Mathematics as an Infinite Game

ALTON T. OLSON

Recently while reading James P. Carse's wonderful book *Finite and infinite games* it occurred to me that thinking of mathematics as either a finite or infinite game was a helpful way to sort out the issues that swirl around the teaching and learning of mathematics. For instance, much of our teaching efforts are directed toward making mathematics a finite game for our students. We emphasize the mastering of finite skills and our students naturally come to see mathematics as a finite game. Of course, the great mathematicians of history saw mathematics as the quintessential infinite game. That was precisely its attraction. This is the paradox of mathematics teaching. How can students be expected to recognize and value the finiteness, important as it is, that is embedded in the more essential infinitude of mathematics?

When mathematics is thought of as a *finite* game it is played for the purpose of finding an ending. Successfully finding an ending in doing mathematics is winning the finite game. In contrast, mathematics as an *infinite* game is played for the purpose of continuing the play. As an infinite game mathematics is potentially recursive. A finite piece of mathematics becomes the input for mathematics at a higher level thus insuring its continuance. It must be thus for otherwise it would have died long ago of over-familiarity.

In most cases students come to see mathematics as a series of finite games and, admittedly, they can even enjoy it as such. Success at finite tasks becomes the vision of mathematics as finite games. Unfortunately many students never even get that far. These students feel too much coercion over their participation. Paradoxically, to the extent that players feel that they *must* play, they *cannot* play. As Carse states, "It is an invariable principle of all play, finite and infinite, that whoever plays, plays freely." [p. 4] For teachers this is indeed a paradoxical situation.

Infinite and finite glames are distinguished in their use of boundaries. Finite games have definite boundaries. For most students mathematics has a definite spatial and temporal boundary defined by the mathematics classroom. At some level finite games also have rule boundaries that define what is acceptable, whereas infinite games have boundaries that are subject to change from within the game as it is played.

It is time for an example.

A finite game player challenges an infinite game player to a game of Nim. (One version of this game requires that the players begin with 21 matches and alternately pick up 1, 2, or 3 matches with the loser being the person forced to pick up the last match.) The finite player will aim at maximizing his chances of winning at this game and ending the play as the winner. The infinite player will not be interested in ending this game but will begin to see this game as one of a family of Nim-type games. She will note that: the number of matches can be varied, the number of matches to be picked up can be varied, the number of players can be varied, the definition of winning can be varied, etc. How do these variations change the nature of the winning strategies? The infinite player will immediately begin to play with the boundaries. The boundaries are a necessary part of the playing but they are a part of the play as well. The boundaries create little patches of finiteness for the infinite player but these boundaries are movable.

When mathematics is an infinite game then it cannot be bounded by externally applied rules or be bounded by spatial or temporal considerations.

The infinite player is not interested in games that seem to be inherently finite for a game that is finite defines a definite world for engagement. An infinite player has to have the freedom to choose her world of engagement for the infinite game. Where was the infinite player in the Nim game playing? In the world of modular number systems? Perhaps, but, in general, it is not possible to know where an infinite mathematics player is playing.

The infinite Nim player is still subject to constraints that define the boundedness of the game. The basic rules are agreed upon before playing the game and are not broken willy-nilly; but they *can* be changed. To see mathematics as an infinite game is to understand the rules that govern the game but to know that those rules are validated only by the players who play the game and that they can change the rules. How can we teach the rules of mathematics *and* the sense of possibility that the rules can be changed? Another paradox?

What is the nature of this paradox? Why do the finite players in mathematics believe that whatever they do they must do?

* There is a sense that they have been selected to be there. They do not have a sense that they have chosen to be there. From that station whoever *must* play *cannot* play.
* Mathematics as a finite game is played to be won. Anything that does not contribute to that winning is not to be considered further.
* Everyone tells students that the prizes for that winning are indispensable to them and by extension to whole societies.

It is a paradoxical situation, but recognizing it as such is the first step toward being able to extricate oneself from its effects.

Infinite players recognize and willingly participate in the finite parts of mathematics. However, they also know that the finite parts are an abstraction from the whole of mathematics. They are an abstraction in the sense that parts have been substituted for the whole The practicing of finite skills in mathematics is only a part of the whole piece from which the finite skills are abstracted Everyone approaches this practicing of finite skills as serious business. However, if it is done as part of an infinite game it is not serious — it may be required but it is not serious. When the practicing is done only for the skills which are abstractions from the whole then the practicing is serious and cannot be done in a playful way. In a curious way nothing of consequence can come out of the serious efforts of the finite player. Only infinite play can produce consequential results.

The infinite player *chooses* to be a mathematician; the finite player *adopts the role* of a mathematician. During the doing of mathematics the finite player can be a mathematician but always with the intention of finishing the mathematics and finishing the role of mathematician. A student will readily admit to being a *reader* but not to being a *mathematician.* Unfortunately, much of our teaching of mathematics inadvertently restricts a student to the role of a mathematician.

In the ideal world of the finite player, mathematics should not be surprising. The finite player plays to predict and control the element of surprise. In contrast, the infinite player plays for the chance that surprise will be a part of the playing. The infinite Nim player actively seeks the surprises that emerge as a result of playing with the rules. In the words of Carse:

> To be prepared against surprise is to be *trained*. To be prepared for surprise is to be *educated*. [p 19]

Infinite and finite players in mathematics differ in other ways Since infinite players are not driven by the need to win they are not concerned about comparative *power* As a logical necessity the number of people with power in mathematics is limited because of its connection with winning Infinite players are more concerned about *strength* because strength is available to all who seek it in mathematics Power is required for success in finite games while strength is needed for the playing of infinite games in mathematics

Infinite and finite players also differ in the ways that they see mathematics Finite players see mathematics as something that is always repeatable whereas for infinite players mathematics is essentially nonrepeatable because infinite games have no particular ending. Finite players look for the boundaries in mathematics and are concerned about staying within those boundaries. Infinite players look for the horizons knowing that horizons change depending on the vantage point.

In summary, infinite players "do" mathematics in the course of being mathematicians Finite players "do" mathematics in order to bring it to a close.

Reference

Carse, James P *Finite and infinite games* New York: The Free Press, 1986