Do We Rob Students of a Chance to Learn?

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We often present students with the solution of a problem even if they themselves are quite ready to think of a solution on their own. Just as an elevator to the third floor robs us of our chance to use our muscles, we should be alert of times when we rob our students of a chance to think.

Take the case of constructing regular hexagons with only straightedge and compass — surely one of the most natural and easy constructions possible with these tools. Most texts, both elementary and secondary, immediately show students how to do it — and usually in only one way. But, starting at an early age, students can have opportunity for activities during which they can experiment and find things out for themselves — things which are related to and involve hexagons. The more students know about hexagons, the more situations in which they have met them, the more problems they have solved in connection with hexagons — including constructing them by means other than ruler and compass, the more chance the student has of divising at least one (and probably more) ruler and compass constructions by himself. Even during activities at an early age students can experiment and find out by themselves; they can make and test hypotheses thereby strengthening their mathematical muscles.

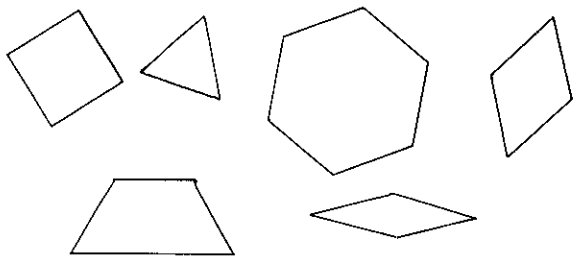

Figure 1

One of the earliest experiences with regular hexagons (other than examining bathroom floors!) may come through the use of Pattern Blocks [1]. The Pattern Blocks consist of a variety of blocks all with the same edge length, including equilateral triangles, regular hexagons, and isosceles trapezoids and diamonds. Each shape has its own color. (Figure 1) While freely building with the blocks, students sooner or later run out of yellow hexagons and spontaneously find ways to substitute other blocks for them. This they can do long before they learn the words "regular hexagons". (Figure 2) Later they may wish to find out how many different ways they can make (or cover) a yellow regular hexagon [2]. Other problems can be suggested by them or by the teacher. In any case, the children gain experience with the fact that 6 equilateral triangles make up a regular hexagon.

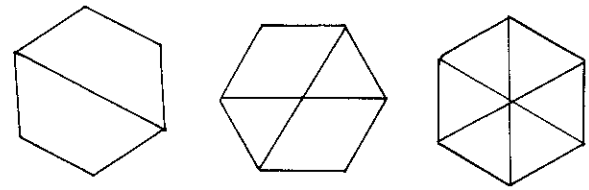

Figure 2

To get back to constructing regular hexagons. There are many avenues to take. One might do some preparatory work as follows: *What different shapes can you see if you draw in one, two, etc., diagonals of a regular hexagon?* (You can pose this problem without using or defining the word "diagonal", by using the word but not defining it or by defining it first and then using it.) This leads naturally to asking how many essentially different ways can one draw in 2 diagonals of a regular hexagon and then to how many different diagonals are there altogether. (Figure 3) It also leads naturally to the names of various shapes and to the idea of congruence.

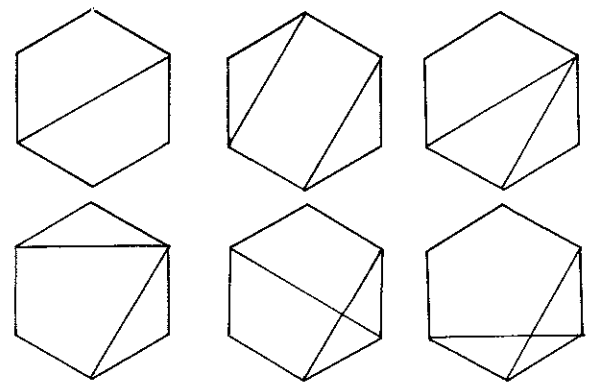

Figure 3

Eventually, when three diagonals are drawn in, one drawing will provide six equilateral triangles (Figure 4) At another time students can be asked if they could, using ruler and compass, *construct* six equilateral triangles to obtain a regular hexagon. After they have done this the "long way" the "usual" construction by using a circle can be seen as short cut if it has not already been discovered by accident while experimenting with ruler and compass or while making geometric designs and patterns.

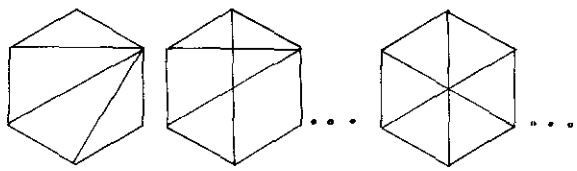

Figure 4

Of course there are many other ways of constructing or making regular hexagons by using ruler and compass and by paper folding We will briefly discuss or indicate some of these. *Take two paper equilateral triangles. How do you have to place them to get a regular hexagon in the overlapping region? When you have found the position fold over the corners of the single layers to obtain it (Figure 5)* Explore to see what other types of hexagons you can get for the overlapping region *What other shapes can you get. A quadrilateral? A parallelogram? A triangle?*

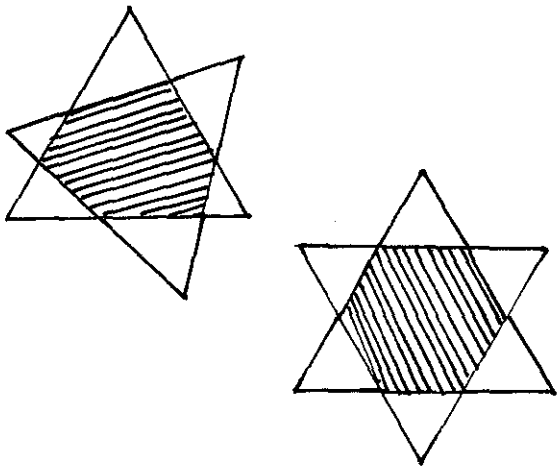

Figure 5

If you put two equilateral triangles base to base, could you fold them in order to obtain a regular hexagon? (Figure 6) There are other questions you can ask at different levels. For example, if the area of one equilateral triangle is one unit, what is the area of the hexagon? What is an easy way to see that the perimeter of the hexagon is less than the total perimeter of the two triangles taken *separately?*

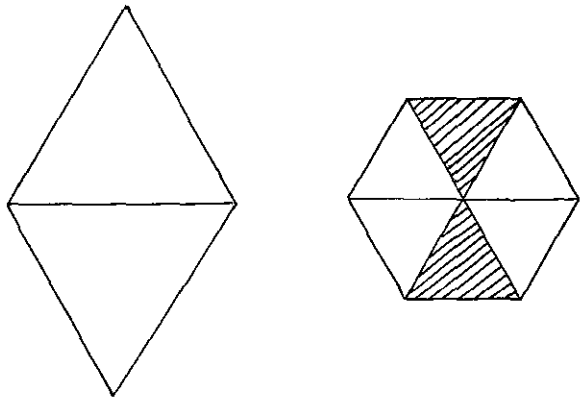

Figure 6

Can you take just one equilateral triangle and turn it into a regular hexagon? (Figure 7)

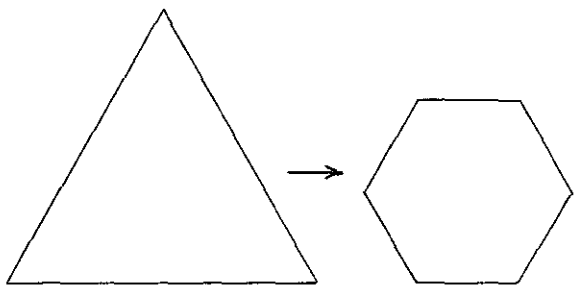

Figure 7

There are many other ways of thinking about regular hexagons. Take a handful of equal coins, border one of them and draw in the tangents. Or if you are in Denmark, for example, and have coins with holes in, mark the center of each coin and join up the dots! (Figure 8)

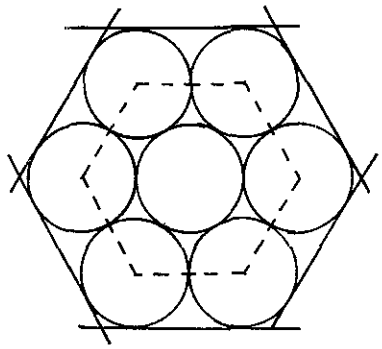

Figure 8

If you have a circular geoboard with, say, 12 nails you can make two regular hexagons on it. (Figure 9) Of course a board with 6 nails also enables you to make a regular hexagon. What about a board with 18 nails? 15? Explore!

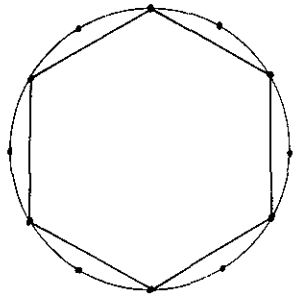

Figure 9

Take three strips of paper. Can you lay them on top of each other to get a regular hexagon at the intersection? Can you join any of the points obtained to get another regular hexagon? (Figure 10)

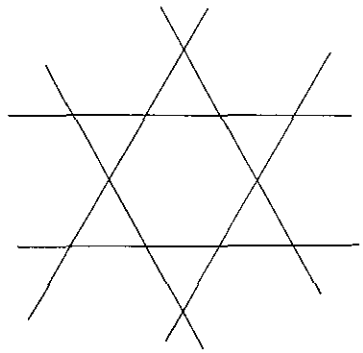

Figure 10

To return to ruler and compass constructions. The usual "step off the radius six times" method is of course based on the fact that six equilateral triangles form a regular hexagon. But even with ruler and compass constructions students will come up with other ways [3]. The drawings in Figure 11 show ways students have constructed regular hexagons.

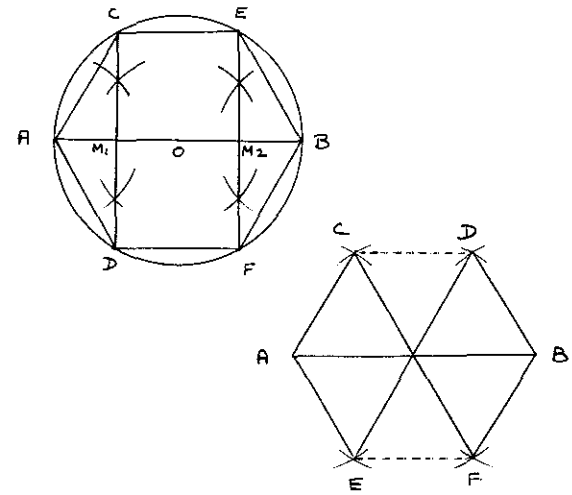

Figure 11

In Fig. 11(a) M_1 and M_2 are midpoints of OA and OB respectively and CD and EF are perpendicular to AB. In Fig. 11(b) four equilateral triangles were constructed on segment AB and then CD and FE were drawn.

I have taken the ruler and compass construction of a regular hexagon only as an example — you can take a problem or a theorem or topic of your own choice. You can assemble a variety of approaches dealing with your chosen topic at several age levels by thinking of related activities, prior activities, activities involving different media and by exploring new questions that come to your mind.

You may wish to examine your textbook to see (a) what ground work the text lays for the construction of a regular hexagon and where such groundwork, if it exists, appears; (b) if the construction is based on any experimental work with ruler and compass and (c) if more than one way of constructing a regular hexagon is shown; (d) if the text indicates or suggests that more than one method is possible! Of course you may wish to examine the text for other topics along these dimensions. Maybe if we did more in the years before high school and during high school than merely showing the students the solution to a problem they would not feel so helpless when confronted with geometry! They might even like it!

Notes

[1] Pattern Blocks were created and developed at the Elementary Science Study in the 1960's. A Teacher's Guide was produced. Since then many commercial publishers have marketed Pattern Blocks and a variety of Teacher's Guides and student materials are available.

[2] This is one of the problems from the original Teacher's Guide for Pattern Blocks (Elementary Science Study)

[3] Walter, M. "Constructing Regular Hexagons" *Oregon Mathematics Teacher* Oct. 1978, 17-18