"You only do Math in Math": A Look at Four Prospective Teachers' Views about Mathematics

MARTA CIVIL

> I guess there always seems to be a way to plug in numbers or something in order to solve a problem, and I didn't use any, I mean I didn't even use an equation or anything, I just kind of used logic . . . If somebody gave me this problem in social studies, then I would say fine, this is a fine way to do it, but you know, if I had this problem in a math class, then you feel like you have to do it using addition or subtraction, something that you learn in math . you only do math in math. [Barb]

The above quote is from a preservice elementary teacher in her senior year. It illustrates some of her views about what doing mathematics entails. She was in the top group, by school standards, in a mathematics course for elementary education majors. What were her views about mathematics? How did she react to different problem-solving strategies?

The goal of the study reported here was to discover the views about mathematics of four preservice elementary teachers in a problem-solving context. The four subjects had all taken a mathematics course for elementary education majors. They represented different levels of mathematical ability as measured by their performance in that course. None of them had received any instruction in problem-solving based on Polya [1957]. My goal was not so much to find out about their problem-solving ability as it was to find out their ideas about doing mathematics. Thus the problems (see appendix) that the subjects worked on were essentially used to prompt them to talk about mathematics. The subjects were encouraged to ask for help if they got stuck on any of the problems. Each strategy, such as drawing a diagram, making a guess, making a list, looking for a pattern, was introduced as appropriate after the subject had worked on (and maybe even solved) the problem. I then encouraged the subject to solve the problem using that strategy. Each subject was individually interviewed in a tape-recorded session lasting about two hours. Though the subjects were interviewed only once, some patterns of behaviour emerged. These will be the subject of this paper.

What is proper in mathematics?

The four subjects, regardless of their ability level and their degree of success with mathematics, had a well established idea of what is proper in mathematics. The quote at the beginning of this article is an example of what Barb meant by a math way *vs* her way. A proper way to solve a mathematics problem involved the concepts of:

Neatness: writing the problems in terms of equations was viewed by subjects of different ability levels as something which was organized and clear to look at. David solved the watermelon problem in a very clear way. He talked it through, got a difference of $5.50 which he correctly attributed to two watermelons, and solved from there. When he tried to use algebra for this same problem, he was really insecure about the set up and needed a lot of help all along. Yet, he said:

> You see, here [pointing at his own method] it's not very clear, if someone wanted to look at it, you know there is no formula. I don't write everything clearly; so, if someone wanted to follow . while this [algebra] is somewhat easier to follow, mine is like shortcuts

Speed: trial and error was dismissed as being slow; Celia pointed out, in relation to what she would do in her own teaching:

> This [trial and error] is just as a last resort; I wouldn't encourage this; it's too time-consuming and they [the students] have to learn to do problems quickly, you know, for the SAT and so on

She did say, though, that trial and error "makes you understand the problem better, you did it yourself."

Professionalism: I am borrowing this term from the excerpt below in which Ellen is talking about the watermelon problem:

> It's a better way [algebra] to go about it than my prehistoric way, I know that. It's probably more, it's more (pause), it's uh, not professional, but it's probably just a better way to go about it than me sitting here, and doing "OK, 1 and 2, and 3," . I don't know.

It is interesting to note that she described her way as being at the eighth grade level, and the algebraic approach as being more advanced and at the college level. On the other hand, David viewed his method as a "more natural, adult-like way", and the algebraic approach as "the mathematical way they teach you to think in high school". He still gave preference to the algebraic method in terms of getting the right answer.

The subjects tended to accord a higher status to certain mathematics problems and ways to approach them, and to dismiss others as childish, not mature enough. For example, the ice cream problem made them feel uncomfortable, not because of its difficulty (it was ranked as the easiest by all the subjects), but because of its method of solution: drawing and counting. This method of solution would be fine for a social studies class, as Barb remarked, but not so in a mathematics class. And she added, "that's how I was taught, I mean I, you only do math in math."

The subjects did not seem comfortable with the use of informal techniques (drawing, listing numbers, trial and error) in mathematics. This is consistent with the studies that point out that students as early as second or third grade have developed a sense of what is legitimate and what is not in mathematics. Though they often start school with a rich baggage of informal techniques, their use to solve a mathematics problem is usually discouraged in school. The result of schooling seems to be the creation of two distinct behaviours, one inside the school and one outside [Baroody & Ginsburg, 1986; Cobb, 1985; Resnick, 1987].

In this study, the "proper" way to do mathematics problems revolves around algebra. This was to be expected given the nature of the problems used and the typical mathematics experience of an elementary education major at this university. This experience makes the association of algebra with any word problem almost immediate on their part. A lack of knowledge of other problem-solving strategies, or, if that knowledge was there, the fact that it did not have a label attached to it (such as "algebra"), and that it was usually not encouraged in their school mathematics, probably accounts for most of these feelings about the superiority of algebra. Algebra is certainly a powerful tool that should be made available to all students by eighth grade [Usiskin, 1987], if not earlier [Davis, personal communication, March 1986] Krutetskii [1976] points out that some mathematics educators actually question the point of solving certain problems with arithmetic tools. Their view is that these same problems can be solved later on in one's schooling in a more elegant, general, and efficient way when the student is exposed to algebra. But, adds Krutetskii, although the solving of arithmetic problems with algebra is simpler and more general than other ways, the value of these other ways is that they develop the ability to reason, to think logically.

The role of thinking in mathematics

The subjects agreed that they were thinking more about the problems when using strategies other than algebra.

> Taking the difference, you have to think about it a lot more, while this one [algebra] is just rote, you know, I can just pull it and plug it [Barb, on the watermelon problem]

> I mean, I understand why I'm doing it [using algebra], it makes sense, but I didn't really think of it myself, I did 20 million problems like this in high school [Celia, on the brandy problem]

This "thinking more" usually involved some reference to logic. The subjects seemed to make a distinction between thinking in terms of logic, and thinking in terms of mathematics. For example, here is what Barb said on this issue:

> That's probably why I had problems with calculus; I mean, I don't know, because you have to learn to think logically and not just how to do this, and then get the answer; and that was harder, I mean to think logically

Does that mean that until she got to calculus, mathematics for her had just been "how to do this and then get the answer"? What about those students (a majority among preservice elementary teachers), who never got to calculus?

Barb's comment below summarizes and illustrates very clearly my point:

> When I think of math I think of a formula that goes with it. A story problem, you set it up, you solve it, that's math.

She further added that she did not consider the thinking that goes on in one's head to be as much mathematics as the formulaic, written algebra approach. In her opinion the chickens and rabbits problem was more like problem-solving, logical thinking, than mathematics. Interestingly, this was the only problem for which all the subjects needed help. Barb and Celia had no difficulty at all with the two standard algebra problems (brandy and watermelon). When I introduced them to other approaches they carried them through and acknowledged they were thinking more about the problems then. Yet they showed little interest in these various problem-solving strategies.

On the other hand, the other two subjects seemed to think more about the problems and devised their own ways to solve them. These were not always "successful" in the sense of leading to a solution. But what I found troublesome was that even when these ways were indeed successful, the subjects did not trust them. They did not believe in their own ways of doing mathematics. They felt more confident when they could solve the problem using some pre-established mathematical procedure (even if they might not quite understand it).

> Actually I probably should use algebra because I'd come with proper numbers; this [his solution] could be wrong, I'm not so confident [David, on the watermelon problem]

Playing by the rules

What do these subjects view as doing mathematics? Applying procedures? Getting the answer? Thinking the problem through? Based on these interviews, I would not hesitate in describing the subjects' views of mathematics as very close to what Davis [1986] calls "a wrong view of mathematics". According to this view, doing mathematics would only refer to the actual writing down of equations, and other symbolic operations, rather than to the thinking process involved.

Unfortunately, typical school programs reinforce this wrong view of mathematics. As Grieb and Easley [1984] say, an authoritarian attitude prevails in elementary school mathematics classes. This attitude probably goes hand in hand with a rule oriented conception of mathematics instruction in which the stress is on executing algorithms. Some "successful" students in school mathematics may in fact owe their success to being good at following the rules. As Schoenfeld [1985] says, "Good student that she is, she knows how to play by the rules" [p. 183]. This may have been the case with the two subjects who had a better command of mathematics as determined by typical school standards. As I probed Barb and Celia's understanding of their algebraic solutions, the influence of school surfaced.

The watermelon problem could be solved more efficiently and probably more elegantly through elimination of one of the unknowns by subtraction than through substitution. Celia was the only one who did it via elimination. When I probed this, it turned out that she had not done it that way because she had realized it was a "better" way for this problem, but just because that is how they had taught her to solve systems of simultaneous equations. I probed Barb, also "good" at algebra, on this same problem. She acknowledged that she would never analyze for which unknown to solve if the problem only asked for one of them She would go ahead and find both. When explaining her process, she said:

> I did it this way because the first one [first equation] is $w + 2c = 4\ 65$, and I was taught that you always solve for the one that doesn't have any number by it

Mathematics and reality

The four problems used in this study were rather typical of school mathematics. Though they use vocabulary from everyday life they do not represent real life situations. Barb and Celia received them as "just school math problems" Yet David and Ellen did try to make a connection between these problems and the real world

When solving the ice cream problem, Ellen made a remark about the order of the flavors making a difference, and wondered whether she should do something about it. Then she added:

> Well, you know, like in Baskin-Robbins they ask you which one you want on the top; it does make a difference.

Later on she said that she did not approve of the wording in the chickens and rabbits problem. She said that it should not have used the word "farmer", but rather "a person passing by"; a farmer would know how many chickens and rabbits he had, she argued, while this problem had no definite answer.

I think that David provided the nicest illustrations of attachment to reality. He checked his reasoning (which was flawless) on the watermelon problem several times He had a hard time accepting his answer because, he told me, the watermelon turned out to be too expensive When solving the brandy problem, he tried some sort of trial and error with the calculator. He started mentioning percentages; so, I asked him about what he was doing: he was taking guesses at what the sales tax might be in order to take it into account for his solution. Needless to say, none of the other subjects showed any concern for overpriced watermelons, or sales tax on brandy However, as soon as David had solved the problems with algebra, his worries about whether the answers made sense in real life seemed to diminish.

Conclusion

Though neither of the four subjects had a particularly strong background in mathematics, Barb and Celia had been more successful (by school standards) than David and Ellen They seemed to take less interest in the different ways to look at the problems; they made fewer references (hardly any) to their feelings about mathematics. Both David and Ellen often expressed how inadequate they felt with respect to their knowledge of mathematics David was especially worried about having to teach mathematics; Ellen insisted on how much she disliked mathematics.

Barb and Celia appeared more successful with the process of acculturation to school mathematics than David and Ellen. The former seemed to know better how to play by the school rules; they knew, for example, that " . . . in school, you don't look at meanings, you know the problems are not real." [Whitney, 1985, p. 221]. Maybe David and Ellen were more resistant to dropping their informal approaches when they started schooling; maybe they insisted on wanting mathematics to make sense from an everyday life perspective. Barb and Celia's behaviour reflected the notion of sense-making within the culture of school [Schoenfeld, 1987, in press]; David and Ellen's behavior gave more indication of sense-making within the outside-school culture. I believe that in teacher education programs an effort should be made to make prospective teachers aware of these two cultures and to try to bridge the gap between the two

References

Baroody, A J , & Ginsburg. H P. [1986] The relationship between initial meaningful and mechanical knowledge of arithmetic In J. Hiebert (Ed), *Conceptual and procedural knowledge: the case of mathematics* (pp 75-112). Hillsdale, NJ: Lawrence Erlbaum Associates

Cobb, P [1985] Two children's anticipations, beliefs, and motivations. *Educational Studies in Mathematics, 16*, 111-126

Davis, R B. [1986] Conceptual and procedural knowledge in mathematics: a summary analysis In J Hiebert (Ed.), *Conceptual and procedural knowledge: the case of mathematics* (pp 265-300) Hillsdale, NJ: Lawrence Erlbaum Associates

Grieb, A , and Easley J [1984] A primary school impediment to mathematical equity: case studies in rule-dependent socialization In M W Steinkamp and M R Maehr (Eds), *Advances in motivation and achievement. Vol 2 Women in science* (pp 317-362) Greenwich, CT: JAI Press

Krutetskii V.A [1976] *The psychology of mathematical abilities in schoolchildren* (Kilpatrick, J and Wirszup, I (Eds.), J Teller, Trans) Chicago: The University of Chicago Press

Polya G. [1957] *How to solve it* (2nd ed) Princeton, NJ: Princeton University Press

Resnick L. [1987] Learning in school and out *Educational Researcher 16*(9), 13-20

Schoenfeld, A H [1985] *Mathematical problem solving* Orlando FL: Academic Press

Schoenfeld, A H. [1987] What's all the fuss about metacognition? In A H Schoenfeld (Ed). *Cognitive science and mathematics education* (pp. 189-215). Hillsdale. NJ: Lawrence Erlbaum Associates

Schoenfeld, A H [in press] On mathematics as sense-making: an informal attack on the unfortunate divorce of formal and informal mathematics. In J F. Voss, D.N Perkins. and J Segal (Eds). *Informal reasoning and instruction* Hillsdale, NJ: Erlbaum

Usiskin, Z [1987] Why elementary algebra can, should, and must be an eighth grade course for average students. *Mathematics Teacher 80.* 428-438

Whitney, H [1985] Taking responsibility in school mathematics education. *The Journal of Mathematical Behavior 4.* 219-235

Appendix

1 A watermelon and two cantaloupes cost $4 65 Three watermelons and two cantaloupes cost $10 15 How much does one watermelon cost?

2 If an ice cream shop sells eleven different flavors of ice cream, how many different double dip cones could they make?

3 A bottle of imported brandy costs $21 The bottle by itself (without the brandy) costs $9 less than the brandy How much does the brandy cost?

4 A farmer has chickens and rabbits. One day he counts all the heads and feet and finds that there are 70 all together How many of each animal does he have?