democratic competence, and this raises a set of new aims for education. Traditionally a major concern of education has been to prepare pupils and students for later participation in work processes in society, but different trends in education have stressed that education must also prepare for dealing with the aspects of social life outside the sphere of work, including political and cultural aspects In short, one of the aims for education must be to prepare for a critical citizenship. The pursuit of such aims was a strong trend in German education after World War II. [18] Also in Scandinavian countries such aims have been put on top of the agenda — underlined by the the use of the German term *Algemeinbildung* (general education), meaning that education must focus on more than the conditions for work possibilities. Education must prepare pupils and students for (political) life in society We are now back to the idea, also stated by Giroux, that education has a specific obligation with regard to democracy, but now we are able to say more about the importance and nature of mathemacy.

The idea which I have tried to provide with meaning (not to prove) is: *If mathemacy has a role to play in an education — similar to but not identical with the role of literacy — in trying to develop a democratic competence, then mathemacy must be seen as composed of different competences: mathematical, technological and reflective. And especially: reflective knowledge has to be developed to provide mathemacy with a radicalized power.* Reflection on the application of formal methods is one important element in the identification of the conditions for social life and, therefore, a part of a democratic competence. That means that the guiding principles for mathematical education have to be raised to a metalevel, they are no longer to be found in pure mathematics, nor in any epistemological theory which focuses on the development of mathematical knowledge as such [19] That means that the whole nature of the discussion of mathematical education has to be changed. The focus must be on the *functions* of the applications of mathematics in society — and not just on modelling as such.[20] The discussion of the content of mathematical education has to be guided by the question whether or not it will be able to clarify the actual function of formal methods in today's societies.

7 Reflective knowing in small steps

Does it make sense to try to develop reflective knowledge as part of an educational task? Has such an intention any educational meaning? One implication seems to be a criticism of all those epistemologies which concentrate on the development of mathematical knowledge in itself. The principal example is found in the genetic epistemology of Jean Piaget, who concentrated on the nature of the development of mathematical knowledge. The main idea in Piaget's constructivism is that immanent schemes of operations can become objects for abstraction and made into explicit logico-mathematical patterns of thought. In this way the architecture of mathematics is constructed. This preoccupation with mathematical knowledge is characteristic also in the further development of constructivism. Reflective knowledge has as its object the use of mathematics, and therefore it becomes important to step outside the cathedral of formal knowledge to take a more general view of this construction.

Let us try to take some steps in *reflective knowing*, but now without any guarantee that the steps lead into the centre of a democratic competence, nor into the concept of reflective knowledge in its general formulation related to the evaluation of technologies in society. We are only talking about steps put in some analytical order, not about the steps which actually may be taken by children and students (when I talk about the first, the second, etc., step, it just has to be read as different steps). We may hope that this may provide a bit more educational meaning to "mathemacy".

A *first* group of questions formulated by pupils, students and teachers, about their work in the mathematical classroom could be: Have we done the calculations right? Have we followed the algorithm accurately? Are there different ways of controlling the calculation? Such questions are all addressing the mathematical aspects of the problem solving process, and any attempt to answer these questions immediately moves us back into the area of mathematics. Still they may be seen as rudimentary steps in making reflections about what has been done. In school all meta-reflections seem to be concentrated on questions of this type. The dominance of such questions supports also the widespread true-false ideology which is incorporated in much of school mathematics. This ideology tells that any answer to a problem or exercise must either be right or wrong. These are the only possibilities when we do mathematics.

A *second* step has to be taken, and we could ask questions like: Have we done the right calculation? Is it possible to chose between different algorithms? Is the algorithm reliable in all circumstances? Is it sound? More generally we could ask: Have we used an appropriate algorithm? Is the algorithm reliable for pursuing our aims?

Meta-reflections need not be confined to the correctness and consistency of the methods used, we could take a *third* step. Reflections could have to do with the reliability of the solution in the specific context. Even if the calculations are done correctly, and the accountability of the techniques is established, it need not be the case that the result should be trusted. We could ask questions like: Even if we have calculated in the right way and used algorithms in a consistent way, do we then find a result which we can actually use? Are the results reliable for the purpose we have in mind? This third group of questions starts attacking the true-false dichotomy and takes into consideration the context of the use of mathematics. The questions have to do with means and aims. In this case we are looking at the technological aspect while at the first two steps we have been addressing the mathematical tools. If it is possible for students and children to raise questions of this type in school, it is important that mathematics is contextualized in a way that they see the value in such investigations.

This turns us towards a *fourth* step in developing reflective knowing. We could raise questions like: Is it appropriate to use a formal technique at all? Do we in fact need mathematics? Is it important to introduce a formal method? Could we find out without mathematics? Is the result based on a mathematical calculation more or less reliable than intuitive interpretations of the situation in question? These questions draw attention to the fact that formal techniques and mathematics need not be necessary tools for obtaining the technological aim. In some cases an intuitive way of handling a specific problem may be preferred. It is an important experience for the children that they sometimes are able to find out without mathematics. The fourth group of questions attacks that variant of the true-false ideology which tells that formal methods must be preferred. Formal methods may reach further in some situations, but they do not always work to give an appropriate answer. By contrasting formal techniques with intuitive ones it becomes possible to see formalization as only one possible way of handling a problem, and this experience is important in developing reflective knowing. The question, sometimes raised after a successful contextualization, is: where has the mathematics gone? It sometimes seems to disappear in the middle of a project But this could also be interpreted as a positive condition for reflections on the necessity of formal tools

We take a *fifth* step towards reflective knowing when we look for broader consequences of the use of specific techniques for solving a problem In steps three and four the reflections concentrate on the technological aims of the task, but now we look outside the original aim for our action and we try to find the general implications of pursuing such a task with formal means How does the application of an algorithm affect our conception of a part of the world? This is the question addressing the formatting power of mathematics But how do we handle this question in an educational context? In taking our fifth step we must not forget that we still try to be settled in a classroom However, before coming back to this question, we shall take a step even further

The *sixth* step is to try to reflect upon the way we have reflected upon the use of mathematics. Reflective knowing must address its own status. That rounds off our way towards reflective knowing.

Let me summarise the six steps by key questions — keeping in mind that the steps are logically ordered but not more: (1) Have we used the algorithm in the right way? (2) Have we used the right algorithm? (3) Can we rely on the result from this algorithm? (4) Could we do without formal calculations? (5) How does the actual use of an algorithm (appropriate or not) affect a specific context? (6) Could we have performed the evaluation in a other way? Questions (1) and (2) focus on the mathematical tool; questions (3) and (4) on the relation between the tool and the task; question (5) on the general effect of pursuing the aim with the chosen tool; and (6) looks at the way we have been looking at (1) to (5)

By the fifth step we definitely seem to have left the classroom Does it make sense to try to illustrate how mathematics is formatting society? Could it make sense at all to introduce the idea of the formatting power of mathematics in elementary education? Let us try to imagine that children/students are involved in a project about child benefit, and let us imagine that, as a sub-project, we try to look at more fundamental questions about child benefit. The actual payment of child benefits in Denmark has varied according to different sets of rules, so let us try to participate in these considerations. Let a specific amount of money M be fixed How might we distribute that amount between a number F of families? Depending on the age of the children/students involved in the project, the questions can be specified in different ways. The number of families could be so small that a description of each family could be given: income of the parents, number of children, jobs, age, etc The children/students could then be asked to suggest reasonable ways of distributing child benefits. One answer could naturally be $cb=M/F$, but at least only families with children should receive child benefit And the amount of money given to each family must naturally depend on the number of children in the family And what about the income of the family? What does it mean if a parent is living alone with the children? etc.

The important question is of course what sort of information about the families is to be used. At one extreme, only a very little, at the other, the whole situation of the family could be taken into account To design a formula in the first case is not so difficult, but to imagine a design in the second becomes nearly impossible. A design of an algorithm presupposes that some simplifications have to be made, and that could be realized by the children/students during a project. They could get an experience of the difficulties raised by bringing more and more aspects into a formal system. The discussions will inevitably change from what is reasonable and fair to what is possible relative to the technological tool. And to get an impression and understanding of that sort of switch over is quite fundamental to an understanding of the role of formal methods in today's societies

When it is decided that some sort of formal methods are to be used, then it is also decided that only a limited set of factors can be taken into account The children/students could come to realise some of the basic features of the formatting power of mathematics — and also that in some cases no alternative to the process of formatting exists although alternative ways of formatting exist. Once an algorithm for the distribution of money is fixed, a little part of social life is also fixed The formatting then becomes hard frozen reality for lots of people. These considerations indicate what it could mean to illustrate, at a rather basic level, that mathematics is formatting society. However, I am not asserting that this may imply pedagogical success I have just tried to indicate in what direction we may find some educational meaning to the idea that mathemacy is of importance in today's technological society as part of a general democratic competence.

8 Conclusions

Driving skill demonstrates only technological competence but it is crucial to know *about* motoring and its consequences. Such meta-reflections are important to society, and that presents us with an educational task — in fact, environmental education is becoming more and more widespread. In the same sense, the guiding principles for mathematical education have not any longer to be found exclusively in pure mathematics nor in applied mathematics but in a broader perspective aiming at reflective knowing. It is possible to connect empowerment and mathematical education to the degree that mathematical education may help in clarifying the formatting role of formal methods in society Empowerment is not connected to any isolated ability in carrying out mathematical calculations as such, but to an understanding of how mathematics is applied and functioning. To be able to carry out democratic obligations and rights it is necessary to be able to understand the main principles in the mechanisms of the development of society. We have to know about the risk structures which accompany social development (and some of these risk structures are in fact created by formalization). [21] We must be able to see the constructive as well as the destructive forces linked to technological development Especially, we must be able to understand what mathematics does to society That provides (some) sense to the formulation: mathemacy as a radical construct has to be rooted in a spirit of critique and project of possibilities that enables people to participate in the understanding and transformation of their society

I see this as a condition for developing the dimension of participation in democracy If people are to be not only receivers of information and instructions but also able to criticise, evaluate, understand, i.e. to provide an input to the democratic institutions, then they must get an understanding of some of basic structuring principles in society And ideological structure is no longer the only structure of importance: formal structures play a role hand-in-hand with ideological formattings That is the reason that I find that mathemacy and literacy have similar roles to play I am not saying that these are the only formatting powers in society (my discussion has not touched on the economy), but the intention has been to see parallels between literacy and mathemacy Perhaps it is a recent historical fact that mathematics education can play a critical role having to do with the nature of the formattings of today's societies: "mathemacy" can now become a critical power That gives (analytical) *meaning* to the project of relating mathematics education and democratic development, although the project may not be realisable: it is by no means obvious that education can be turned into a strong, progressive social force However it *is* possible [22]

Notes

[1] See the remarks about Gramsci in Giroux, [1989], p. 147

[2] The term 'mathemacy' has been used by Ubiratan D'Ambrosio in a number of his works on ethnomathematics

[3] See also Frankenstein [1987] and [1989]

[4] In *Models of Democracy* David Held writes: " .. nearly everyone today says they are democrats no matter whether their views are on the left, centre or right Political regimes of all kinds in, for instance, Western Europe the Eastern bloc and Latin America claim to be democracies Yet what each of these regimes says and does is radically different Democracy seems to bestow an 'aura of legitimacy' on modern political life: rules, laws, policies and decisions appear justified and appropriate when they are 'democratic'." [Held, 1987, p 1].

[5] A classical discussion in analytical philosophy of democracy is found in Benn and Peters [1959]

[6] See for instance Purvise and Hales (eds) [1983] In Peters [1966] is found a discussion of 'democracy and education' and also a discussion of the meaning of 'equality' and 'freedom' in relation to education

[7] See for instance Bowles [1983]

[8] See Skovsmose [1990a]

[9] Democracy need not be linked to the question of governing a country or a community In what follows we shall have a more general approach in mind meaning that democracy is seen as a characteristic of governing some sort of organization, be it a large or small community or a private or public organization However we shall still refer to these as 'society'

[10] We are witnessing a de-personalized and de-humanized conquering of our *Lebenswelt* (lifeworld). This phenomenon has been discussed in detail in Habermas [1984, 1987]

[11] In my terminology the concept 'technological knowledge' refers to the competence needed in developing and applying a technology The concept will de discussed further in section 6, "Reflective knowledge".

[12] A little later Bell concludes: "The fear that a knowledge elite could become the technocratic rulers of the society is quite far-fetched and expresses more an ideological thrust by radical groups against the growing influence of technical personnel in decision making." [Bell, 1980, p 543].

[13] See Skovsmose [1988]

[14] Ideas described in this section are developed in co-operation with Christine Keitel and Ernst Kotzmann as part of the work in the BACOMET-group See also Keitel [1987]

[15] See the discussion of the prescriptive function of applied mathematics in Davis and Hersh [1988].

[16] See Taylor [1947]

[17] See also Skovsmose [1989a], [1989b] and [1990b]

[18] See Adorno [1971], which contains the important essay from 1966 "Erziehung nach Auschwitz".

[19] See also Davis (1989)

[20] A similar conclusion has also been pointed out by Mogens Niss: "It is of democratic importance to the individual as well as to society at large, that any citizen is provided with instruments for understanding the role of mathematics [in society] Anyone not in position of such instruments become 'victim' of social processes in which mathematics is a component. So, the purpose of mathematical education should be to enable students to realize, understand, judge, utilize and also to perform the application of mathematics in society, in particular in situations which are of significance to their private social and professional life" [Niss, 1983, p 248] Of special importance is also the discussion in Ernest [1991] of the public educator programme

[21] See Boos-Bavnbek [1991]

[22] It has to be noticed that the line of analysis in this paper is far from the only one possible. For instance, although reflective knowledge can be related to reflections about the use of mathematics in society, it can also, in an educational context, be related to reflections about what is happening in mathematics classrooms. Further, I have analysed the technological development as a threat to democratic development, but it is obvious also that information technology has a democratic possibility as it can facilitate the distribution of information However, this argument does not exclude the dangers build into the technological development

Acknowledgments

In 1988 the Danish Research Council for the Humanities decided to set up the general research initiative "Mathematical Education and Democracy in Highly Technological Societies". The initiative is planned to run during the period 1988-1993 The members of the group directing the initiative are Gunhild Nissen, Roskilde University (chairperson) Morten Blomhoej (secretary), Peter Bollerslev, Ministry of Education, Vagn Lundsgaard Hansen, Technical University Copenhagen, Mogens Niss, University of Roskilde, Ebbe Thue Poulsen, University of Aarhus, and Ole Skovsmose, University of Aalborg

My work forms a part of this overall initiative It is supported by the Research Council for the Humanities, and additional financial support is received from Den Obelske Familiefond and University of Aalborg My overall project of developing a philosophy of a critical mathematical education gets inspiration from some experimental work and research carried out in relation to the overall initiative The school and the teachers involved are Klarup skole, Henning Boedtkjer; Aalborg Friskole, Ole Dyhr, Jens Joergen Andersen and Thue Oerberg; Nyvangskolen i Randers, Ib Trankjaer Bjarne Würtz Andersen and Ane Marie Krogshede Nielsen; and Roesenbaekskolen i Hinnerup Joergen Vognsen In addition Andreas Reinholt from Aalborg Teacher Training College is involved as co-ordinator for the experimental work The involved researchers are Morten Blomhoej, Iben Maj Christiansen, Dan Eriksen, Kirsten Groenbaek Hansen and Lena Lindenskov

For advice and comments on this paper I would like especially to thank Evelyn Arizpe, Alan Bishop, Iben Maj Christiansen, Paul Dowling, Marilyn Nickson, Ruth Loshak and Kenneth Ruthven Mistakes are, as usual, caused by myself

References

Adorno, T [1971] *Erziehung zur Mündigkeit*, Frankfurt am Main: Suhrkamp

Bell D [1980] "The Social Framework of the Information Society" in T Forester, *The microelectronics revolution* Oxford: Blackwell

Benn, S I., and P S Peters [1959]: *Social principles and the democratic state* George London: Allen and Unwin.

Boos-Bavnbek B [1991] "Against Ill-founded, Irresponsible Modelling", in M Niss *et al* (eds) *Teaching of mathematical modelling and applications*. Chichester: Ellis Horwood.

Bowles, S. [1983] "Unequal Education and the Reproduction of the Social Division of Labour", in B Cosin and M. Hales, *Education policy and society*. London: Routledge and Kegan Paul, 27-50

D Ambrosio, U [1985] "Mathematics Education in a Cultural Setting" *International Journal of Mathematical Education in Science and Technology*, Vol 16 469-477

Davis P J. [1989] "Applied Mathematics as Social Contract", in C. Keitel (ed.): *Mathematics, education and society* Paris: UNESCO, 24-27

Davis P. J and R Hersh [1988] *Descartes dream: the world according to mathematics*. London: Penguin Books

Ernest P [1991] *The philosophy of mathematics education* Brighton: The Falmer Press

Frankenstein M. [1987] "Critical Mathematics Education: An Application of Paulo Freire's Epistemology", in Ira Shor (ed) *Freire for the classroom*. New York/London: Heineman, 180-210

Frankenstein, M [1989] *Relearning mathematics* London: Free Association Books

Giroux, H A [1988] *Schooling for democracy. critical pedagogy in the modern age*. London: Routledge.

Habermas J. [1984, 1987] *The theory of communicative action I-II* London: Heinemann and Cambridge: Polity Press.

Held D [1987] *Models for democracy* Cambridge: Polity Press

Keitel, C [1989] "Mathematics and Technology" *For the Learning of Mathematics*, Vol 9, 7-13

Mill, J S [1975] *Three essays on liberty, representative government and the subjection of women* Oxford: Oxford University Press

Niss, M [1983] "Considerations and Experiences Concerning Integrated Courses in Mathematics and Other Subjects", in M Zweng *et al* (eds) *Proceedings of the Fourth International Congress on Mathematical Education* Birkhauser 247-249

Peters, R S [1966]: *Ethics and education* London: George Allen and Unwin.

Purvis, J. and M. Hales (eds) [1983] *Achievement and inequality in education* London: Routledge and Kegan Paul.

Rousseau, J-J [1968] *The social contract*. London: Penguin Books

Schumpeter J A [1987] *Capitalism, socialism and democracy* London: Unwin Paperbacks

Skovsmose, O. [1985] "Mathematical Education versus Critical Education" *Educational Studies in Mathematics*, Vol 16 337-354

Skovsmose O. [1988] "Mathematics as Part of Technology", *Educational Studies in Mathematics*, Vol 19, 23-41

Skovsmose, O [1989a] "Towards a Philosophy of an Applied Oriented Mathematical Education". in W Blum et al. (eds) *Applications and modelling in learning and teaching mathematics* Chichester: Ellis Horwood, 110-114

Skovsmose, O. [1989b] "Models and Reflective Knowledge" *Zentralblatt für Didaktik der Mathematik*, Vol 21 No 1 3-8

Skovsmose O [1990a] "Mathematical Education and Democracy", *Educational Studies in Mathematics*, Vol 21, 109-128

Skovsmose, O [1990b]: "Reflective Knowledge: Its Relation to the Mathematical Modelling Process", *International Journal of Mathematical Education in Science and Technology*, Vol 21, 765-779

Taylor, F W [1947] *Scientific management* New York: Harpers and Brothers

---

You should not forget however, that there are two kinds of generalizations One is cheap and the other is valuable It is easy to generalize by *diluting*; it is important to generalize by *condensing* To dilute a little wine with a lot of water is cheap and easy To prepare a refined and condensed extract from several good ingredients is much more difficult but valuable Generalization by condensing compresses into one concept of wide scope several ideas which appeared widely scattered before Thus the Theory of Groups reduces to a common expression ideas which were dispersed before in Algebra Theory of Numbers Analysis Geometry Crystallography and other domains

Georg Polya
