A Kinder, Gentler Socrates: Conveying New Images of Mathematics Dialogue

EILEEN FERNANDEZ

As part of my preparation for a degree in mathematics education, I took a course in philosophy of education that was required for the prospective mathematics teachers in my department This course began with the reading and discussion of Plato's *Protagoras* and *Meno,* the latter of which contains the famous dialogue between Socrates and Meno's slave. In this passage, Socrates demonstrates (to Meno) how his questioning of a slave boy awakens the boy's desire to explore certain relationships between a square's side length and its area by first disclosing the boy's lack of understanding on this matter

Although this was my first experience reading either of these Platonic dialogues, the excerpted *geometry experiment* in the *Meno* between Socrates and the slave boy was not altogether alien to me: years earlier, as a graduate student in mathematics, I recalled student colleagues hailing this portion of the dialogue as a model by which to teach. I additionally recalled a reference to "what Socrates taught us" in a paper endorsing aspects of Socrates' teaching as a partial solution to the problem of learning to teach mathematics.[1] In my current studies, some of the mathematics education literature appears to reinforce these earlier, favorable references: Cooney, Davis and Henderson cite the geometry experiment as "the first and most celebrated" example of the discovery method in their book on mathematics teaching [2] Along with a brief reference to the "Socratic teaching style" in an article on mathematics dialogue traditions,[3] these sources created a favorable, first impression of Socrates' ability to enable a slave boy to discover certain geometric relationships.

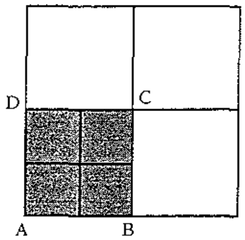
My first doubts concerning this impression were produced by a summary included in our translation of the dialogues entitled "The geometrical experiment with Meno's slave." This summary refers to the "value" in Socrates' questioning method of clearing away the "conceit of false knowledge" so that the desire to learn is instilled as a result of "coming to realize one's ignorance."[4] This association of *student misunderstanding* with *conceit* or *ignorance* seemed a harsh interpretation with possibly damaging implications to a teacher's role in addressing student error. My eventual reading of the excerpted dialogue suggested both desirable and undesirable implications for teaching. Unfortunately, the discussion in our philosophy class failed to acknowledge the latter In the end, my student colleagues and I were left with various unresolved questions and undesirable images concerning the kind of dialogue advocated by the literature through the example of Socrates and the slave boy. This concern seemed particularly relevant given recent articles and standards acknowledging dialogue as a powerful medium in conveying ideas about mathematics and its teaching.[5]

Since this introductory reading, I have read and reread the *Meno* with a focus on understanding the purpose of the excerpted passage in the context of the entire dialogue This initial study continually disclosed new aspects to the meaning of this passage and its usefulness to Socrates' attempts at illustrating various issues to Meno. I have additionally read detailed analyses of the passage by educators wishing to endorse certain aspects of Socrates' method to our mathematics classrooms.[6] The viewpoint adopted in the later analyses (and some of the earlier references), of studying the excerpted passage as a dialogue in its own right, is the one I wish to apply in this paper.[7] My objective, however, is to acknowledge and document how, in the absence of the literary perspective provided by my initial study, the excerpted dialogue draws certain pedagogical pictures to be avoided—not emulated—by current students of mathematics teaching

In this paper, I begin with a detailed analysis of the excerpted dialogue which attempts to unveil some of the negative images portrayed by Socrates' questioning of the boy As already mentioned, the first step in Socrates' method is disclosing his student's lack of understanding so that the desire to learn is instilled in this student. Through my initial analysis, I hope to illustrate how certain aspects of Socrates' manner and reasoning may have *unnecessarily* confused the slave boy, thus bringing into question how much of the boy's confusion was genuine and how much was unnecessarily provoked by his teacher A short analysis of the types of questions favored by Socrates suggests images of mathematics as a calculating subject with right or wrong answers and a teacher-dominated dialogue The concluding section reinforces the importance of considering the excerpted passage in the context of the larger dialogue by discussing it in this light. Finally, suggestions are made for a study of this dialogue in teacher preparation programs that serves to convey an image of mathematics teaching based on promoting a more congenial, and less confusing, desire to learn.

Analyzing Socrates' questioning of Meno's slave

Socrates begins his questioning of Meno's slave by drawing a square in the sand with a stick and ensuring the boy understands the geometric definition of this object. This original square, whose side length is two feet, is shown to have an area of four square feet (82C - 82D).[8] Socrates proceeds to double the area of this square and asks the slave boy how doubling this area will affect the new side length, to which the slave boy responds, "It will double, Socrates, obviously."[9] By adjusting the original diagram (Figure 1), Socrates next reveals how doubling the square's side length results in a fourfold figure, (83A - 83C), after which, he asks the boy, "So doubling the side has given us not a double but a fourfold figure?"[10]



ABCD is the original square

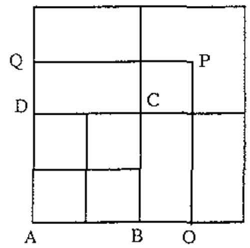
Figure 1

It is important to remark, at this juncture, on the subtle "leap" in logic illustrated by this introductory sequence Letting s and A denote the original side length and corresponding area, and s' and A' the *new* side length and corresponding area, the above exchange can be represented schematically as follows:

Socrates and slave boy:	If the side length is s, the area is A
Socrates:	If $A' = 2A$, what is s'?
Slave boy:	If $A' = 2A$, then $s' = 2s$.
Socrates:	If $s' = 2s$, then $A' = 4A$ (with the help of a diagram)

Using this representation, it is easy to see how the contrapositive of Socrates' concluding remark (If $A' \neq 4A$, then $s' \neq 2s$) contradicts the slave boy's assertion What remains uncertain is the slave boy's understanding at the end of this exchange: is it clear to the boy that Socrates' argument contradicts the boy's assertion? or does the boy only see, as an entity in its own right, that doubling the side length quadruples the area? Unfortunately, Socrates does not follow his demonstration with a discussion of this delicate point in reasoning. Having taught mathematics for several years during and between graduate programs, I am familiar with my own students' difficulties concerning the subtleties of such reasoning. A famous series of experiments also illustrates respondents' difficulties seeing the equivalence of a conditional and its contrapositive.[11] To a modern reader, therefore, Socrates' oversight may be interpreted as confusing to a slave boy with little or no experience in mathematics.

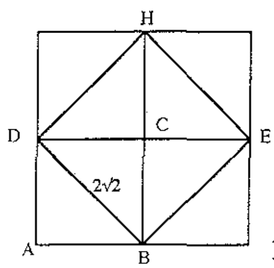
Still searching for the side length of the square with an area of eight square feet, Socrates next narrows the range of values in which this side length falls: his line of questioning reveals the desired side length to lie between two and four feet since the eight foot square lies between the corresponding areas of four and sixteen square feet (83C - 84A) Using this new information, Socrates asks the boy to guess what the side length might be. Although the boy incorrectly guesses the desired side length to be three feet, his guess narrows the range of the unknown side length to between two and three feet since the eight foot area lies between the corresponding areas of four and nine square feet (again, Socrates accomplishes this by drawing the diagram in Figure 2). After observing that the eight foot square is not obtained from the three foot side, Socrates demands, "Then what length will give it? Try to tell us exactly. If you don't want to count it up, just show us on the diagram.[12]



Socrates adds in QP and PO to illustrate the desired side length is between 2 and 3 feet since the 8 foot square lies between the corresponding areas of 4 and 9 square feet

Figure 2

This latest remark carries the implication that the slave boy (and the reader) should now be in a position to either correctly guess the *exact* value of the unknown side length or to identify it on the diagram. Concerning the latter position, the diagram referred to in this remark (Figure 2) may not be very helpful to the boy given his inexperience. With regard to the former position, the side length of a square with 8 square feet is $2\sqrt{2}$—an irrational number—and the *exact* value of an irrational cannot be "counted up" as Socrates seems to imply It is interesting to note that at the time this dialogue was written, as well as the year in which it was supposed to have taken place, mathematicians themselves were still struggling with the concept of irrationals.[13] In fact, a notation for irrationals had not yet been invented.[14] Considering Socrates' latest question in this historical context reinforces its confusing effect on a slave boy with little or no training in the field of mathematics



Socrates makes a new picture with the desired square BEHD and the desired side length given by the diagonals.

Figure 3

Following Socrates' last question, the slave boy admits to "complete ignorance" on this topic. Socrates concludes his investigation with the boy by *constructing* the square (and desired side length) whose area is eight square feet (84D - 85C; see Figure 3). If the reader now looks back on the dialogue, she will note three changes in the "object" Socrates seeks: he begins by searching for the numerical value of the side length of a particular square; he then searches for a range of values within which this side length falls; and he finally draws a picture of a square with the desired side length. Mathematics teachers and mathematicians are familiar with the usefulness of seeking various representations of a solution in solving a particular problem. Nevertheless, Socrates' failure to inform the slave boy (or the reader) of these changes makes it difficult to determine which objective he is moving toward at any point in the episode. The reader may claim that such unannounced changes make this passage more "realistic." In response, I concede that mathematicians and students sometimes overlook such announcements while engaged in the moment-to-moment decision making of problem solving. However, these latter situations apply to groups whose participants are all unacquainted with the solution they seek. In the present situation, Socrates' complete understanding of the problem and its solution gives him a certain control over the dialogue that is inaccessible to the slave boy. Consequently, Socrates' unannounced changes in objective may be interpreted as causing unnecessary confusion for his inexperienced student.

Negative characteristics and implications of the Socratic method

By contradicting the boy's initial claim and demonstrating the boy's initial incapacity to see this problem's solution, there is little doubt of Socrates' successfully completing the first step in his method (as evidenced by the boy's admission of perplexity). In the analysis above, however, I have tried to illustrate that it may not be clear, to a modern reader, how much of the boy's confusion is due to his *genuine* lack of understanding and how much is *unnecessarily* provoked by Socrates. Does Socrates' failure to discuss the subtleties of reasoning unnecessarily confuse the slave boy? Does the reference to an "exact" solution imply that it exists and, consequently, confuse the boy? Why doesn't Socrates refer to a less cluttered diagram in assisting his student? Why wasn't Socrates clearer concerning his direction in orchestrating this dialogue? The modern reader's uncertainty concerning responses to these questions suggests the need for a "modern dialogue" on the application of *Socrates' method* to today's classrooms as modeled by this excerpted passage.

In addition to this analysis, the types of questions favored by Socrates throughout this episode have significant implications for the teacher's role in orchestrating discourse (as well as societal views about mathematics). For example, of the 50 questions Socrates directs to the slave boy, 36 are "yes/no" questions: that is, their responses entail only an affirmation or refutation on the part of the slave boy. Of the 14 remaining questions: 9 require the slave boy to do some "counting" or "calculation" to respond and one question requires an estimation. Only 4 questions attempt to embrace the slave boy's perspectives and, lamentably, these perspectives are not explored beyond the boy's initial response. To the modern reader, the dominance of "yes/no" questions conveys an image of mathematics discourse in which ideas flow primarily from teacher to student; the teacher is the sole possessor of knowledge; and answers are either right or wrong. Such images perpetuate the unrealistic expectation that "the teacher is always right" which, in turn, undermines the student's knowledge, inhibits the student's thinking, and minimizes the student's role in classroom discourse. The reader should note, for example, that the slave boy asks *no* questions during his exchange with Socrates. Coupled with the prevailing computational nature of the remaining 14 questions, Socrates' questions suggest the traditional misconceptions of mathematics as a "calculating" subject and its classroom as a place where only teachers get to think and reason mathematically.

The geometry experiment in its context

I noted that my initial reading of the excerpted passage suggested both desirable and undesirable images of dialogue for current mathematics teachers. For example, the "question/answer" format of this dialogue provides a *desirable* contrast to the teacher who states mathematical propositions and directs his students to memorize them. In addition, Socrates' assumption that anyone (including this boy) is capable of discovering mathematical ideas is a praiseworthy one. In this paper, I have tried to illustrate that favorable references to the excerpted passage might also suggest negative images to a modern reader unfamiliar with Socrates' objectives in the context of the entire dialogue. In fairness to Socrates and the reader unfamiliar with the larger dialogue, I now consider how the episode's interpretation changes within this context and how, in fact, this context discloses some of the very characteristics it is hoped a modern teacher might emulate.

Prior to Socrates' exchange with the slave boy, Socrates applies his *Socratic method* on Meno in their search for a definition of *virtue*. Although Meno complains that Socrates' methods are "numbing" him into a state of perplexity, Socrates convinces Meno that, in fact, neither of them knows a satisfactory definition of virtue. With this knowledge, they can now seek out such a definition together. First, Meno presents Socrates with his famous paradox concerning the quest for knowledge: if a person knows something, he will not seek it since he already knows it; if a person does not know something, how will he know he has found it since he does not know the thing he seeks? In effect, Meno argues that the quest for a definition of virtue (or any kind of knowledge) is, *a priori*, an empty one.

To address Meno's Paradox, Socrates appeals to Plato's philosophy of the soul's immortality: since the soul is immortal and "has seen all things both here and in the other world,"[15] it possesses *all* knowledge. Thus, Socrates explains, we recognize "new" knowledge since "learning" merely entails "remembering" what we already know.[16] Meno then asks Socrates to explain how "learning" is "remembering." To illustrate how his questioning method can elicit "remembering", Socrates constructs a working model of it. First, Socrates must select a topic familiar to Meno so that Meno recognizes the various stages of the method as they are applied to a third person unfamiliar with the selected topic. Enter the slave boy and the topic of geometry: by illustrating the slave boy's initial perplexity and incapacity to solve the geometry problem (which parallels Meno's and Socrates' state on the definition of virtue) and his eventual resolution of this problem, Socrates illustrates how his questions enable the boy to "remember" the problem's solution and, thus, demonstrates the value in searching for the unknown

Effectively, then, the geometry experiment is a *model* of Socrates' method and, as with any model, it is sometimes difficult to eliminate certain undesirable characteristics resulting from the attempt to simulate the original phenomenon (as evidenced by this paper's initial analysis). Likewise, it is difficult to maintain many of the *actual* phenomenon's desirable characteristics. It is important to point out, for example, that in the method's application to Meno, neither Socrates nor Meno (teacher nor student) controls the dialogue since neither participant knows the answer to the question they are exploring. In contrast to the situation in the geometry experiment, Meno (the student) selects the topic of virtue for his discussion with Socrates, and both Meno and Socrates (student and teacher) ask questions of each other during their exchange. There is also a certain "playfulness" between Socrates and Meno that does not translate to the exchange between Socrates and the slave boy. Thus, an analysis and discussion of the excerpted passage in its (appropriately) larger context conveys images of (more) student-centered, congenial discussions between a teacher and his student

Suggestions for a study of the geometry experiment for prospective mathematics teachers

This paper's analysis illustrates how blanket, favorable references to the excerpted geometry experiment, in the literature and our schools, might reinforce certain misconceptions about teaching to students, teachers and students of teaching. Students of *mathematics* teaching, in particular, should be encouraged by their educators to consider this episode's implications for mathematics dialogue and the teaching of mathematics. They could be encouraged to think and write about alternative scenarios between Socrates and the slave boy that more closely resemble the application of *Socrates' method* between Socrates and Meno. For example, what if Socrates had asked the boy why he believed, as he originally did, that doubling the area results in doubling the side length? What if, at the first juncture, the slave boy turned to Socrates and asked, "I claimed that doubling the area of the square implies a doubling of the side length; yet you have shown that doubling the side length implies a doubling of the area. How does your demonstration contradict my claim?" How might Socrates respond? Could a prospective teacher imagine a situation in which Socrates and the boy discover something about the incommensurability of the square's diagonal and its side? What might have happened if Socrates let the slave boy draw some of the diagrams?

In addition to these ideas, the experiment with Meno's slave provides a rich setting for discussing such topics as open-ended problems; discovery learning; student self confidence; or the teacher's and student's role in discourse Discussing and writing about these and the aforementioned ideas contributes to images of more congenial teacher-student discussions in mathematics classrooms. Images of teachers and students empowering one another are central to generating the kinds of dialogue that promote teacher *and* student discovery of mathematics. As a setting in which to explore such images, it is hoped this paper illustrates the importance of seeking out Socrates' "kinder, gentler" side in applying his method to current classrooms of mathematics.

Acknowledgement

I wish to thank Professor Anneli Lax for her helpful comments on an earlier version of this article

Notes

[1] Paul R. Halmos, "The problem of learning to teach," Paper presented at the Annual Meeting of the American Mathematical Society and the Mathematical Association of America, San Francisco, 1974

[2] Thomas J Cooney, Edward J. Davis, and K.B Henderson, *Dynamics of teaching secondary school mathematics* (Boston: Houghton Mifflin Company, 1975), pp 136-7

[3] John Richards, "Mathematical discussions," in *Radical constructivism in mathematics education*, ed Ernst von Glasersfeld (Netherlands: Kluwer Academic Publishers, 1991), p. 36

[4] Plato, *Protagoras and Meno,* trans W K C Guthrie (London: Penguin Group, 1956), p 108

[5] National Council of Teachers of Mathematics, *Professional standards for teaching mathematics* (Reston, Va.: 1991), pp. 34-54
Deborah Ball, "What's all this talk about discourse?" *Arithmetic Teacher,* (November 1991), pp. 44-48
Paul Cobb, Terry Wood, Erna Yackel, and Betsy McNeal, "Characteristics of classroom mathematics traditions: an interactional analysis," *American Educational Research Journal,* 29 (Fall 1992), pp 573-604
Magdalene Lampert, "When the problem is not the question and the solution is not the answer: mathematical knowing and teaching," *American Educational Research Journal,* 27 (Spring 1990), pp. 29-63

[6] David Wheeler, "Teaching for discovery," *Outlook* 14 (Winter 1974)
Rainer Loska, "Teaching without instruction: the Socratic method," Paper presented at the International Congress on Mathematics Education, Quebec City, Canada, 1992

[7] This excerpted passage contains two "asides" in which Socrates talks to Meno. In these asides, he refers back to a previous discussion with Meno in which Socrates had applied his method to Meno Given this paper's perspective of studying the excerpted passage as a dialogue in its own right, this paper's analysis will not consider the dialogue contained in the asides between Socrates and Meno, but focus on the dialogue between Socrates and the slave.

[8] This dialogue can be found on pages 130-137, lines 82B-85C, in Guthrie's translation of the *Meno*. Although a reading of the dialogue is not necessary for an understanding of this paper, it would enrich the reader's appreciation of the points cited throughout the paper. Line numbers are cited throughout this paper for the reader who wishes to follow the dialogue in Guthrie's translation.

[9] Plato, page 132

[10] Plato, page 133

[11] P N Johnson-Laird and P C Wason, "A theoretical analysis of insight into a reasoning task," in *Thinking: reading in cognitive science,* ed. P N Johnson-Laird and P.C Wason (Cambridge: Cambridge University Press, 1977), pp. 143-157

[12] Plato, page 134

[13] The composition and dramatic dates of this dialogue are, respectively, 387 B C and 402 B.C. Irrational numbers were first discovered by the Pythagoreans between 500 and 400 B C and their discovery