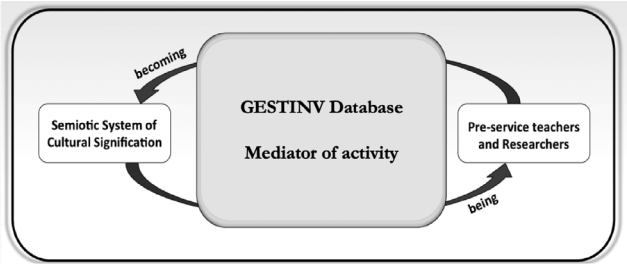

Figure 1. The model for pre-service teachers' professional development.

the design of an activity for students *etc.,* which should highlight teachers' reflections, convictions and beliefs.

General discussion. The sub-groups present their materials in a written or oral form to the whole group. Each presentation is discussed with a researcher, in order to highlight beliefs and convictions, and tackle doubts, difficulties and unclear content regarding both the mathematical content and the analysis of the related learning processes. Another setting for this final phase requires each sub-group to prepare a written presentation that is exchanged, so that each sub-group presents orally the material of another sub-group to the whole group. The final discussion based on the oral presentations is carried out with the same characteristics described above and aims at investigating pre-service teachers' subjectification.

We have implemented the model in a pre-service teacher professional development program that involved 140 students of the Faculty of Education of two Italian universities in 2019-2020. The use of standardized assessment data via Gestinv allows pre-service teachers' process of subjectification and their ensuing positioning in the social and cultural world of education as prospective mathematics teachers.

Concluding remarks

Our aim is to acknowledge the need for operative interventions informed by standardized assessment and a critical approach that addresses its informative and developmental potentials. We focussed our attention on pre-service teachers' professional development as one of the possible routes to realize educational interventions informed by standardized assessment that take advantage of its informational and developmental potentials.

We therefore devised a model for pre-service teachers' professional development that rests on two interacting pillars:

- a theoretical framework cast in Radford's Theory of Objectification that conceives teacher training as a subjectification process;
- Gestinv database, which realizes the entanglement of Di Martino and Baccaglini-Frank's *informational* and *developmental potentials* of standardized assessment.

Our model, based on processes of subjectification, is particularly appropriate to frame pre-service teachers' professional development, since prospective teachers can be seen as individuals who are transitioning from the identity of a student to the identity of a teacher. Thus, the notion of subjectification as a dynamic positioning in a particular social space (the school system) appears effective to frame the development of teachers' professionalism. The model brings a novel understanding of teacher professional development based on the production of new subjectivities according to the dialectical materialist philosophical stance advocated by Radford.

As a final remark, we can say that the need for a critical and operational approach towards standardized assessment quantitative data has opened a possible new trajectory in pre-service teachers' professional development that ascribes a prominent position to cultural-institutional features intertwined with the production of new subjectivities.

Notes

[1] http://www.gestinv.it

[2] Quadro di riferimento delle prove di INVALSI di matematica. https://tinyurl.com/FLM43-2-1

References

Bartolini Bussi, M.G. (1996) Mathematical discussion and perspective drawing in primary school. *Educational Studies in Mathematics,* **31**(1-2), 11–41.

Di Martino, P., & Baccaglini-Frank, A. (2017) Beyond performance results: analyzing the informational and developmental potentials of standardized mathematics tests. *For the Learning of Mathematics,* **37**(3), 39–44.

Ferretti, F., Gambini, A., & Santi, G. (2020) The Gestinv database: a tool for enhancing teachers professional development within a community of inquiry. In Borko, H & Potari, D. (Eds.) *Proceedings of the Twenty-fifth ICMI Study School Teachers of mathematics working and learning in collaborative groups,* 621–628. University of Lisbon.

Ferretti, F., Santi, G.R.P., & Bolondi, G. (2022) Interpreting difficulties in the learning of algebraic inequalities, as an emerging macro-phenomenon in Large Scale Assessment. *Research in Mathematics Education* **24**(3), 367–389.

Radford, L. (2015) Methodological aspects of the theory of objectification. *Perspectivas da educação matemática* **8**(18), 547–567.

Radford, L. (2016) Mathematics education as a matter of labor. In Peters, M.A. (Ed.) *Encyclopedia of Educational Philosophy and Theory* (unpaginated). Springer.

Radford, L. (2020) Play and the production of subjectivities in preschool. In Carlsen, M., Erfjord, I. & Hundeland, P.S. (Eds.), *Mathematics Education in the Early Years: Results from the POEM4 conference 2018*, 43–60. Springer.

False friends in mathematics education: Implications for meaning making in multilingual classrooms

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In this short communication, I focus on the linguistic barriers that make teaching and learning mathematics challenging for students who are neither native speakers nor are communicatively competent in the medium of instruction. When learning a new language, learners may encounter

new words that strongly resemble words from their native language. Word pairings in two languages that appear to have similar phonetic forms but have entirely different meanings are known as 'false friends'. The term 'false friends' was introduced in 'Les Faux Amis' by Koessler and Derocquigny (1928) illustrating the betrayals of English vocabulary for francophone speakers learning English. For example, the word 'magasin' in French or *'maghaze'* in Persian refers to a shop or store, but native English speakers could easily mistake it for a published periodical because of its close phonetic resemblance to the word 'magazine'.

In the past, there have been discussions in mathematics education on homophones and polysemy (Zazkis, 1998), which are different from false friends. Homophones are words that sound alike (having different meanings) within the same language, such as 'eight' and 'ate', 'weight' and 'wait', 'won' and 'one', but false friends originate from different languages. For a more mathematical example of false friends, German offers an interesting term *'zirkel'*, sounding close to the English 'circle' or Spanish *'círculo',* but actually means 'compass'. The Spanish *'compás'* refers to 'a time signature' or 'a metre signature' indicating the number of beats in each musical measure, while in English, a compass has a completely different mathematical meaning.

English and Norwegian offer interesting examples of false friends. The term *'gift'* in Norwegian has got nothing to do with what one might receive for Christmas, but instead means 'married' in English. Similarly, the term *'akkurat'* in Norwegian sounds very much like accurate, but it means 'just' in English. In mutually intelligible languages such as Spanish and Portuguese, the same word may have a diametrically opposite meaning. For example, *'tirar'* in Spanish means 'to throw' and in Portuguese it means 'to take' or 'obtain'. A more mathematical example could refer to *'largo'*, which means 'long' in Spanish but 'wide' in Portuguese. Farsi (commonly known as Persian) and Dari are other examples of mutually intelligible languages spoken in Iran and Afghanistan, respectively. What Iranians call *'sisad'* means 300, but to the speakers of Dari in Afghanistan it mean 3000. "How far do we have to climb up the mountain? Oh it is only *sisad* meters". A similar numerical confusion happens between English and most other European languages. An English 'billion' is 10^9, but in Norwegian and French 'billion' means 10^{12}.

English and Spanish also offer different affordances of false friends, both outside and within the mathematical context. In particular, I want to refer to the Spanish term *'molestar'* meaning 'to bother' or 'to disturb', while in English, 'molest' has a very different connotation referring to a sexual assault or sexual abuse. I remember an event where I was in Patagonia (in the south of Argentina), visiting animals that were exotic to me coming from the deserts of the Middle East. Many tourists were visiting the penguins, and bilingual signposts said *'No molestar a los pinguinos'* along with its English translation 'Do not molest the penguins'. Next to me was a middle-aged Irish couple. I saw and heard the woman turning to the man and saying, "What kind of sick people would ever wanna do that? Ew!" pulling down her eyebrows and wrinkling her nose. Similarly, some terms in mathematics may not be mutually intelligible across languages. Hirigoyen (1997) observed that, in Romance languages, "The word *trapezoid* is reserved for a quadrilateral without any parallel sides, whereas *trapezium* is used when there is one pair of parallel sides" (p. 167). In UK English the words have the same meanings, while in US English, the meanings are the reverse; 'trapezoid' refers to a quadrilateral with two sides parallel, while 'trapezium' refers to a quadrilateral with no sides parallel. It appears that false friends, if not paid particular attention to, can cause not only errors and breakdowns in communication in bi/multilingual mathematics classroom settings, but also afford different conceptual understandings.

Even the concept of time can offer different affordances to the native speakers of the same language. For example, in Spanish, the term *'ahora'* meaning 'now' has a diminutive form 'ahorita' meaning 'right now' in Chile; however, in Colombia or Venezuela, it means 'in 30 minutes or so'. Furthermore, a language like Spanish allows for double diminutives. In this case, *'ahoritita'* would mean 'right right now', whereas in Colombia or Venezuela, it would convey the meaning of 'in an hour or so'. Similarly, 'just now' in South Africa means 'later' in the UK. Given how the concept of time is expressed in the same language may provide different affordances to its interlocutors. It can also be ambiguous to bilingual learners. For example, to many English and Spanish speakers, 'half nine' means half-past nine, or 9:30. However, this particular interpretation may not be in line with Norwegian, Swedish and German, where 'half' takes a subtractive form from the following number. For instance, in Norwegian, the time *'halv ni'* (half nine) or in modern German, *'halb neun'* (half nine) does not mean 'half past nine' but 'half to nine' or 8:30. Due to the nature of subtractive counting, one might get the timing wrong but would hope people can get the days right, right? Surely, Monday would be Monday, Tuesday would be Tuesday and so forth. It is surprising how Persian and Norwegian languages offer a different linguistic conceptual understanding of the days of the week based on subtractive counting, compared to English and Spanish. One can express 'Friday night' in English or '*viernes noche'* (*viernes* = Friday, *noche* = night) in Spanish, but in Norwegian Friday night is *'natt til lørdag'* (*natt* = night, *til* = to, *lørdag* = Saturday) and in Persian *'shab-e shanbe'* (*shab-e-* = night of, *shanbe* = Saturday). Both Norwegian and Persian use subtractive counting in qualifying the days of the week.

Examining mathematical discourse practices enable us to raise awareness of culturally responsive teaching (Moschkovich, 2007). Mathematical discursive practices can be verbal, vocal or visual. Texts and writing materials contribute significantly to meaning-making practices. Whenever there are potential affordances for meaning-making practices, there are also potentials for mismatches, too. For example, in most English, Norwegian and Spanish textbooks, the names of kings and queens are written in roman numerals but are pronounced as ordinal numbers. For example, Henry VIII: Henry the Eighth (not Henry Eight). How something is written does not necessarily correspond to how it is read. French, on the other hand, offers consistency between the verbal and visual. For example, Louis XIV is pronounced as Louis quatorze and not quatorzième.

Another false friend in writing mathematical texts can be observed by examining the thousands separator (or digit group separator). In England, commas are used to divide large numbers into groups of three figures by separating the thousands and the millions: 79,521,989. In Chile dots are used instead of commas: 79.521.989 and in Iran, 'subscript slashes' are used instead 79⁄521⁄989. Furthermore, how numerals are written can also cause confusion among speakers of different orthographic systems. The way '7' is written in English "looks very much like what many cultures write as a 1" (Hirigoyen, 1997, p. 165). The numerals that are employed in most Middle Eastern countries represent 6 as ٦ which resembles 7 is employed in the West. Gorgorió and Planas (2001) have reported a case where a Bangeladeshi student in Spain confusingly said, "My six is your seven" (p. 257).

This short communication reminds us that while formulating teaching strategies teachers need to be alert to possible mismatches that mathematics learners can draw from the existing false friends in the languages which they are familiar with. Although building on students' prior knowledge is perceived to be an acceptable practice, it is equally essential to be aware of how bilingual (or multilingual) learners' prior knowledge can interfere with new learning and understanding in mathematics classrooms. Therefore, the teaching practice needs to take into account possible confusion in mathematical meaning making or incorrect inferences that can emerge while connecting between and across languages. The language, both as a medium and message, can influence the form and function of what is perceived. It appears that false friends, if not paid particular attention to, can cause not only errors and breakdowns in communication but also afford different conceptual understandings in mathematics. However, language is not primarily limited to the verbal, but also vocal and visual. We have seen how false friends can affect the linguistic resources for making mathematical meaning. There are other multimodal components of mathematical meaning (O'Halloran, 2015), such as symbols and visual representations, that are affected by false friends to a lesser extent. Therefore, by integrating an eclectic approach towards foregrounding multimodally in multilingual mathematics classrooms could be perceived as a short solution towards avoiding false friends in multilingual mathematics contexts.

Linguistic differences can be 'language resources' in bi/multilingual classrooms whereby students could go back and forth using their linguistic and semiotic resources drawing from different languages, registers and modalities, as an opportunity for learning. However, it is equally as important for educators to think about and be aware of how certain words/embodied actions/gestures could be perceived totally different as students go back and forth using their linguistic and semiotic resources in mathematics classrooms (Farsani, Lange & Meaney, 2022). Knowing and raising awareness of how our mathematical discourse and our embodied actions can influence students' understanding is the first step towards solving this ongoing struggle.

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References

Farsani, D., Lange, T. & Meaney, T. (2022) Gestures, systemic functional linguistics and mathematics education. *Mind, Culture, and Activity* **1**, 75–95.

Gorgorió, N. & Planas, N. (2001) Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics* **47**(1), 7–33.

Hirigoyen, H. (1997) Dialectical variations in the language of mathematics: a source of multicultural experiences. In Trentacost, J. & Kenney, M.J. (Eds) *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity,* 164–168, National Council of Teachers of Mathematics.

Koessler, M. & Derocquigny, J. (1928) *Les Faux Amis, ou les Trahisons du Vocabulaire Anglais (Conseils aux Traducteurs).* Vuibert.

Moschkovich, J. (2007) Examining mathematical discourse practices. *For the Learning of Mathematics*, **27**(1), 24–30.

O'Halloran, K.L. (2015) The language of learning mathematics: a multimodal perspective. *Journal of Mathematical Behavior* **40**(A), 63–74.

Zazkis, R. (1998) Divisors and quotients: acknowledging polysemy. *For the Learning of Mathematics* **18**(3), 27–30.

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