A Kinder, Gentler Socrates: Conveying New Images of Mathematics Dialogue

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As part of my preparation for a degree in mathematics education, I took a course in philosophy of education that was required for the prospective mathematics teachers in my department. This course began with the reading and discussion of Plato’s *Protagoras* and *Meno*, the latter of which contains the famous dialogue between Socrates and Meno’s slave. In this passage, Socrates demonstrates (to Meno) how his questioning of a slave boy awakens the boy’s desire to explore certain relationships between a square’s side length and its area by first disclosing the boy’s lack of understanding on this matter.

Although this was my first experience reading either of these Platonic dialogues, the excerpted *geometry experiment* in the *Meno* between Socrates and the slave boy was not altogether alien to me: years earlier, as a graduate student in mathematics, I recalled student colleagues hailng this portion of the dialogue as a model by which to teach. I additionally recalled a reference to “what Socrates taught us” in a paper endorsing aspects of Socrates’ teaching as a partial solution to the problem of learning to teach mathematics[1]. In my current studies, some of the mathematics education literature appears to reinforce these earlier, favorable references: Cooney, Davis and Henderson cite the geometry experiment as “the first and most celebrated” example of the discovery method in their book on mathematics teaching[2]. Along with a brief reference to the “Socratic teaching style” in an article on mathematics dialogue traditions,[3] these sources created a favorable, first impression of Socrates’ ability to enable a slave boy to discover certain geometric relationships.

My first doubts concerning this impression were produced by a summary included in our translation of the *Meno* which attempts to unveil some of the negative images portrayed by Socrates’ questioning of the boy. As already mentioned, the first step in Socrates’ method is disclosing his student’s lack of understanding so that the desire to learn is instilled in this student. Through my initial analysis, I hope to illustrate how certain aspects of Socrates’ method is disclosing his student’s lack of understanding so that the desire to learn is instilled in this student. Through my initial analysis, I hope to illustrate how certain aspects of Socrates’ manner and reasoning may have unnecessarily confused the slave boy, thus bringing into question how much of the boy’s confusion was genuine and how much was unnecessarily provoked by his teacher. A short analysis of the types of questions favored by Socrates suggests images of mathematics as a calculating subject with right or wrong answers and a teacher-dominated dialogue. The concluding section reinforces the importance of considering the excerpted passage in the context of the larger dialogue by discussing it in this light. Finally, suggestions are made for a study of this dialogue in teacher preparation programs that serves to convey an image of mathematics teaching based on promoting a more congenial, and less confusing, desire to learn.

**Analyzing Socrates’ questioning of Meno’s slave**

Socrates begins his questioning of Meno’s slave by drawing a square in the sand with a stick and ensuring the boy understands the geometric definition of this object. This original square, whose side length is two feet, is shown to have an area of four square feet (82C - 82D)[8]. Socrates proceeds to double the area of this square and asks the...
slave boy how doubling this area will affect the new side length, to which the slave boy responds, "It will double, Socrates, obviously."[9] By adjusting the original diagram (Figure 1), Socrates next reveals how doubling the square's side length results in a fourfold figure, (83A - 83C), after which, he asks the boy, "So doubling the side has given us not a double but a fourfold figure?"[10]

\[ \text{Figure 1} \]

\[ \text{ABCD is the original square} \]

It is important to remark, at this juncture, on the subtle "leap" in logic illustrated by this introductory sequence. Letting \( s \) and \( A \) denote the original side length and corresponding area, and \( s' \) and \( A' \) the new side length and corresponding area, the above exchange can be represented schematically as follows:

- **Socrates and slave boy:** If the side length is \( s \), the area is \( A \)
- **Socrates:** If \( A' = 2A \), what is \( s' \)?
- **Slave boy:** If \( A' = 2A \), then \( s' = 2s \)
- **Socrates:** If \( s' = 2s \), then \( A' = 4A \) (with the help of a diagram)

Using this representation, it is easy to see how the contrapositive of Socrates' concluding remark (If \( A' \neq 4A \), then \( s' \neq 2s \)) contradicts the slave boy's assertion. What remains uncertain is the slave boy's understanding at the end of this exchange: is it clear to the boy that Socrates' argument contradicts the boy's assertion, or does the boy only see, as an entity in its own right, that doubling the side length quadruples the area? Unfortunately, Socrates does not follow his demonstration with a discussion of this delicate point in reasoning. Having taught mathematics for several years during and between graduate programs, I am familiar with my own students' difficulties concerning the subtleties of such reasoning. A famous series of experiments also illustrates respondents' difficulties seeing the equivalence of a conditional and its contrapositive.[11] To a modern reader, therefore, Socrates' oversight may be interpreted as confusing to a slave boy with little or no experience in mathematics.

Still searching for the side length of the square with an area of eight square feet, Socrates next narrows the range of values in which this side length falls: his line of questioning reveals the desired side length to lie between two and four feet since the eight foot square lies between the corresponding areas of four and sixteen square feet (83C - 84A). Using this new information, Socrates asks the boy to guess what the side length might be. Although the boy incorrectly guesses the desired side length to be three feet, his guess narrows the range of the unknown side length to between two and three feet since the eight foot area lies between the corresponding areas of four and nine square feet (again, Socrates accomplishes this by drawing the diagram in Figure 2). After observing that the eight foot square is not obtained from the three foot side, Socrates demands, "Then what length will give it? Try to tell us exactly. If you don't want to count it up, just show us on the diagram.[12]

\[ \text{Figure 2} \]

This latest remark carries the implication that the slave boy (and the reader) should now be in a position to either correctly guess the exact value of the unknown side length or to identify it on the diagram. Concerning the latter position, the diagram referred to in this remark (Figure 2) may not be very helpful to the boy given his inexperience. With regard to the former position, the side length of a square with 8 square feet is \( 2\sqrt{2} \)-an irrational number-and the exact value of an irrational cannot be "counted up" as Socrates seems to imply. It is interesting to note that at the time this dialogue was written, as well as the year in which it was supposed to have taken place, mathematicians themselves were still struggling with the concept of irrationals.[13] In fact, a notation for irrationals had not yet been invented.[14]

[11] Considering Socrates' latest question in this historical context reinforces its confusing effect on a slave boy with little or no training in the field of mathematics.

\[ \text{Figure 3} \]

Socrates makes a new picture with the desired square BEHD and the desired side length given by the diagonals.

Following Socrates' last question, the slave boy admits to "complete ignorance" on this topic. Socrates concludes his investigation with the boy by constructing the square (and desired side length) whose area is eight square feet (84D - 85C; see Figure 3). If the reader now looks back on the dialogue, she will note three changes in the "object" Socrates seeks: he begins by searching for the numerical value of the side length of a particular square; he then searches for a range of values within which this side length falls; and he finally draws a picture of a square with the...
desired side length. Mathematics teachers and mathematicians are familiar with the usefulness of seeking various representations of a solution in solving a particular problem. Nevertheless, Socrates’ failure to inform the slave boy (or the reader) of these changes makes it difficult to determine which objective he is moving toward at any point in the episode. The reader may claim that such unannounced changes make this passage more “realistic.” In response, I concede that mathematicians and students sometimes overlook such announcements while engaged in the moment-to-moment decision making of problem solving. However, these latter situations apply to groups whose participants are all unacquainted with the solution they seek. In the present situation, Socrates’ complete understanding of the problem and its solution gives him a certain control over the dialogue that is inaccessible to the slave boy. Consequently, Socrates’ unannounced changes in objective may be interpreted as causing unnecessary confusion for his inexperienced student.

**Negative characteristics and implications of the Socratic method**

By contradicting the boy’s initial claim and demonstrating the boy’s initial incapacity to see this problem’s solution, there is little doubt of Socrates’ successfully completing the first step in his method (as evidenced by the boy’s admission of perplexity). In the analysis above, however, I have tried to illustrate that it may not be clear, to a modern reader, how much of the boy’s confusion is due to his genuine lack of understanding and how much is unnecessarily provoked by Socrates. Does Socrates’ failure to discuss the subtleties of reasoning unnecessarily confuse the slave boy? Does the reference to an “exact” solution imply that it exists and, consequently, confuse the boy? Why doesn’t Socrates refer to a less cluttered diagram in assisting his student? Why wasn’t Socrates clearer concerning his direction in orchestrating this dialogue? The modern reader’s uncertainty concerning responses to these questions suggests the need for a “modern dialogue” on the application of Socrates’ method to today’s classrooms as modeled by this excerpted passage.

In addition to this analysis, the types of questions favored by Socrates throughout this episode have significant implications for the teacher’s role in orchestrating discourse (as well as societal views about mathematics). For example, of the 50 questions Socrates directs to the slave boy, 36 are “yes/no” questions: that is, their responses entail only an affirmation or refutation on the part of the slave boy. Of the 14 remaining questions: 9 require the slave boy to do some “counting” or “calculation” to respond and one question requires an estimation. Only 4 questions attempt to embrace the slave boy’s perspectives and, lamentably, these perspectives are not explored beyond the boy’s initial response. To the modern reader, the dominance of “yes/no” questions conveys an image of mathematics discourse in which ideas flow primarily from teacher to student; the teacher is the sole possessor of knowledge; and answers are either right or wrong. Such images perpetuate the unrealistic expectation that “the teacher is always right” which, in turn, undermines the student’s knowledge, inhibits the student’s thinking, and minimizes the student’s role in classroom discourse. The reader should note, for example, that the slave boy asks no questions during his exchange with Socrates. Coupled with the prevailing computational nature of the remaining 14 questions, Socrates’ questions suggest the traditional misconceptions of mathematics as a “calculating” subject and its classroom as a place where only teachers get to think and reason mathematically.

**The geometry experiment in its context**

I noted that my initial reading of the excerpted passage suggested both desirable and undesirable images of dialogue for current mathematics teachers. For example, the “question/answer” format of this dialogue provides a desirable contrast to the teacher who states mathematical propositions and directs his students to memorize them. In addition, Socrates’ assumption that anyone (including this boy) is capable of discovering mathematical ideas is a praiseworthy one. In this paper, I have tried to illustrate that favorable references to the excerpted passage might also suggest negative images to a modern reader unfamiliar with Socrates’ objectives in the context of the entire dialogue. In fairness to Socrates and the reader unfamiliar with the larger dialogue, I now consider how the episode’s interpretation changes within this context and how, in fact, this context discloses some of the very characteristics it is hoped a modern teacher might emulate.

Prior to Socrates’ exchange with the slave boy, Socrates applies his Socratic method on Meno in their search for a definition of virtue. Although Meno complains that Socrates’ methods are “numbing” him into a state of perplexity, Socrates convinces Meno that, in fact, neither of them knows a satisfactory definition of virtue. With this knowledge, they can now seek out such a definition together. First, Meno presents Socrates with his famous paradox concerning the quest for knowledge: if a person knows something, he will not seek it since he already knows it; if a person does not know something, how will he know he has found it since he does not know the thing he seeks? In effect, Meno argues that the quest for a definition of virtue (or any kind of knowledge) is, a priori, an empty one.

To address Meno’s Paradox, Socrates appeals to Plato’s philosophy of the soul’s immortality: since the soul is immortal and “has seen all things both here and in the other world,”[15] it possesses all knowledge. Thus, Socrates explains, we recognize “new” knowledge since “learning” merely entails “remembering” what we already know.[16] Meno then asks Socrates to explain how “learning” is “remembering.” To illustrate how his questioning method can elicit “remembering”, Socrates constructs a working model of it. First, Socrates must select a topic familiar to Meno so that Meno recognizes the various stages of the method as they are applied to a third person unfamiliar with the selected topic. Enter the slave boy and the topic of geometry; by illustrating the slave boy’s initial perplexity and incapacity to solve the geometry problem
(which parallels Meno’s and Socrates’ state on the definition of virtue) and his eventual resolution of this problem, Socrates illustrates how his questions enable the boy to “remember” the problem’s solution and, thus, demonstrates the value in searching for the unknown.

Effectively, then, the geometry experiment is a model of Socrates’ method and, as with any model, it is sometimes difficult to eliminate certain undesirable characteristics resulting from the attempt to simulate the original phenomenon (as evidenced by this paper’s initial analysis). Likewise, it is difficult to maintain many of the actual phenomenon’s desirable characteristics. It is important to point out, for example, that in the method’s application to Meno, neither Socrates nor Meno (teacher nor student) controls the dialogue since neither participant knows the answer to the question they are exploring. In contrast to the situation in the geometry experiment, Meno (the student) selects the topic of virtue for his discussion with Socrates, and both Meno and Socrates (student and teacher) ask questions of each other during their exchange. There is also a certain “playfulness” between Socrates and Meno that does not translate to the exchange between Socrates and the slave boy. Thus, an analysis and discussion of the excerpted passage in its (appropriately) larger context conveys images of (more) student-centered, congenial discussions between a teacher and his student.

Suggestions for a study of the geometry experiment for prospective mathematics teachers

This paper’s analysis illustrates how blanket, favorable references to the excerpted geometry experiment, in the literature and our schools, might reinforce certain misconceptions about teaching to students, teachers and students of teaching. Students of mathematics teaching, in particular, should be encouraged by their educators to consider this episode’s implications for mathematics dialogue and the teaching of mathematics. They could be encouraged to think and write about alternative scenarios between Socrates and the slave boy that more closely resemble the application of Socrates’ method between Socrates and Meno. For example, what if Socrates had asked the boy why he believed, as he originally did, that doubling the area results in doubling the side length? What if, at the first juncture, the slave boy turned to Socrates and asked, “I claimed that doubling the area of the square implies a doubling of the side length; yet you have shown that doubling the side length implies a doubling of the area. How does your demonstration contradict my claim?” How might Socrates respond? Could a prospective teacher imagine a situation in which Socrates and the boy discover something about the incommensurability of the square’s diagonal and its side? What might have happened if Socrates let the slave boy draw some of the diagrams?

In addition to these ideas, the experiment with Meno’s slave provides a rich setting for discussing such topics as open-ended problems; discovery learning; student self-confidence; or the teacher’s and student’s role in discourse. Discussing and writing about these and the aforementioned ideas contributes to images of more congenial teacher-student discussions in mathematics classrooms. Images of teachers and students empowering one another are central to generating the kinds of dialogue that promote teacher and student discovery of mathematics. As a setting in which to explore such images, it is hoped this paper illustrates the importance of seeking out Socrates’ “kinder, gentler” side in applying his method to current classrooms of mathematics.

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Notes


[13] The composition and dramatic dates of this dialogue are, respectively, 387 B.C and 402 B.C. Irrational numbers were first discovered by the Pythagoreans between 500 and 400 B.C and their discovery.