TALKING ABOUT SUBJECT-SPECIFIC PEDAGOGY

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In comments from readers and reviewers, I have begun to notice statements such as, "interesting and thought-provoking but the editor will have to judge whether the writing is for the learning of mathematics", "Wouldn't these ideas apply to any subject? What has this to do with mathematics?" In the suggestions to writers on the inside back cover of each issue, as editor, I have to interpret the words of the founding editor, David Wheeler, writing that articles "are welcomed provided their content bears on the learning *of mathematics. This might be achieved directly, or indirectly through offering a significant perspective to teachers of mathematics."*

The following lightly edited conversation is offered to generate further international discussion on subject-specific teaching methods and pedagogy. Barbara Ball, the Professional Officer of the Association of Teachers of Mathematics (ATM) in the UK, invited responses to part of an article she has written for the September, 2005 issue of Mathematics Teaching*, MT192. Towards the end of July, 2005 there was a flurry of responses to the writing that will be posted on ATM's website www.atm.org.uk. You can respond to me as editor (flm-editor@bris.ac.uk) and I will pass your writing to Barbara Ball to contribute to the on-going debate. I would be interested in publishing some considered responses in future issues of this journal. [ed.]*

Barbara Ball, 25/07/05: In July 2005, the National Council for School Leadership (NCSL) in the UK organised a conference *'Making mathematics count' in school networks*. [1] When she opened the NCSL conference, Celia Hoyles reminded us of the Department for Education and Skills (DfES, 2005) advice suggesting that necessary components of effective CPD are:

- developing a depth of personal subject knowledge to underpin teaching and learning
- enhancing your repertoire of subject-specific teaching methods and pedagogy
- applying general strategies for teaching and learning.

The second suggestion particularly intrigues me. Through my role, I have discovered that the concerns of mathematics teachers are very similar to the concerns of teachers of English, science, technology, geography, history and modern languages. Thinking about what Celia Hoyles said, I wonder what subject-specific teaching methods and pedagogy are for mathematics. My first thought is of my young step grandson who learns about mathematics and language and geography and so on as if they were all part of the same network of knowledge – surely a good thing! But how can we have subject-specific pedagogy for knowledge that is seen as integrated? My second thought is that what makes mathematics different is that you can be certain that you are right when you have produced a proof of a mathematical result. So what teaching methods should I employ to encourage both the understanding of proof, and also the conviction that, because I can prove, I can be in control of what I know?

Alf Coles, 25/07/05: I don't think (but will be happy to be persuaded otherwise) I have any subject-specific teaching methods or pedagogy.

As I read Barbara's article, I tried to think what I might consider to be a subject-specific teaching method that I use. The example that came to mind as possibly mathematics specific was to do with ways of working with classes on watching mathematical films. With any class of students having watched a film, I will first invite the class to try and reconstruct in as much detail as possible what they observed. In the reconstruction, questions are inevitably raised by different students' differing accounts. These differences can lead to fruitful mathematical activity. At first, this way of working seemed very tied to the mathematical content of the animations.

However, this year I have been working with a group of teachers in my department on watching videos of each others' lessons. I found myself inviting the group to begin by trying to reconstruct in as much detail as possible the short clip I had selected. Again, differences always arose in people's accounts and these differences led to fruitful discussion of what teaching strategies people saw in operation. It seems to me that I am employing exactly the same teaching methods in both situations. And the pedagogy behind both offerings is the same (for me, something about the power of focusing attention on what we see rather than what we think we see).

In whatever I do, I have subject-specific awarenesses about content that inform my decisions (*e.g.*, some key questions that I will probably offer at some point in the discussion of a film), but I am struggling to think of any 'teaching method' (*e.g.*, having some key questions I will probably offer at some point in a discussion) that I couldn't imagine using in a different subject (in fact couldn't imagine not using).

Dave Hewitt, 26/07/05: Interesting, many thanks Alf. I find myself agreeing with you whilst not agreeing with you at the same time! This is because it is a matter of what each of us understands by this term 'subject-specific methods', as it comes without in-built meaning, of course, and so each of us puts meaning into this phrase. It seems to me from your lovely examples that you see subject-specific methods as exclusive (*i.e.*, used only for mathematics and not in other subjects), whereas I see them as inclusive where the same method might be employed across subjects but the issue is 'when' and 'to what degree'.

I suggest that the 'when' (including 'if at all') is determined by the subject. I am thinking here of the *local* level, where each moment in time is considered ... so, the exact moment you ask a question and the exact nature of that question will relate to the subject as well as the person(s).

Regarding the 'to what degree', I feel this is also subject dependent. For example, over the years I have been struck by the relative clarity of working with the *Silent way* for language teaching (developed by Caleb Gattegno for the learning of languages where the teacher does not actually speak in the language being taught) compared with the less clear (although still insightful and informative) offerings he made for mathematics. It seems to me this has a lot to do with the difference in the subject matter.

Within the learning of a language, there is a lot that I would describe as arbitrary in the sense that words within a language are choices made a long time ago and to someone learning that language today there is no sense that a chair has to be called 'chaise'. There are plenty of structures within a language but these are not 'necessary' structures in that they could have been other structures if different choices were made. Furthermore, there are often exceptions to these structures anyway. Thus, there is much within language that is arbitrary (in this sense) and so gives a clarity for the role a teacher has to play. There are many techniques employed which are concerned with helping students memorise sounds, words and phrases, particularly in the early stages of learning a new language.

In comparison, mathematics is more concerned with those things I describe as necessary (they have to be how they are and there are reasons why that must be so – it was never a matter of choice), although there are also plenty of arbitrary things (such as conventions and names) that exist as well. So, the role for a teacher of mathematics has less clarity compared with teaching a new language (which is why I suggest Gattegno never really developed such a comprehensive 'way' of teaching mathematics as he did for languages). Although there could be similar techniques employed within teaching mathematics as there are for languages (in Gattegno's *Silent way*), it will be mainly for those things which are arbitrary, such as learning names and conventions, and not so much for those things in mathematics which are necessary (which constitutes most of mathematics as far as I am concerned). Thus the degree to which those teaching methods are employed within mathematics are likely to be less than they might be employed within teaching languages.

Dave Wilson, 26/07/05: My first thoughts are of what seem to be general notions likely to be of concern, or at least relevant, to any teacher: being aware of the powers of the learners; the role of generalising and specialising; attempting to direct the attention of learners to specific features of whatever; stressing certain aspects; deliberately using classroom tactics to manage learners' behaviour (and I'm not thinking here of the 'discipline and order' issue but more about ways of, for example, encouraging participation and handling contributions).

However, I realise that I haven't got much sense, if any, of the other disciplines. From my own language learning and musical instrument learning it seems that my list will apply easily, but I don't know what the items in the list might mean for the study of literature, geography and history. The last two subjects always seemed to me, as a schoolchild, to consist of learning lots of particulars (as did the study of poetry which seemed to consist of learning poems by heart). I certainly was not aware of a role for generalising in them – but then, neither was I aware of that importance in language learning as a schoolchild.

The particular subject comes into explicit play when I utilise my subject-specific knowledge to create initial stimuli for learners to do, to make or to observe; to make interventions, questions and comments; to prepare myself to notice learners' comments and gestures (because geometrical reflection is not the same as reflection in real mirrors I am prepared to notice intakes of breath or "ooh" when an object is dragged across a reflection axis).

I remember wanting my school pupils to write about what they had been doing and finding this a very hard thing to do. Watching the pedagogic moves of English teachers helped me a great deal and I needed to generalise from those specifics.

Laurie Jacques, 26/07/05: A further thought, turning this idea on its head. I wonder whether our pupils perceive subject-specific *learning* strategies?

From a primary (4-11 year olds) teacher's perspective, conflicting issues are raised. In a day we shift from one subject to the next, sometimes teaching four different and unique subjects. In many cases, each lesson takes its different format. If we are to have a repertoire of teaching methods and pedagogy for each subject, then we are going to have to multiply each by 12 (since we are required to teach 12 subjects)! Or are we?

Dave Hewitt's considered response made me think about why I choose certain approaches for certain subjects and furthermore approaches for certain topics within that subject. I know there are some subjects that I find easier to teach than others. Mathematics comes top of this list. I consider that I do it well based on how my pupils emerge as mathematicians: pupils who are prepared to question, argue, justify and explain and see this as intrinsic to their learning of this specific subject. To some extent, this methodology (or pedagogy?) is extrapolated into science. But I am not sure why I don't use it for teaching English.

I have always hoped to see a non-subject-specific National Curriculum, where process skills are the educational outcomes. In which case, teaching approaches become generic rather than specific. Primary teachers in the UK are now enthusiastically returning to a cross-curricular teaching approach. How then does a subject-specific teaching method and pedagogy fit? For primary teachers, I am not sure that the enhancement of 'subject-specific' teaching methods and pedagogy is where we should be heading. Rather, what teachers need is time to reflect on and challenge their thinking about their pedagogical practices. Providing time for classroom-based research could assist in this.

Kath Cross, 26/07/05: When I first read Barbara's comments, I was reminded of work we did when [the inspection framework] Ofsted was created. The HMI with responsibility for subject areas [I was responsible for mathematics at that time] were asked to write guidance for inspecting the subject and particularly for lessons. When we had all done this, it was decided that much was generic, so a general section was written and I remember feeling a bit put out that what remained for mathematics did not feel right without the other part! This was over ten years ago.

I have a feeling, however, that teaching mathematics is different. I think we are trying to teach more concepts than in other subjects. I could be wrong, but when I think back to my A-level teaching, for every file of notes students had for mathematics, they had 2 or 3 for physics and 4-6 for biology. In the latter, they were listing far more facts and then having to remember them. The 'blobs and links' method (ATM, 1987) of learning (see Figure 1) did not seem to apply much there.

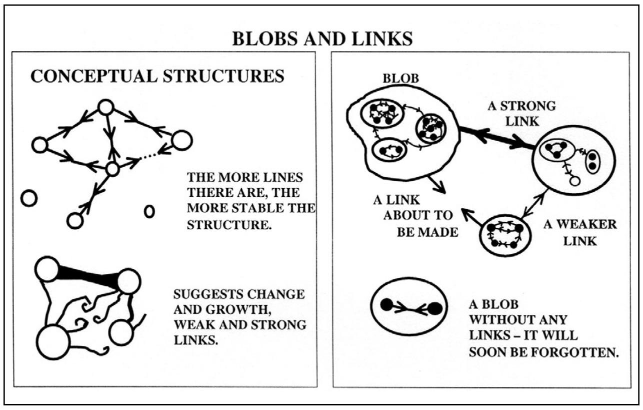

Figure 1: Image created from an Association of Teachers of Mathematics working group (ATM, 1987)

Conceptual structures are very important in mathematics, and I don't pretend to know enough about other subjects to comment on their importance in those subjects. Helping pupils to engage with ideas and then develop an understanding of them is a key to successful learning of mathematics. Maybe the 'quantity' of this kind of work is what makes the difference.

I found myself saying "yes" to Dave Hewitt's comments.

Dick Tahta, 26/07/05: How do you learn the rules of a game? Some people (lawyers?) like to first study and discuss the written rules. Others start playing straight away. (Sometimes that's a difference between dads and mums.) How do you teach the rules of a game? What game?

There are various games taught in schools. For instance, the game of being a chemist, a geographer, a reader, a writer ... a mathematician. Such games all have a specific vocabulary that has to be provided and then learned. Some of them have large vocabularies. And you can't really play the associated game until you have learned a lot of words. Some games are quite parochial – being a geographer in Canada is to work on different material than a geographer in Uzbekistan.

Of course, to play these various games you need more than words or facts. You need to know the rules of the game. But it's not easy when you are a child – indeed often not all that interesting – to play being a geographer, say, or being a chemist and the associated rules can be quite complex (and not always universally agreed). But, hey, you can pick up the rules of being a mathematician. And, my goodness, when and if you do so, you are free of the providers – as long as they have done their job and given you sufficient vocabulary. (The aim of all learning is to become free of the teaching?)

Teaching mathematics involves making the rules of the game available from the start. Ah! How do you do that? Well, that's another issue – and Alf and Dave have already made some interesting comments. All I want to do here is to emphasise my own view that teaching mathematics does indeed involve very subject-specific methods.

(PS: What about the game of being a teacher of mathematics. How do you learn the rules of that game? Whose rules? Do you first pick up the lid and read the provided rules? Or do you just start playing the game? Are there subject-specific ways of teacher-training?)

Laurinda Brown, 27/07/05: I am thinking about a number of interconnecting strands that have partly developed in awareness because of my role as a teacher educator. The question for me is always 'What is learning?' This is a generic question of course but it seems to me that your range of answers to this question will shape how you envisage teaching and learning. Learning about teaching means that the ways of answering 'What is learning?' extend and grow and change.

Similarly, the strategies used to support students' learning of mathematics will depend upon the range of responses each individual would make to the question 'What is mathematics?'

To extend Dick's questioning about how students in classrooms learn to play the game, whatever student behaviours that teachers see as mathematical can be commented on to their students as being mathematical – some such behaviours have emerged already in this conversation, for example, stressing and ignoring, but there are many others. We do not all see the same behaviours as mathematical though, so the 'rules of the game' are the 'rules of the games of doing mathematics in this classroom'. There can be teacher comments to students individually, in groups or to the whole class on the conjecturing atmosphere, being a community of inquirers (asking why), getting organised, turning conjectures into theorems. These teacher 'metacomments' can be common at the start of the teaching year, supporting the students to develop the behaviours they will find adequate for doing mathematics together. Once the behaviours are established the need for metacomments reduces.

Now, metacommenting is not a subject-specific pedagogical behaviour but what is metacommented upon is arguably subject specific because those comments are coming from the awareness of mathematical behaviours of the teacher. I do not believe that all mathematics teachers do this metacommenting, or if they do would comment on the same behaviours, so I am therefore thinking of a personal subject-specific pedagogy.

As a teacher educator, I comment to student teachers on my decision-making as a teacher of mathematics whilst teaching mathematics through the same process of meta-commenting but this time the awarenesses I am using to comment with are not only about the mathematics but also about teaching mathematics.

Gill Hatch, 26/07/05: I was encouraged by Dick's contribution, which seemed to be coming at the topic from my kind of direction. Cockcroft (DES, 1982) said that "mathematics is a difficult subject both to teach and to learn" (para. 228). Well, if this is correct then subject-specific methods may be to do with overcoming the difficulties involved in any way we can.

Firstly, mathematics expresses itself in a language that sometimes uses words which pupils think they understand like volume and difference when they actually mean something quite different. When I sit in lessons I am always being struck by new examples of the double use of words. I videoed a year 6 lesson (10-11 year old students) a year or so ago when the class, and to some extent the teacher, got tied in knots about the meaning of diagonal. To sort this, you need to realise that the 'sloping' idea of diagonal uses the word as an adjective whereas the mathematical use of diagonal is as a noun. Only the open way of teaching that this teacher used allowed the misconception to emerge and be discussed so that Figure 2 in fact had diagonals even if they did go straight up and down!

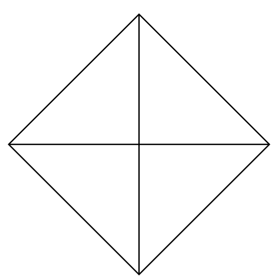

Figure 2: A diagonal going straight up and down

Secondly, mathematical language is highly symbolic and condensed and a lot of meaning can be included in a simple set of symbols. Consider the meaning of 2097 or of the equals sign – this symbolism and its underlying concepts creates huge problems for pupils, especially if the teaching takes short cuts and does not come back to unpacking meaning.

These two points seem to imply that there are perhaps commonalities with language teaching. It makes me think of the active learning of language in which pupils are expected to express themselves in the new language and see if what they say makes sense to others. I had a sense when I was playing algebra games with pupils that the speaking of algebra aloud was significant in it becoming an accepted language – one of my pupils talked of doing mental algebra … But of course there is a sense in which mathematics is a created language and not an organic one like English or French and this makes it different

Thirdly, mathematics is not a subject which is about people like almost all others are. Even the sciences are about objects in the real world. Here I am discarding the pretence of 'word' problems and even the call to teach pupils through real-world problems – I have never believed that mathematics is, in essence, to be seen only for its value as an applied science, vital though its applications are.

Fourthly, mathematics is essentially, in my view, a way of thinking, of ordering information and making logical deductions from what is known in order to establish new truths. Now, this process often seems to becomes less natural to pupils as they move up the school – sad really when you think how babies and toddlers learn to make logical deductions from what they discover about the world.

This process can be to some extent separated from the technical language of mathematics. I have never forgotten the discomfort of a mathematics graduate on a Masters level module on proof when I produced proofs which used no algebra. Most pupils are quite the reverse, getting lost in the technicalities of the algebra so that they lose the meaning of what is happening. But how to teach the appreciation and a simple mastery of the process seems to be a specific requirement of teaching mathematics.

I am interested by those who say that solving a *Su Doku* is not mathematics since the sometimes very complex, logical process by which one establishes what number goes where seems to me to be essentially mathematical – I admit I am not interested in deciding where logic ends and mathematics starts.

Fifthly, the teaching of mathematics, as we know it, creates 'misconceptions' – actually I think these are creative as long as you realise that they cannot be avoided and that exposing them can lead to deeper learning. But there must be a subject-specific teaching skill concerned with an awareness of this aspect of teaching and learning mathematics.

Finally, there is a ongoing tension between trying to create fluency in the use of mathematical language and the presentation of the need for logical argument. If some level of fluency is not created, then the simplest calculation or algebraic manipulation can become so laborious that understanding of purpose is destroyed. However, much of mathematics teaching has been directed at fluency without purpose ….

So, where does this get me, am I just writing about the problems of teaching mathematics?

I hope not, I think many of the above problems can be addressed by what are, I suppose, the skills of getting pupils to talk, to listen to one another, to present arguments without fear of being wrong. Perhaps the subject specificity lies in awareness of the reasons for these approaches and what aspects of mathematics we are valuing and expecting them to expose for pupils to respond to.

Pete Griffin, 27/07/05: These are my conjectures.

I feel that when I am planning a lesson and I am asking myself questions of the "How am I going to do this?" type then I am being informed by a set of principles and beliefs which transcend subjects:

- Learning is an active process of assimilation, not a passive reception of fully formed ideas.

- Talking (in the form of, for example, convincing, arguing, describing) is essential to learning because talk can be thought of as externalised thought.

- "Only awareness is educable"; I am working on and with students' awareness.

- That we all learn in the context of a social situation (education is a social activity) and, particularly young learners find it difficult to separate what they are learning from the situation that they are learning it in.

- Activity without reflection on that activity rarely produces deep learning.

- Learning is often optimised if I can train and then call upon a set of shared behaviours.

- Learning is a personal thing; it is bound up with emotion. To be a successful teacher you need to tap in to this and provide learners with the motivation to learn ("Why do I need/want to think about this?").

Now, this may be made mathematically specific when we talk about these beliefs and principles using the example of mathematics:

- Mathematics is a way of thinking, not a body of knowledge to be transmitted from expert to novice.

- Mathematics isn't just being able to perform certain techniques; it's about understanding big ideas.

- Seeing connections between concepts and actively making links between different mathematical topics is important.

- The symbols aren't the mathematics; there is a need to approach the symbols through activities that enable the learner to contact the mathematical idea and then to work on symbolising it.

But these statements are not really specific to mathematics. You could replace "mathematics" with something else and they would still work.

It is only when I am asking myself questions of the "What is the essence of the mathematical idea, here?" type that I entertain subject-specific considerations:

- Fractions – 3/5 (for example) means '3 of those things called fifths' and 3/5 + 2/5 is a particular example of 3 things add 2 of the same thing.

- Algebra is generalised arithmetic and so a technique like multiplying out brackets [*e.g.* $(x + 3)(x + 2)$] emerges from seeing the general in a number of particular examples where two numbers are multiplied together: 23×22, 33×32, 43×42, ….

- Ratio – any number can be transformed into any other by multiplying.

- Graphs – a rule that transforms numbers into other numbers can be represented as a picture (using coordinates). If there is link in the numbers (*i.e.*, there is a rule) then this will show itself in the position of the plotted coordinates.

… and I think that it is in these sorts of areas that the idea of subject-specific pedagogy exists.

Notes

[1] National College for School Leadership, *'Making mathematics count' in school networks*, NCSL, 2005: a pack of 12 booklets available free: e-mail nlc@ncsl.org.uk.

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[These references follow on from page 15 of the Younggi Choi and Jonghoon Do article beginning on page 13. (ed)]

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