THE WRITING MATHEMATICIAN

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Writing isn’t like math; in math, two plus two always equals four no matter what your mood is like. With writing, the way you feel changes everything.
—Stephenie Meyer [1]

I feel for my students when I hand them their first essay assignment. Many are mathematicians—students and teachers—who chose to study mathematics partly to avoid writing. But in my mathematics education courses, and in the discipline more generally, academic writing is part of our routine practice.

Mathematicians face some challenging stereotypes when it comes to writing. As the Meyer quote suggests, writing is often seen as ephemeral, subjective and context-dependent, whereas mathematics is seen as enduring, universal, and context-free. Writing reflects self, mathematics transcends it: they are essentially unlike each other. This false dichotomy of writing versus mathematics can discourage mathematicians from writing, especially when combined with similarly coarse dichotomies such as right brain \| left brain, creativity \| logic, and art \| science. Taken together, these dichotomies suggest that writing is outside the natural skillset of the mathematician, and that one’s mathematics training not only neglects one’s development as a writer, but also actively prevents it.

Where does this writing \| mathematics dichotomy come from? It is profoundly unhelpful for our discipline of mathematics education: it assumes a gatekeeping role, turning away potential newcomers who might otherwise have a lot to offer. This essay deconstructs the writing \| mathematics dichotomy by identifying similarities between the practices of academic writing and mathematics. I offer three writing-mathematics metaphors based on these similarities: writing as modelling, writing as problem-solving, and writing as proving (see Table 1). I propose that these metaphors might encourage students who identify more as mathematicians than writers to recognize and replace unproductive writing beliefs and practices with more productive ones that are grounded in familiar mathematical experiences.

I wish to delimit my intentions around this offering. I am not suggesting that mathematicians who read this essay will automatically become highly competent writers, simply by acknowledging similarities between writing and mathematics. Nor am I suggesting that one’s skill in mathematical modelling, problem-solving and proving transfer effortlessly to the domain of writing. My goals are more modest and realistic. I want my students to work on their writing, just as they have worked on their mathematics. The three metaphors are offered as encouragement to begin this process, rather than being intended as a remedy or solution. I propose that mathematicians do not have to see themselves as starting from nothing when they engage in academic writing. Rather, they can view their writing development as building on competencies they have already honed in their mathematical training, but which they may not have formerly recognised as writerly.

The imperfect ideal: writing as modelling
Let us consider a prototypical mathematics education student who has spent weeks thinking, reading and talking...
about her essay topic, but only starts writing it the night before it was due. She writes one draft only—the one she hands in, and is disappointed with the low grade her essay receives. She wishes she had started earlier, but rationalises to herself that she was still trying to figure out what she wanted to say up until the moment she started writing. It was only the pressure of the deadline that forced her to start writing: without it, she would have spent even more time thinking and reading to develop her ideas. After all, she reasons, there is no point writing when you do not know what to write about!

This “think first, write after” approach, sometimes known as the “writing up” model is a dangerous trap that many students fall into, and is at odds with the way writing often works (Menary, 2007). The approach allows no room for an iterative drafting process, whereby imperfect drafts are written that are not intended as final copy, but are necessary steps towards subsequent improved drafts. Writing experts trade on the generative power of imperfect writing—they encourage writers to turn off their internal critic and allow themselves to write badly as a way of overcoming writing inertia and discovering new ideas (Elbow, 1985). Lamott heralds the “shitty first draft” (1994) as an ideal (and achievable!) first goal in the writing process: anyone can produce a sketchy first draft that generates material that can be worked on, improved, and eventually rewritten into a more sharable form.

Mathematical modelling offers a compelling metaphor for the generative power of imperfect writing in the creative process. Like polished writing, polished mathematical models are seldom produced in a first attempt. A modeller typically begins with some understanding of the real situation to be modelled. The modeller mathematizes variables and relationships from his or her understanding of the real situation, and ‘writes’ it into an initial mathematical model. Next, the modeller runs the model to test it, and interprets the results back into the real situation, comparing the mathematical output with real data. At this point, the modeller may notice information in the real situation that was previously missed, create a revised model based on this enhanced interpretation of the situation, and subject the model to further testing and revision, ultimately going through the full modelling cycle multiple times until s/he is satisfied (Borromeo-Ferri, 2006). The model is his or her mathematical description of the situation, written in mathematical notation, and the modeller who publishes a mathematical model has typically created and discarded multiple models along the way, just as the writer who publishes a piece of writing has typically written and discarded multiple drafts along the way.

Novices to modelling might regard this iterative process as a waste of time: Why bother creating models that will only be discarded? The novice might try to bypass the iterations, thinking that if one thinks harder in the first place, one might produce a better model in an initial attempt. But the experienced modeller takes the more efficient approach of entering into modelling cycles, not avoiding them. Models self-propagate: the model one produces to express one’s interpretation of a situation becomes a conceptual lens through which one can review the situation (Lesh & Doerr, 2003); on this re-viewing, one can notice deeper levels of structure that can be incorporated into a more powerful model. The modeller who views a real situation through multiple different models will typically notice more than one who spends the same amount of time trying to understand the real situation from an initial, untested point of view in the hope of producing a perfect first model. Similarly, the writer who writes, reads, and revises multiple drafts will likely develop his or her ideas further than the writer who only thinks about his or her ideas in the hope of producing a perfect first draft. Modelling, like drafting, expects imperfection. This can be liberating: one can create a bad first model without worrying that it means one is a bad modeller. Modelling can be a useful metaphor for the generative role of writing for the mathematics education student who does not know what to write for her essay. Instead of waiting to figure out her ideas before writing, she can allow herself to write a bad first draft—full of contradictions, unfinished thoughts, ideas that are not structured well—in order to generate material that can be worked on and improved. Rather than viewing her bad first draft as evidence that she is a bad writer, she can view it as a useful tool for figuring out what she wants to say. She can enter into the drafting cycle knowing that the writing she does now will help her figure out what she wants to say.

The thinking laboratory: writing as problem solving

The generation of ideas is just one of writing’s roles; writing also plays a role in the analysis and organisation of ideas that have been generated. Our prototypical student comes close to experiencing this second role when she knows what she wants to write about, but struggles to write it. She has a thesis and outline for her intended essay, but the writing is painstaking. She spends hours writing the first sentence, only to delete it the next day. She throws up her hands and complains that she knows what she wants to say, but does not know how to say it—that she is ‘bad at writing’, and that if she were ‘good at writing’ she would be able to convey the same ideas more effectively, more eloquently. Good writers, she thinks, would not get stuck like this; their writing would flow elegantly from their pen.

It is an intimidating expectation. Even the most ‘talented’ writer can be scared into inaction by the demand for elegant sentences created on the spot: “Be brilliant. Now!” Rather than finding it easy, many writers approach writing as an act of problem solving where getting stuck is a natural and expected part of the process. They may have a clear goal in mind, but do not know how to get there. They write carefully, analysing their writing as they go, shifting their attention back and forth between the writing that has been done and the goal that is beyond their reach. They scrutinise their inscriptions,
but not 'writing' [3]. Of course, we are talking about differ-
'writing up' is 'working', 'thinking', or 'figuring things out',
up one's results', and the semiotic activity that precedes the
most mathematicians, writing is synonymous with 'writing
ing ideas around the page or board (Livingston, 2015). To
and pattern—involves scratching out symbols, marks, mov-
relationships and objects to notice new levels of structure
and to create and work towards new ones.

Mathematical problem solving has been characterised as
involving a similar process of working back and forth
between givens and goals and posing new problems while
exploring existing ones (Kontorovich, Koichu, Leikin &
Berman, 2012). Mathematical problems require the solver to
combine known ideas in previously unknown ways to create a
solution method that was not already known. Problems have
been distinguished from mathematical exercises (Schoen-
feld, 1985), which are strings of structurally similar questions
that require students to practice a recently learned procedure.
A student who has mastered a procedure will complete exer-
cises quickly, easily and accurately, much like the mythical
"good writer" who issues a stream of perfect sentences in one
sitting without breaking a sweat. This difference is impor-
tant: exercises require less mental effort because the script is
already known; problems are more demanding, as they
require the solver to create the script as they go.

Writing an original essay is like trying to solve a prob-
lem—there is no script to follow, it must be created by
analysts and writers ensnaring themselves to re-evaluate their
goals and figuring out how to achieve them. In both essay writing and mathemati-
cal problem solving, getting stuck is natural and
expected—it is even a special kind of thrill. When mathe-
maticians get stuck, they engage in metacognitive activity
(Garofalo & Lester, 1985), re-viewing their desired goal and
comparing it to their current ways of thinking in order to
identify their domain of validity (Broussard, 2002)—the
domain where their ways of thinking work and the point at
which they break down. They also try to shift their ways of
attending (Mason, 2003) to the problem, often by employing
heuristics such as working backwards, drawing a diagram,
or solving a related problem (Polya, 1945) in the hope of
finding productive ways of thinking. They usually conduct
such activities with a writing implement (pencil, chalk,
whiteboard marker, keyboard) in hand, creating a "coded
system of visible marks [...] whereby a writer could deter-
mine the exact words that a reader would generate from a
given text" (Ong, 1982, p. 83). That is, they write.

This last observation may come as a surprise to mathe-
maticians who do not think of their problem solving activity
as writing. Doing mathematics—that is, the ordinary every-
day concrete details of manipulating mathematical
relationships and objects to notice new levels of structure
and pattern—involves scratching out symbols, marks, mov-
ning ideas around the page or board (Livingston, 2015). To
most mathematicians, writing is synonymous with 'writing
up one's results', and the semiotic activity that precedes the
'writing up' is 'working', 'thinking', or 'figuring things out',
but not 'writing' [3]. Of course, we are talking about differ-
ent kinds of writing. The 'writing up' of results is
writing-as-reporting, performed at the end of the problem-
solving. In contrast, the semiotic activity that precedes
'writing up' is a more analytic form of writing used to
restructure one's ways of thinking during the problem-solv-
ing process. In academic writing, this writing-as-analysis is
often performed without the expectation that it will appear
intact in the final draft; it is often performed on scrap pieces
of paper, with scribbles and arrows used to help the writer/thinker restructure his or her help mathematicians see
new levels of structure. The archetypal story of mathematical
discovery tells how Poincaré unexpectedly realised the link
between non-Euclidean geometry and complex function the-
ory while stepping on a bus, resumed his previous
(non-mathematical) conversation for the rest of the bus ride
with quiet certitude of his discovery, and only verified it when
he got home—presumably with pen and paper (Hadamard,
1945). The story creates a powerful image of pure thought (or
divine inspiration) as the source of mathematical discovery,
and diminishes the role of writing to verification. Mathemati-
cians' reluctance to call their analytical writing 'writing' may
also be motivated by a mathematical aesthetic that depends
on the purity, simplicity, and abstraction of mathematics from
the materiality of the so-called real world: ink and chalk dust
are substantial, but the ideas they express endure and tran-
send. Perhaps this aesthetic prevents some mathematicians
from acknowledging their role as writers who deal in messy
physical marks, preferring instead to consider themselves
thinkers reflecting on the ideal forms those marks express.

Why do I care that mathematicians acknowledge their
semiotic activity as 'writing'? Quite frankly, because they are
good at it. They have spent years honing their ability to use
writing to restructure their thoughts, to dissect their ideas,
identify new arguments—they possess an analytic discipline
that most writers struggle with. Yet few of my mathematics
education students take advantage of this in their academic
writing. They want their writing to come out in consecutive,
polished sentences, and become discouraged when it does
not, rather than using their writing to analyse and probe their
arguments as they do when they are stuck on mathematical
problems. By viewing writing only as a medium for communicating perfectly formed thoughts, they deny themselves their own laboratory, their own thinking tools.

I am not suggesting that one’s success in solving mathematical problems automatically translates into successful essay writing. But the metaphor of writing as problem solving may encourage a mathematics education student not to give up too easily when she finds herself stuck in her writing. Perhaps she can approach her writing metacognitively, reviewing her central thesis in light of her arguments, articulating her goals and identifying limitations in her current ways of thinking like she does when working on a mathematical problem. She may even try using problem-solving heuristics to break her current ways of attending to her writing, and attend to her writing (and ideas) in new ways. At the very least, she may come to view her stuckness as a natural and expected state (Burton, 2004), just as mathematicians do with mathematics, and recognise it as an opportunity to analyse and restructure her ideas.

**Entering into dialogue: writing as proving**

Our prototypical student now has a good draft that she is happy with. She is satisfied it represents her knowledge of the subject matter, and has read extensively to check the accuracy of its content. A friend reads the draft and finds it difficult to understand. Unperturbed, our prototypical student attributes the reader’s difficulty to insufficient subject knowledge. She is confident that her essay demonstrates her mastery of the topic, and that this will be recognised by her more knowledgeable teacher.

But the essay is not merely an inert record of a writer’s knowledge, and its quality is not merely judged on the number of correct facts it contains. The essay is also a rhetorical act that seeks to engage a public. It has to do real work as a dialogic tool: it must address an audience; it must convince or persuade a public; it must inspire some kind of response or action. Writers take considerable care to shape their writing’s addressivity (Bakhtin, 1986) by seeking insight into their readers’ experience of the writing.

Mathematical proofs are like expository essays in this regard: they must also convince an audience. When an undergraduate mathematics student switches from memorising other peoples’ proofs to constructing proofs of her own, she becomes concerned with how her proof (writing) performs under dialogic conditions. She is encouraged to test her proofs on different audiences: “convince yourself, convince a friend, convince a public; it must inspire some kind of response or action. Writers take considerable care to shape their writing’s addressivity (Bakhtin, 1986) by seeking insight into their readers’ experience of the writing.

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**Notes**


[2] Readers may object to the use of the term ‘real’—is mathematics not real? Some writers on modelling us the term ‘extra-mathematical’ instead of ‘real’ to acknowledge the ontological slipstream; Borromeo-Ferri (2006) points out that some demarcation between mathematical and ‘real’ or ‘extra-mathematical’ worlds is necessary and the use of ‘real’ to describe the ‘extra-mathematical’ can be a pragmatic choice.

[3] See also Latour and Woolgar’s (1979) anthropological observation of laboratory scientists as “compulsive and manic writers […] who spend the greatest part of their day coding, marking, altering, correcting, reading, and writing” (pp. 48-49).

**References**


Austin, TX: University of Texas Press.


