

Cartesian and Euclidean Rhetoric

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I gotta use words when I talk to you
But if you understand or if you don't
That's nothing to me and nothing to you
T.S. Eliot, *Sweeney Agonistes*

Is there a best way of communicating mathematics? Those who have spent their formative years attending lectures had a good opportunity to reflect on the variety of communication styles. In one time-hallowed teaching mode, the learner sat day after day, watching the back of someone writing up on the board, for fifty minutes at a stretch, a string of things like *Lemma, Theorem 4.2.1, Proof, Remark* and so on. The idea was that what the lecturer copied from her notes onto the board should be recopied into the student's notebook (and there stored, perhaps, until it was committed to memory shortly before the examination).

By contrast, another style of lecturing is to enthuse and inspire, stimulating the student to believe that this is a really exciting subject and fun to learn. This style leaves the learner with a warm, happy feeling, but carries the risk of imparting not enough of substance for consolidation later; or giving a feeling that things are easy, which in the privacy of one's room turns out to present unexpected difficulties, if possible at all.

Each of these styles of communication can also be found in written mathematical texts. Indeed, we can see as the paradigms of these extremes two highly influential texts from the past: Euclid's *Elements* of about 300 B.C., and Descartes' *Geometry* published in 1637. I describe mathematical communication as having a *Euclidean* or a *Cartesian* rhetoric, according as a text looks and reads more like the *Elements* or the *Geometry* [1].

To speak of *rhetoric* in connection with mathematics may at first sound a somewhat strange notion. But what is meant is just a concern for how language is used in communicating mathematics; I have in mind some triangle of writer, text and reader, whereby the writer is taken to be trying to communicate something to the reader *via* a text. It is the rhetorical form of this text, the result of the choices the writer can be presumed to have made, which is our focus of interest.

Euclidean rhetoric

The style of Euclid's *Elements* is too familiar to need detailed description here. Its definition-axiom-proposition-proof form is the ultimate inspiration for a thousand

similar productions. What is of special interest to us in assessing Euclid's rhetoric is his tone towards the reader. Euclid's attitude is perfectly straightforward: there is no sign that he notices the existence of readers at all. Rather, he seems engaged in laying down inexorable eternal truths. The reader is never addressed. The nearest Euclid seems to get to recognising the existence of readers is in his "let such-and-such be done" mode — but that strengthens our awareness that at such moments Euclid is talking about abstract possibilities validated by the axioms, not about an activity that the reader was supposed to carry out.

This rhetorical style has proved highly successful. No other mathematical text has had so many readers over such a long time. Evidently a studied ignoring of the reader has not been seen as user-unfriendly.

Cartesian rhetoric

To turn to Descartes' *Geometry* is to enter quite another world of writer-reader relations. The mathematics described is clearly created, not unveiled, in rhetoric which veers from grabbing the reader by the lapels to treating you with utter disdain (sometimes achieving these two effects simultaneously). The book which proved so influential on the course of seventeenth-century mathematics — and thus on our own — is a strange and perverse piece of work.

The *Geometry* was an essay appended to Descartes' *Discourse on method*, published in Leiden in 1637. No author's name appeared on the titlepage, the author appearing to prefer the pseudo-mystery of a not-well-guarded anonymity. The *Discourse* too is a curious work, and looking briefly at it helps prepare us for what Descartes might have been up to in writing the *Geometry* as he did.

In the *Discourse on method* Descartes has apparently let the reader overhear an autobiographical rumination, delivered with all the artless simplicity of a milkmaid:

I should be glad in this discourse to describe for the benefit of others the paths I have followed, to paint a picture, as it were, of my life, of which each one may judge as he pleases; and I should be happy, too, to learn what public opinion has to say of me, and so discover a fresh mode of instruction for myself [2].

Such sweet reasonableness must have gladdened the heart of kindly readers; though if they rushed to their pens to help instruct the eager author they will soon have discovered their mistake.

I offer this work only as a history, or, if you prefer, a fable, in which there may perhaps be found, besides some examples that may be imitated, many others that it will be well not to follow [3]

The author could hardly be detaching himself more from his text. The reader is simultaneously alerted to this being an artful construct, and won over by the engaging quality of the author's tone

The *Discourse* turns out to be a finely constructed story about the past persona (called "I") of a narrator (also called "I"), structured so as to bring out an imaginary intellectual journey — a fictional narrative case in the form of an autobiography. Jonathan Rée has shown [4] how throughout the *Discourse* Descartes exercises most careful control of the different time dimensions: time of writing, time of the reader's progression through the text, time of events described. In short, Descartes was a shrewd literary craftsman who went to immense pains to define and structure what he wanted the reader's response to be. And this was a most successful strategy. The *Discourse on method* made the author's name immediately — as soon as the secret of the anonymous authorship was carefully revealed — in precisely the evaluation he had laid down, as a bright and knowledgeable young man who had arrived at deep understanding

It was in the final section of the *Discourse* that Descartes referred to the three appended essays illustrating his method, on optics, meteorology and geometry. His discussion of these essays was wrapped around in the most Uriah Heep-like of tones: he's quite indifferent to his reputation, except that he'd rather not have a bad one; he dislikes becoming known, but is too upright a man to conceal his actions, he's merely trying to open a few windows and let in the daylight; please will everyone let him know where he has gone wrong, and he will confess his errors freely. In this way the reader is both disarmed and also encouraged to believe unquestioningly what Descartes says. The author's proclamation of his own fallibility is so loud that anything which has finally made it to paper — such as the three essays — must, we infer, surely be true:

I know that I am extremely subject to error, and almost never rely upon my first notions . . . ; but I have rarely encountered any objection to my views from any source which I had altogether failed to foresee. [5]

Descartes made a further point in the final section of the *Discourse* which has a bearing on the reader's response to the *Geometry*. This concerns the effort that the reader should put in to compensate for the fact that Descartes does not always spell things out very fully. Although he made various remarks about people distorting his views and taking them over before fully understanding them, the main reason for his skating over details was, he said, the didactically caring one that

one cannot conceive a thought so well, and make it one's own, by learning it from another, as when one discovers it oneself

The reader was doubtless impressed by now with the author's thoughtfulness — the leaving out of difficult steps was for the reader's higher benefit. This would be some consolation, perhaps, for the neophyte struggling through the trickier passages of the *Geometry*.

But the reader who, wound up to a fever-pitch of anticipation, made it as far as the final essay was to find a further clear signal of the author's expectations of the reader. The "Advertisement" at the start of the *Geometry* stated:

So far I have tried to make my meaning clear to everyone; but I doubt if this treatise can be read by anyone not familiar with geometry books, for I've thought it superfluous to repeat the demonstrations contained in them [6]

So this essay was to be for initiates, in a way that the *Discourse* and the other two essays were not. The author seems to be wishing to establish that most of his readers would not be able to understand the final essay; and this despite its being in the vernacular and written, indeed, in rather a chirpy style. It is not clear why he did this, but it has the effect of a rhetorical act of intimidation, both elevating Descartes further in his readers' eyes and bolstering the scientific ideology that science, and mathematics in particular, it too hard for ordinary folk to understand.

Certainly the *Geometry* was found extraordinarily difficult by his contemporaries, and it is clear that that is the way Descartes wanted it. We can see how skilfully Descartes constructed his reputation and gloried in the fact that his work was hard to understand. He wrote to Marin Mersenne in 1637 (publication year) that

I do not enjoy speaking in praise of myself, but since few people can understand my geometry, and since you wish me to give you my opinion of it, I think it well to say that it is all I could hope for . . . [7]

And he was still writing nearly a decade later (1646), again to Mersenne, that

I have omitted a number of things that might have made *La Géométrie* clearer, but I did this intentionally, and would not have it otherwise.

The text does indeed testify to an author in full, intentional, control of his material and the mode of expressing it. Let us look at some aspects of his remarkable rhetorical production.

Descartes' Geometry

Descartes started his essay in a splendidly bold and attention-grabbing way, in a trumpet-call of exultation:

All problems in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is all that is needed for their construction [8]

The first few pages, amiable and straightforward, set a rather enchanting tone. It is the story of young René and his instruments, of a craftsman inculcating you into the skills of his trade. The author tells you how to do things, as a furniture-maker might:

Suppose for example AB is unity & it is wanted to multiply BD by BC, I have only to join the points A & C, then draw DE parallel to CA, & BE is the product of this multiplication.[9]

The first sign we get that René has more on his mind than sharing mathematical craftsmanship with the reader is a few pages in. He has reached the point of explaining one of the key ideas of the book, that geometrical problems are to be analysed by labelling everything in sight with letters and coming up with an equation. Then he breaks off to observe, in a direct address to the reader,

But I shan't stop to explain this in more detail, because I should deprive you of the pleasure of mastering it yourself, & the advantage of training your mind by working over it, which is to my mind the main benefit to be drawn from this science. Also I find nothing here so difficult that it cannot be worked out by anyone slightly versed in ordinary geometry & algebra, & who considers carefully all that is in this treatise.[10]

So the reader has entered the story as a recognised participant who is expected to pull her weight, with this reminder of the thesis about learning mathematics that arose at the end of the *Discourse*. The remark is actually somewhat redundant in terms of the mathematical exposition of the essay. It follows a rather bland generalisation which does not really call for “mastering”, so seems like a deliberate insertion for its own sake.

Descartes then moved on to more particular cases, notably the problem of Pappus, the problem of the locus of three, four or more lines which he was so proud — justifiably — of solving with his new method. In the middle of this exposition there is a splendid “this is hurting me more than it's hurting you” remark, namely

I will try to give the demonstration in a few words, for I am already wearied by so much writing [11]

This is quite a significant remark rhetorically, for the continuity it shows with the style of the *Discourse*. There the narrator was very conscious of time and kept the reader in touch, as part of the fictional structure, with the relation between the events described and their being written down. Here, the narrator wants to keep the reader aware of the physical act of writing, and of the fact that the caring author is becoming worn out in the service of his readers.

We might note also that as in the *Discourse* there are two “I's” involved in the *Geometry*: the “I” of the narrator, who tells the story, has views on education and becomes wearied; and the timeless “I” of the craftsman in charge of the instruments. These “I” have different ontological statuses. The narrator is a real person (within the narrative fiction), whereas the craftsman is conceptual. The latter point is confirmed in, for example, the well-known story that Aubrey told:

[Descartes] was so eminently learned that all learned men made visits to him, and many of them would desire him to shew them his Instruments . . . he would

drawe out a little Drawer under his Table, and shew them a paire of Compasses with one of the Legges broken; and then, for his Ruler, he used a sheet of paper folded double.[12]

By careful control of his readers' responses Descartes built up the sense of an omniscient author having to restrict what he could put down on paper. It is no time before we are lulled into believing anything he cares to dream up, because it is put forward in the author's cheery don't-bother-me-with-details kind of way. Thus he says shortly that certain statements about classes of curves “are easily proved by actual calculation”[13] when the statements are actually false. Another example of the author's hustling the reader along in his excitement occurs later, when Descartes is explaining his method for finding normals to curves:

... as you can easily see by experience, but if I stopped to prove all the theorems of which I make some mention I should have to write a much larger volume than I want.[14]

So Descartes was still eager to remind the reader of the author's presence and voice — and that he had found yet another reason for not filling in the details, this time that the book was too short for that. The very next paragraph continues with subtle intimidation of the reader who has survived thus far:

I shall not give the constructions for the required tangents and normals in connection with the method I have just explained, for it is always easy to find them: although one often needs a little ingenuity to make them easy and simple.

So any readers unable to find the constructions would just have to accept that they were lacking in ingenuity. Descartes did in fact give some examples of his method here, but he chose the three for which the solution does come out quite straightforwardly. Readers were soon to find that trying to apply his method to other curves landed them in mindboggling or impossible calculations.

René, however, was up and away. By halfway through the final Book (of the three which comprise the essay) his weariness had clearly increased even more, for he announced in what was by now a familiar strain

I've omitted here the demonstrations of most of my statements, because they seem to me so easy that if you take the trouble to examine them methodically the demonstrations will present themselves to you: & it will be more useful to you to learn them in that way than by reading them.[15]

The theme of pedagogic generosity had clearly taken hold of Descartes' rhetorical imagination, for this was how he ended the work. The final two sentences of the *Geometry* typify the rhetoric of the whole. Notice the astonishingly bold inductive handwaving of the first sentence, and the arguably somewhat disingenuous quality of the last,

For in the case of mathematical progressions, when one has the first two or three terms it is not hard to

find the rest. And I hope that posterity will judge me kindly, not only for the things which I have explained here; but also for those which I have intentionally omitted, so as to leave to others the pleasure of their discovery [16]

The success of Cartesian rhetoric

Posterity has indeed judged Descartes kindly. This is especially true of his mathematical achievements, which have never attracted the obloquy subsequently visited upon his cosmology, or indeed his philosophy, by various interested parties. It is worth reflecting upon how his *Geometry* came to be so successful. The effect of the various rhetorical strategies described above was certainly to establish him in *everyone's* eyes as very clever. This was established in the eyes of the young ladies who made up much of the vernacular audience for the *Discourse*, and whom he told that the *Geometry* was too difficult for them; and in the eyes of the Latin-speaking intelligentsia whom he told to discover the details of *La Géométrie* for themselves.

Nor is it hard to analyse how the rhetorical form of Descartes' production contributed to its success. Instead of a clear, linear mathematical account in a Euclidean form, Descartes put out an extraordinary blend of hints, procedures, assertions, truths and falsehoods — a carefully controlled stream of consciousness. And because it wasn't a closed text but cried out for explanation, commentary, spelling out, it served as a mathematical treasure chest for the rest of the century and beyond. By the time of its second Latin edition in 1659-61, Descartes' little text was accompanied by nearly 900 pages of commentary and development by younger Dutch mathematicians.

This fact helps to put the "success" of *La Géométrie* in perspective. Although Descartes' own text left readers in little doubt of his genius, it took the endeavours of his Latin translator, Frans van Schooten, to make it a success in the mathematical community. For one thing, Descartes had not written his essay in a language understood by most European mathematicians; it is, on reflection, an extraordinarily perverse action to have written, in French, a book which one advertises as suitable only for people who have studied the other geometry books — which meant, predominantly, Latin speakers. Nor were Descartes' rhetorical gambits best devised to help busy people understand the contents, as opposed to admiring its author. As Jan van Maanen has shown [17], the successful diffusion of Cartesian mathematics was very largely the work of van Schooten, whose private study left him in no doubt both of the potential power of Descartes' approach and the unsuitability of the *Geometry*, unmediated, as an instructional text.

Reflections on rhetoric

Descartes was not in fact the first to use what I've called *Cartesian rhetoric* in a mathematics text. The great algebra text of nearly a century earlier, Cardano's *Ars magna* (1545), was also written in a fairly chatty, button-holing kind of way. Cardano's exposition was much clearer, and less unhelpfully self-conscious. He was a better teacher

altogether, one might say. Goethe commended his approach to exposition in terms which summarise the advantages of this style of rhetoric at its best:

Cardano always considers the sciences in connection with himself, his personality, and his life. And so his works speak to us with a naturalness and liveliness that attract, inspire, refresh us and set us into action. He is not a professor in his gown lecturing us *ex cathedra* but a man who goes this way and that, listens, is amazed, is seized by joy and pain, and forces upon us a passionate account of it all [18]

Any mathematical text can be assessed in terms of what its author seems to have been doing through choosing the words he did. Eliot's remark "I gotta use words when I talk to you" is rather more consequential than it may appear. Sometimes, for example, a text seems right out of the author's control, with unhappy consequences. The nineteenth century was especially full of people whose expository rhetoric did not live up to their clarity of perception. Lobachevskii, Grassmann and Kronecker are but three who contributed to their own lack of public success by appallingly shoddy accounts of their work which made no attempt to think through what an act of communication involves.

One factor in the varied attention paid by mathematicians to the rhetoric of communication may be their conception of mathematics. If one is simply recording eternal truth then perhaps there is no great call to do it in a more or less user-friendly way. Certainly Euclidean rhetoric can be seen as laying out a description of the Platonic world of real mathematical objects, and is entirely compatible with Proclus' claim that Euclid was a Platonist. Descartes, however, as a Cartesian (presumably) was dealing with clear intuitions from God, which may help account for why things seemed so clear to him that he did not need to spell out all the proofs.

Euclidean and Cartesian rhetorics are by no means the only styles in which mathematics texts have been written. The father of the English mathematics textbook tradition, Robert Record, used what one might call *Platonic rhetoric* in his great series of texts written in the 1540s and 1550s: a dramatic dialogue form in which deliberate mistakes or misunderstandings play an important part in leading to eventual understanding. But Record's work is evidently the product of someone who thought intensely and carefully about how to write instructional texts for learning from.

Examining the rhetoric of others can perhaps bring us face to face with our own practices. What kind of rhetoric do we use, for different purposes, as teachers and writers? How does this relate to the kind of communication we perceive as going on? This is a way in which the history of mathematics can show itself to be neither an antiquarian activity, divorced from the concerns of mathematics teachers, nor a frill to enliven mathematics teaching in an icing-on-the-suet-pudding kind of way. But the study of history can claim to provide a perspective and a grounding which enables us to stand outside and reflect critically on our practices, and learn the realm of other possibilities.

Notes

- [1] This is not to claim that every piece of mathematics has to be one or the other merely that this distinction is interesting and fruitful
- [2] René Descartes *Discourse on method* (tr. Arthur Wollaston), Penguin 1960, p. 38
- [3] Elizabeth Anscombe and Peter Thomas Geach (tr. and ed.) *Descartes philosophical writings*, Nelson 1970, p. 9
- [4] Jonathan Rée, *Philosophical Tales*, Methuen 1987
- [5] Descartes *Discourse on method* Penguin 1960, p. 89
- [6] Charles Adam & Paul Tannery (eds.), *Oeuvres de Descartes* vol VI: Discours de la Méthode & Essais, Vrin 1982, p. 368, tr. J G F
- [7] David Eugene Smith and Marcia L. Latham (trs.), *The geometry of René Descartes* Dover 1954, p. 10
- [8] *La Géométrie*, p. 297. The text of the first edition appears in facsimile in the Smith-Latham edition cited in note 7
- [9] p. 298
- [10] pp. 301-2
- [11] p. 309
- [12] Oliver Lawson Dick (ed.) *Aubrey's Brief Lives* Penguin 1962, p. 185
- [13] *La Géométrie* p. 322
- [14] pp. 350-1
- [15] p. 389
- [16] p. 413
- [17] Jan van Maanen, *Facets of seventeenth century mathematics in the Netherlands*, Utrecht 1987, pp. 23-31. Van Maanen also points to the irony of the fact that the very effort which van Schooten and his students put into making Descartes' mathematics accessible ultimately misdirected Dutch mathematics and led it to stagnate, by regarding mastery of the contents of Descartes' *Geometry* as the ultimate goal for a mathematician rather than fresh research using it as a stimulus or tool.
- [18] J. W. von Goethe *Theory of colours* 1810

In free group discussion, those parts of each individual's store of information which are for him relevant to the scientific topic under consideration, and which help or hinder him get information of predictive value, become clear. What a student says he sees in, for example, hand radiographs, depends on the way his schemata of recently received, specialized information about techniques of radiography are related to older, more generalized, schemata of the kind that "things which are alike in some respects are alike in others". The recently received information is easily verbalized; much of it has been received verbally. The older schemata are far less easily verbalized; they may have been made and reinforced largely through non-verbal channels. The student is usually not aware that he is using them, but their existence can be inferred from verbal and other behaviour. Learning in free group discussion is a process of identifying, through verbalization, the associations between schemata, so that the new information can be dissociated from those schemata with which it is automatically associated, and can be seen to be potentially relevant to many schemata, instead of to a few only.

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