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LEFT-VENTRICLE REDUCTION THROUGH AN ETHNOMATHEMATICS LENS

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Ethnomathematics is the mathematics of identifiable cultural groups. D'Ambrosio (1985, 1990, 1998) includes national and tribal societies, children of certain age groups, labor groups and professional classes. Borba (1997) agrees with this and states

> even the mathematics produced by professional mathematicians can be seen as a form of ethnomathematics because it was produced by an identifiable cultural group. (p. 40)

The professional class examined in this study was a group of thoracic cardiovascular surgeons. Sixty days were spent in operating rooms and surgical clinics observing and interpreting the mathematics used by this professional group. Overall three surgical procedures were observed, coronary artery bypass grafts (CABG), carotid endarterectomy (CEA), and left-ventricle reduction (LVR). This article focuses on left ventricle reduction. (This surgical procedure physically changes the dimension of the human heart's left ventricle by removing a muscle shaped like an ellipse from the wall of the left ventricle.)

It was one minute past eight in the morning as I enter the operating room and heard (Surgeon, S; Anaesthesiologist, A),

> A: How much ventricle do you decide to take out Doctor?
> S: Hard decision.
> A: How big is it now?
> S: I measure it about 7 centimeters.
> A: What should it be three or four?
> S: Yeah, about. (Surgeon to Anaesthesiologist, operating room, day 8)

This conversation referred to the inside diameter of the patient's left ventricle. While the surgeon knew that this inside diameter was almost double a normal value, he certainly did have a hard decision to make in determining how much ventricular wall to remove:

> How much of the ventricle we took out was certainly not very quantitative. I mean, we had heard that the advice is that you take out as much as you can bear, and then take out some more. (Surgeon, LVR post-operatively, day 8)

To my surprise, the surgeon at the operating table had a marking pen in his hand and used it to draw a dashed-line ellipse on the posterior ventricle surface of the patient's heart. This ellipse went across the ventricle and the heart was physically lifted to allow the continuation of the drawing about the base of the ventricle. Imagine drawing an ellipse on an inflated balloon. As I attempted to estimate the size of the ellipse, the surgeon requested the "measuring stick" and a sterile ruler was handed to him by the surgical nurse. He quickly measured the minor axis to be six and a half centimeters. I was amazed, first the drawing then a ruler. It was 9:09:28 when the first incision was placed in the patient's ventricle after the three surgeons agree on the placement (Fieldnotes, operating room, day 8).

Quantity and space

The first of two questions guiding this research was:

> What mathematics do these surgeons use to deal with surgical issues regarding quantity and space?

Mathematics was not used in the traditional sense. Surgeons relied on mathematics for measurements of quantity and space, but these measurements were done by medical technologists, specialized medical equipment, and by eye. Number sense for these surgeons allowed them to judge the reasonableness of values and understand the context of a numerical value without attaching units of measure (Sowder, 1992; Hope, 1989; Howden, 1989; Garofalo and Bryant, 1992 have each reported similar findings on the importance of context and reasonableness of numbers). Estimation and measurement by eye were critical mathematical elements that I observed in the operating room.

Kastner (1989) discusses the use of real-world items for measurement by students in mathematics classrooms. Textbook illustrations related to measurement are often whole unit measures and may mislead students developing measurement understanding. Measurement done be these surgeons frequently was done to the nearest millimetre, or fraction thereof, in contrast to textbook examples. One surgeon checking this study, reported there must be "a subconscious use of quantification."

Many of the surgical observations presented in follow-up interviews within a mathematical context, for example quantifying an anastomotic site (an anastomotic site is the location where two blood vessels are sutured together) for the placement of sutures, had not been thought of by these surgeons to be mathematics. Mathematics for this group of surgeons was confined to their past classroom experiences as students enduring a lecture in calculus or statistics.

The mathematical perspective presented to these surgeons is a common thread in ethnomathematics research, that is to say, the group being studied frequently do not consider their activities to be mathematics. Like many aspects of mathematics applications, it is appropriate to recognize that mathematics in thoracic cardiovascular surgery is context specific. Both arithmetic and algebra underlie the mathematics used by these surgeons (*i.e.* ejection fraction, cardiac output, body surface area). With the exception of the LVR procedure, when a sterile ruler was introduced into the surgical field these surgeons relied on sophisticated medical technology and the expertise of individuals trained to use these technologies for quantity measurements. For these surgeons "the eye" did their measuring.

Millroy's (1992) study of South African carpenters also revealed that the mathematics used was context specific. She brings out how measurement was often done with a rough board that had two scratches to denote a length measured by a carpenter. In her discussion of the mathematics of these carpenters, an important geometric element rises to the surface when she describes daily carpenter routines using the language of mathematics. Millroy emphasizes the carpenters' intuitive understanding of diagonals bisecting within a rectangle and how they create symmetric woodworking patterns, without using traditional mathematics.

Masingila (1993) provides similar findings in her study of carpet layers and the context within which they work. Carpet layers must pay attention to nap direction, location of doors and closets, and the joining of different floor coverings at these openings. Carpet layers use tape measures and convert units of feet to units of yards. Masingila's (1993) carpet layers allowed a two-inch margin of carpet error at the edges, quite a contrast to thoracic cardiovascular surgery working within millimeters.

The context of thoracic cardiovascular surgery demands accurate measurements often to the nearest hundredth, calculated by technology. These thoracic cardiovascular surgeons use precise surgical instruments to estimate inside diameters of blood vessels and rely on medical technologists to provide accurate, precise, technologically computed values associated with quantity and space as it relates to the human heart. Precision of measurement and use of technology in the context of thoracic cardiovascular surgery is dramatically different than Millroy's (1992) and Masingila's (1993). Introduction of thoracic cardiovascular surgery to the body of ethnomathematics research introduces the role of technology.

Scales of measurement, in some cases less than a millimetre, used by the surgeons in this study are in stark contrast to the scales of measurement of the Marshall Islanders, meters and kilometeres, using mattang for ocean navigation (Ascher, 1995). Saxe's (1991) study of the Oksapmin's use of their body for counting demonstrates another tool used by a particular group for a measurement task. These comparisons point to the recognition that different groups utilize different tools for measurement. Inherent within any tool is a scale of accuracy that varies dramatically from group to group. Stepping away from ethnomathematics and looking into a traditional mathematics classroom, students are often asked to estimate to the nearest half-inch or centimeter, again a contrast to thoracic cardiovascular surgery.

One distinct difference between classroom development of number sense and the number sense of these cardiovascular surgeons is that these surgeons frequently dealt with ranges of values rather than specific values as might be found in classroom mathematical environments. These thoracic cardiovascular surgeons relied on their own internalized number sense to interpret and judge the reasonableness of measurements of quantity and space.

This medical procedure is so complex that, for instance, mathematically modeling real-world phenomena in this surgical context is beyond the secondary mathematics level (*e.g.*, modeling left-ventricle reduction). This group of surgeons were observed to use problem solving, reasoning, communication, geometry, measurement, and estimation in thoracic cardiovascular surgery. Each of these items is brought to the forefront of importance in today's school mathematics.

These thoracic cardiovascular surgeons were fast problem solvers, for obvious reasons. They did not have the luxury of analyzing and re-analyzing patient results, as the consequence could be dire, death. Mortality is a critical element for cardiovascular problem solving, that is, when the surgeon checks his (all participants in this study were male) results a successful solution is a living patient. Calculations are not so crucial in a mathematics classroom.

Roles of procedural, conceptual, and intuitive knowledge

The surgeons possessed adequate procedural mathematical knowledge, as demonstrated in their recollection of formulae related to the heart, but did not have a need for implementing this mathematical knowledge in their daily routines. Calculations that are needed by these thoracic cardiovascular surgeons are performed by sophisticated medical technology with software specifically designed for particular quantities, but they still have the ideas and concepts about their mathematical practices.

A surgeon could illustrate calculations that he could perform, if needed. During one of the formal interviews a surgeon shared his knowledge of "human dynamics," calculations that he makes on a regular basis. Initially, I perceived this to be procedural mathematical knowledge but quickly recognized a demonstration of his conceptual knowledge of oxygen delivery:

> Delivery of O_2 per minute is cardiac output times the oxygen content of arterial blood, umm, that is heart rate times stroke volume and that is hemoglobin, grams per deciliter, times the percent saturation hemoglobin, times this constant one point three four milliliters of O_2 per gram of hemoglobin. Yeah, so when you umm, and this is in liters per minute so when you normalize everything it's thirteen point four times cardiac output times hemoglobin times percent sat. And that's O_2 delivery.
>
> $D\ O_2\ (\text{ml}\ O_2/\text{m}) = \text{C.O.} \times \text{Ca}\ O_2$
>
> $(\text{HR} \times \text{SV})\ \text{Hg}\ (\text{g/dL}) \times \%\ \text{sat} \times 1.34\ \text{ml}\ O_2/\ \text{g Hg l/min}$
> $= 13.4 \times \text{CO} \times \text{Hg} \times \%\ \text{sat}$

These surgeons' conceptual mathematical knowledge related to the various measurements of quantity and space, both preoperatively and intra-operatively, to create a whole picture for each patient and their potential need for surgery. While this study focused on conceptual knowledge of the human heart, the whole picture is not complete unless some attention is given to the vascular system. Understanding an individual patient's vascular system leads to knowledge of their lifestyle and genetics and implications for surgery, this must all be part of the reasoning and problem solving by a surgeon.

Intuition of the attending surgeons is the most difficult to put in words or quantify. My perceptions of intuition were hindered by a lack of background in thoracic cardiovascular surgery. In discussion, one of the surgeons began to question his intuition when he posed these questions for thought:

- How did we get to the point of being able to do it? ["it" refers to a "subconscious use of quantification"]
- What should we tell our protégés about doing it? [how can the attending surgeon 'teach' number sense and/or quantification as this applies to thoracic cardiovascular surgery?]
- What about the early part of our education? [with respect to the learning of quantity, *i.e.*, developing number sense for surgery]
- More mathematics … more statistics … more understanding of quantitative stuff?

Typically, an individual training to become a thoracic cardiovascular surgeon has earned a bachelor's degree, a medical school degree, participated in a surgery rotation, and competed a three-year residency as a Thoracic Cardio Vascular (TCV) fellow, a range of education that exceeds ten years. It was understandable when the surgeons were not able to articulate their development of intuition! Academically, it may be important that the surgeons began to ask these questions in consideration of the future training of surgeons.

A number of assertions were developed in this study. To me, surgical geometry is the most critical aspect of these TCV surgeons' ethnomathematics. The term "surgical geometry" was introduced to me when I was requesting permission for entry to the hospital. "Surgical geometry" (Rosa, personal communication, suggests this may be "ethnogeometry.") is a surgeon's language to sum up the surgical procedure of creating an anastomosis:

> We want to create it [the anastomosis] so it does three things: one, doesn't leak, two, stays open, and three, it's formation will create a hood.

The end of the vein being sewn into an artery has to be "about one and a half to two times bigger than the artery." The surgeon continued,

> length, width, and diameter are measured with our eyes, then we put the stitches in to create that appearance. (Surgeon, phone interview, day 8)

This is the key element of TCV, "that's the whole thing here." The initial definition of surgical geometry was restricted to coronary artery bypass anastomosis, an end-to-side anastomosis but also came to include end-to-end anastomosis and the re-shaping of the left ventricle during the left-ventricle reduction surgery.

Each mention of the left-ventricle reduction procedure became synonymous with LaPlace's law.

> Observations of hearts in different mammalian species, including humans, suggested that heart mass is proportional to body weight. Heart mass and radius are also related. An increase in radius leads to an increase in mass to maintain this normal relationship. When the radius increases without commensurate changes in mass, compensatory mechanisms occur that lead to further dilatation and, eventually, heart failure. Furthermore, Laplace's law states that, $P = kT/R$, where P is the cavitary pressure, T is the wall tension, and R is the radius of the chamber. Thus, an increase in the radius leads to increased wall tension and increased myocardial oxygen consumption. Theoretically, there are three options to reverse this situation: (1) increase the LV mass (this is not possible at the present time but may be the basis for cardiomyoplasty), (2) decrease the wall tension by medical therapy (vasodilators), or (3) reduce the LV radius (cardioreduction). (Batista, *et al*., 1997, p. 636)

These surgeons chose cardioreduction or left-ventricle reduction, frequently referred to as the Batista procedure for the Brazilian surgeon to first perform this surgery.

To reduce the left ventricle is to remove an ellipse-shaped tissue from the left-ventricle wall. The purpose of removing an elliptical shape from the wall is for closure of the wound.

When I asked the surgeon about the size of the ellipse removed for left- ventricular reduction he stated:

> The general idea is based on LaPlace's law, which is that the larger the radius the cylindrical structure is, or spherical for that matter, the more tension there is on the wall for any given pressure. So, the bigger, the more tension there is on the wall, and the tension on the wall of a heart determines blood flow more than any other one thing except the single other possibility of having coronary artery obstruction. Which he did not have. Meaning that tension made the inside of the heart ischemic [inadequate blood circulation] and less able to work. So the more tension there was, the more ischemic it became the less it worked and the more dilated [enlarged]. The premise is that by taking out part of that wall that you reduce the tension and the rest of the heart can then work better. The muscle that remains is okay and can work. There may be other elements besides ischemia that are operative there. (Surgeon, formal interview, day 57)

These surgeons understand the concept of how the pressure and wall tension relate to a heart's ability to work, but do not have a need for formal calculations of LaPlace's law. They need to understand that removal of a portion of the left-ventricle wall may improve the ventricle's ability to work. These surgeons had to rely on relationships and mathematical underpinnings, not on formal mathematical language from their classroom past. Each surgeon participating in this procedure understood the relationships between wall tension, oxygen consumption, ejection fraction, and how they anticipated these values being affected by this surgery. These surgeons did not have a need for numerical calculations, or even details of the law and its algebraic representation, they relied on their conceptual understanding.

Left-ventricle reduction

Measurement of the minor diameter of this ellipse was secondary to these surgeons' understanding of the anatomical boundaries they were working in. Drawing the ellipse then measuring was counter-intuitive from my mathematical perspective. While the advice was to take out a little more, postoperatively I learned that these surgeons had discussed the amount to be removed from an anatomical perspective.

> But the real answer is there are three territories of the heart, supplied by three arteries. We took out one of those territories and we had to see where the territories overlapped, and we could see that because the heart artery is on the surface of the heart. So, we knew going in we were going to take out that vascular area, and we knew it was going to be about one-third of the heart, but it was defined more by anatomical boundaries than it was by a relative sense or quantitative sense or something like that. (Surgeon, formal interview, day 47)

Exploring the premise of this procedure a little further, reducing the radius of the left ventricle, revealed that this radius reduction was not the primary focus in the actual surgery (R stands for researcher and S for surgeon):

R: On that patient's ventricle you removed a six and a half by ten centimeter ellipse?
S: Uh, uh
R: Was that a dead piece of his heart?
S: No it was fine.
R: It was fine?
S: Sick heart.
R: Sick heart, that was just from watching the film and saying well Dr. I think this is,
S: Yeah
R: A good chunk right here.
S: Yeah, this looks right.
R: Okay, now what were you thinking with respect to the radius of that ventricle when you took that out?
S: Wasn't thinking about it at all.
R: Wasn't thinking about it at all?
S: But I'm pretty good at creating images when it's things like this, but we need to be better, maybe I'm pretty good at it but is guy b as good at it?
R: Well, yeah, that's something to consider for your residents.
S: But that's a problem, we need to visualize, but we need more than visualization, truth is we need a template.
R: Yeah, I described that procedure to someone and told them you drew a pen mark on the guy's heart, now was that patient's heart enlarged?
S: Yeah.
R: From previous …
S: Huge.
R: Okay.
S: So we had to get it down in size. That's why we chose the ventricle reduction. The concepts are all drawn in those papers I gave you. So that would be valuable to us, say this is the balloon you want in here so where you want it to be.
R: That's why I was asking you about the pre and post echo.
S: We already know, see in the operating room we got it right, a normal ventricle is four to five centimeters, and this guy's was about eight, so we got it down to five. How can we do that and be consistent? Maybe I can do it, but not everyone could. (Surgeon, informal LVR interview, day 10)

As the surgeon above implied, intuition for cutting-edge surgery is a major component for the surgeon conceptualizing and performing these surgeries. Left-ventricle reduction was in its infancy in TCV surgery. Since the time of this research, the *Batista Procedure* is being referred to as the *Batista Concept* as this invasive procedure has made inroads to new research and new surgical techniques.

There is, therefore, a rich, ethnomathematical body of knowledge for these TCV surgeons. These findings do not diminish the importance of procedural mathematical knowledge, but place conceptual mathematical understanding at the forefront. Surgeons are experts at communicating quantity, connecting past classroom experiences to their surgical realm, and solving problems that have life and death outcomes. Communication, reasoning, problem solving, and connections are the elements of importance in a mathematics classroom and these elements are critical in the 'real-world.'

Implications for mathematics education

Critiques of mathematics education often cite a lack of conceptual and procedural knowledge on the part of students. As a past secondary mathematics teacher, I recall many department meetings and conversations concerned with a lack of conceptual knowledge on the part of students, standing in stark contrast to a curriculum focused on procedural knowledge. This study puts procedural mathematical knowledge in a new perspective, not diminishing its role but demonstrating the importance of conceptual mathematical knowledge for these thoracic cardiovascular surgeons. The importance of conceptual mathematical knowledge lies in these surgeons' understanding what particular numbers imply and where these numbers come from. For these TCV surgeons, how these numbers relate to human anatomy and physiology begins a surgeon's decision making with regard to individual patients. Finally, the above-mentioned elements constitute these surgeons' number sense, which is critical for each surgeon so they may communicate patient needs by relying on the language of quantity and space. These surgeons are able to communicate their knowledge due to having a deeply intuitional number sense.

Number sense begins for youngsters during their developmental years and continues to become enhanced during their formative school years. By the time surgeons have earned the title of attending surgeon they have had in excess of twenty years of education to refine their number sense. An individual's school experience needs to be very rich in mathematical activities to develop a rich conceptual mathematical understanding.

As early as 1989, the National Council of Teachers of Mathematics posed four universal standards for mathematics education: problem solving; reasoning; communication;

and connections. Problem solving begins with initial patient consults and continues through post-operative care. Reasoning is an element of making sense of surgical quantity and implications therein for patient care. Communication has many paths to consider: surgeon to patient, surgeon to surgeon, surgeon to nurse, surgeon to physician, and surgeon to the operating team. Each surgeon must be able to communicate to these groups to delivery quality patient care. Finally, each surgeon was able to connect their past academic experiences to their current surgical experiences.

Implications for ethnomathematics

Ethnomathematics for these thoracic cardiovascular surgeons is

1. measurement of specific quantities
2. ways to interpret these measurements
3. use of these measurements in decision-making
4. successful anastomotic construction
5. understanding spatial relations of anatomy and
6. being able to reshape a heart's geometry,

which are all idiosyncratic to this professional class. As D'Ambrosio's definition suggested, ethnomathematics does exist for professional classes.

An artifact of TCV surgery is how quantity is reported. Technology calculates quantity with appropriately attached units. These surgeons do not attach units of measure to quantity in conversation. Another artifact of these surgeons is the attachment of adjectives to quantified values, *i.e.*, carotid blockages as for example, mild or severe. These artifacts contribute to each surgeon's number sense and how they communicate with one another.

This research supports Masingila's finding of problem solving in the 'real-world' and as exercises in the classroom. The problem solving found in this environment has potentially dire consequences in the death of the patient. Understanding the language of these surgeons is akin to studying a non-English speaking group in some respects. Medical terminology is not commonplace outside of medical facilities and thus has meaning to a sub-group of medical speaking communities. While this study was not linguistic in nature (*cf.* Leap, 1981) these surgeons do use language with underlying mathematical meaning, *i.e.*, tightness and gradient.

One other essential finding worthy of mention deals with ethnomathematics research of communities with no formal mathematics background, writing system, or often times a lack of recognition that their actions can be described with mathematics. These surgeons have a rich mathematics background. They know that mathematics underlies many of their daily routines but at the forefront for these surgeons are the surgical procedures themselves. These surgeons do not have a need to recognize mathematical underpinnings of their activities, just as culture groups studied through an ethnomathematics lens do not have a need to recognize the mathematics underlying their cultural activities.

Future research of professional groups would include more than asking 'how do you use mathematics?' but would investigate how specific groups use mathematics as compared to how mathematics is taught.

So, there are some answers here to the age-old student question of "when am I ever going to use this?" I believe this question and the opportunity to create pedagogical bridges, in this case for surgical education, is at the heart, no pun intended, of the value of ethnomathematics research.

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