Case Studies on the Symbolism of Difference-Finding Problems in First Grade

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The research literature has shown that 'compare' problems are more difficult than 'separate' ('change') problems, since the former refers to a static relation of compared sets as opposed to the latter which describes an action (Nesher, Greeno and Riley, 1982; Carpenter and Moser, 1982, 1983; Carpenter, Hiebert and Moser, 1983; Lean, Clements and Campo, 1990; Fan, Mueller and Marini, 1994; Fuson and Carroll, 1996). Moreover, the degree of difficulty changes according to the wording of problems (Hudson, 1983; de Corte, Verschaffel and de Win, 1985) For example, the problem, "How many more birds are there than worms?" (the 'more' problem) proves more difficult to solve than the problem, "Each of these birds wants to eat a worm. How many of them will not get a worm?" (the 'won't get' problem)

There is some controversy over which factors, either of linguistic difficulty or basic mathematical knowledge, contribute to students' success and failure (Okamoto, 1996) In any case, students acquire various strategies to solve single-step addition and subtraction problems in conformity with their semantic structures (Carpenter, Hiebert and Moser, 1981; Matthews, 1983).

In this article, I discuss a kind of subtraction, one of 'comparison', such as: "There are eight white flowers and five red flowers. How many more white flowers are there than red flowers?" (the difference-unknown, more problem). I will call it the 'difference-finding' problem, following the usual Japanese terminology. The problem of comparison of white flowers with red flowers mentioned above proves more difficult than the one of birds with worms. Since the former is the comparison of two sets of which the objects are of the same kind, the problem situation lacks a matching cue (the 'more' problem where it is difficult to match).

On the other hand, the problem, "How many more birds are there than worms?" involves the comparison of two sets in which the objects are of different kinds and one that implies natural matching (the 'more' problem where it is not so difficult to match). It proves more difficult than the 'won't get' problem, though. Some researchers point out that when teaching the arithmetic symbolism of 'difference-finding' problems, the problems of the 'won't get' type must be treated first and then the 'more' problems which are difficult to match after (Thoyama and Ginbayashi, 1976).

In the next section, after a brief introduction of typical addition and subtraction problems adopted in Japanese mathematics textbooks, I report on three instances of introductory teaching of 'difference-finding' problems in the first grade. The teaching in both case 1 and case 2 was carried out in July 2000, and Case 2 was partially written up in Miyamoto (2000). Case 3 was carried out much earlier, in June 1989, and the teacher concerned was a university student on a teacher-training course. Previously, I reported it briefly (Hasegawa, 1990, in Japanese), but in this article I re-examine it at some length.

The goal of the classes was to have students understand the symbolism of 'difference-finding' subtraction problems. Students in the classes learned about a subtraction number sentence based on the 'separate' problem. However, students could not reach the goal. In each of the classes, students made various comments about the problem presented by the teacher, many of which seemed to deviate from the problem initially posed.

Based on the teaching episodes, I focus on the symbolism of 'difference-finding' problems, since the number of articles on the teaching and learning process of the symbolism of elementary word problems in general, and on the 'difference-finding' problems in particular, seems relatively small. Then, I discuss the implication of the students' utterances about the problem.

Teaching about difference-finding problems

Japanese first-graders learn the numerals from one to ten, zero, ordinal numbers, and then elementary addition and subtraction only involving numbers not exceeding ten. Addition and subtraction are usually taught in the following order. I show here brief, typical examples of problem situations described in mathematics textbooks

(1) *Addition*

(a-1) *Joining*: Three goldfish are in a fish bowl and two goldfish are in the other. How many goldfish will be there if you put them in a big fish tank?

(a-2) *Part–Part–Whole*: There are three boys and two girls on the playground. How many children are there altogether?

(a-3) *Increase*: Three goldfish are in a fish tank and two goldfish are in a fish bowl. How many goldfish will be there in the fish tank if you put the two in the fish bowl in the fish tank?

(2) *Subtraction*

(s-1) *Remainder-finding*: Eight goldfish are in a fish tank. How many goldfish will be there if you scoop five goldfish out?

(s-2) *Complement-finding*: There are eight children on the playground. Five are boys. How many girls are there on the playground?

(s-3) *Difference-finding*: There are eight red cars and five blue cars. How many more red cars are there than blue cars?

The development of the introductory parts of these various problems in mathematics textbooks are similar to each other. The problem is not presented in written form but by a picture. Following it the question, "How many goldfish are there?" for example, is stated. Then, the situation is represented by manipulatives such as blocks. Finally, a number sentence is introduced. Treating addition and subtraction in this way means that the Joining and remainder-finding problems are fundamental to the cases of addition and subtraction, respectively.

Joining, part-part-whole and increase problems are all different if we take into account their problem situations. However, if the problem situations of part-part-whole and increase are represented by manipulatives, they can be reduced to joining with a little modification of the operation on the manipulatives. For example, in the case of joining, a student gathers three blocks and two blocks to the middle with both hands. In the case of increase, he/she pushes two blocks with the right hand toward the left hand where three blocks are. At this time, he/she moves the left hand slightly with the intention of joining all blocks, and so the situation of increase is reduced to the one of joining. Then joining and increase can be unified at the stage of activity based on the operation on the manipulatives. Therefore the situation of increase can be represented by the same symbolism as joining. In the case of addition, its structure is not so complicated that students seem to understand the unified symbolism about these three different types of addition situation without any confusion.

I now present three classroom settings that treated the introductory part of the difference-finding problem in the first grade. All students in each class learned about the remainder-finding problem with its symbolism. The teaching objective was, as I mentioned earlier, to represent the difference-finding problem by a number sentence.

Case 1

After students learned about subtraction in relation to remainder-finding and complement-finding problems, they made up word problems about subtraction situations. Each student drew pictures of the problem situation on two or three sheets of drawing paper and wrote problem sentences on the reverse sides. All but one of the word problems students made were remainder-finding or complement-finding; however, one student made up a difference-finding problem. The teacher adopted it and started the class with the student presenting his problem.

Ka: There are five morning glory flowers in the school garden. [He showed the first picture where five flowers were drawn.] There are two morning glory flowers in my garden. [He showed the second picture of two flowers.] What is the difference?

The teacher encouraged students to represent the situation by blocks. Some put five blocks and two blocks and others put only five blocks on their worksheets.

Fu: I think the two blocks are not necessary, because if you put them, you may think there are seven altogether.

T (the teacher): How many flowers are there?

All: Seven!

T: Are there seven?

Az: There are five in the school garden and two in his garden, so, it looks like there are seven.

Ta: If I join them, there are seven.

T: Let's read the problem again! [Students read the problem written on the blackboard.]

Fu: I think you must put five blocks at the first scene.

T: What is the answer?

All: Three! Five take away two is three!

T: How do you move the blocks?

Some students showed the removal of two blocks from five on the blackboard; however, when the teacher asked why they removed two blocks, they could not explain.

Fu: I grew morning glory and brought two flowers in my garden. So, I took away two.

Ko: There are two in his garden and two in the school garden. [He matched two blocks in five with two blocks.] The remainder is three.

Fu: There were five flowers in the school garden, but the two died. So, I removed [them].

Ta: The story is on subtraction, so I take away the two blocks.

Az: Five flowers bloomed in the school garden but two flowers fell down and two in his garden fell down, too. So, there are three.

Ar: Two flowers in the school garden and two in his garden died, so the flowers are three.

At the end of the lesson, the teacher asked for their impressions of the lesson. Some students said that the problem was difficult and that they could not see whether the two blocks indicating two flowers in his garden were necessary.

It seems to me that almost all students knew that the answer was three and more than a few students knew how to represent the situation by the number sentence: $5 - 2 = 3$. However, since they could not link the subtraction they had learned and the problem situation they had just faced, they made up some stories so as to fill in the missing link. At least, the utterances of Az ("fell down") and Ar ("died") suggested the operation of not only matching but also removing (a core element of many subtraction situations).

Case 2

The teacher told the class that the lesson was on subtraction and showed two drawings of morning glory: eight white flowers were drawn on one and four red flowers on the other.

T: There is a difference between what you are now seeing and the subtraction you learned previously. What is it?

C_1 (a child): The color is different.

C_2: Drawings usually line in a row and there is an arrow But today, there is no arrow.

C_3: The number is different

C_4: We usually take away the same things But today, we think the difference

C_5: Usually, flowers died or butterflies flew away Today, there is no such scene.

C_6: It is probably to take away just the number of red flowers.

Then he asked them to make up a word problem according to the pictures After every student had written a problem on his or her worksheet, some students presented their own problems The following are their problems (identical problems are omitted).

P_1: There were eight flowers of morning glory. Four flowers died. How many flowers are there?

P_2: There were eight flowers of morning glory Four flowers changed their color. How many flowers are there?

P_3: There were eight flowers of morning glory. Four flowers became red. How many flowers are there which are not red?

P_4: There are eight white flowers of morning glory. There are four red flowers of morning glory Which is more and how many?

P_5: There are eight white flowers of morning glory. There are four red flowers of morning glory. I planted four flowers in another place. How many remainders are there?

The teacher also presented his own problem:

There are eight white flowers of morning glory There are four red flowers of morning glory Which is more and how many more are there?

Then he encouraged them to think about his problem through drawing pictures and writing figures. From my observations, I saw many students writing the number sentence 8 - 4 = 4 on their worksheets. After a while, six students drew figures on the blackboard and explained how they thought about the problem one after another. Their explanations were similar: that is, "There were eight white flowers and four turned red So, there are four white flowers " During the last part of the lesson, the teacher obtained the number sentence (8 - 4 = 4) and the answer (four flowers), and the lesson was over

In this lesson, too, morning glory was adopted as the objects of the problem. It is familiar to Japanese students, since teachers often make first-graders cultivate and observe it. The utterances of C_2 ("An arrow" is a symbol of taking away), C_4 ("The same things", which seems to mean a part of the whole), and C_5 show that students were aware of the difference between the type of subtraction they had learned (the remainder-finding) and the problem situation they confronted.

Moreover, although many students wrote the number sentence, they could not link the sentence with the problem situation Their explanations given to the problem in the second half of the lesson seem to be heavily influenced by the student who explained first In other words, six students explained their drawings one after another, but since there was neither negative response to nor negative evaluation of the first (and second, and so forth) speaker(s), subsequent students might imitate preceding speech At the same time, they were influenced by the problems students proposed in the first half (P_1 - P_5). Really, all but one of the problems were remainder-finding problems and their expressions were similar to the six students' comments. It is inferred that they made stories for their explanations so that the answer of the problem and the number sentence would agree

Case 3

In this case, 'bean-jam bun', 'bean bun' and 'cream bun' were employed as objects in the problems They are sweet buns Japanese children like and are called *an-pan*, *mame-pan* and *cream-pan*, respectively in Japanese. Accordingly, there is no possibility of students confusing 'bean-jam bun' (*an-pan*) with 'bean bun' (*mame-pan*) verbally. The teacher started the lesson to present a remainder-finding problem as a review of the last lesson

T: There were eight bean-jam buns [She presented drawings of eight bean-jam buns on the blackboard. The magnet was attached to the reverse of each drawing so as to be movable on the blackboard. All drawings mentioned below were of the same type.] Taro [a boy's name] ate three buns [while presenting a drawing of Taro's face] Now he eats [in response to a student's utterance that Taro did not eat yet and she presented a drawing of a stomach and moved three buns onto the drawing of stomach]. How many buns are there now?

In response to the problem, a student wrote its number sentence (8 - 3 = 5) on the blackboard and another student explained it by moving drawings of three buns into the stomach again. Then the teacher explained the next problem situation.

T: A boy bought eight bean-jam buns and three bean buns. [She presented the drawings of eight bean-jam buns arranged in two rows of four on the upper part of the blackboard and drawings of three bean buns on the middle part of the blackboard.] Which was more?

All: Bean-jam buns!

T: How many more?

All: Five!

Then the teacher encouraged them to show whether their answer was correct or not with using marbles and writing a number sentence. After a while, a student wrote 8 - 3 = 5 on the blackboard. When the teacher asked whether it was all right, quite a few students replied they were not sure.

T: This [the first problem] was subtraction, wasn't it? You take away three from eight because Taro ate. But why is this problem also subtraction?

Sa: We use subtraction when we take away, but this is addition. [She wrote 8 + 3 = 5 on the blackboard. Being asked why, however, she could not explain.]

Though the teacher made students ascertain both the problem and the meaning of addition and subtraction previously learned, students' confusion did not ravel out. For example, a student uttered that the number sentence was 8 + 3 = 11.

T: What do you say when I asked which was more and how many?

All: Bean-jam buns. There are five more than bean buns.

T: Bean-jam buns were five more than bean buns. How do you know?

Ab: I knew when I saw. We can use subtraction. There were eight bean-jam buns and three bean buns. Then Taro ate three.

T: Did Taro eat?

Ok: There were eight buns. I take away three.

T: Why do you take away?

Without reaching the objective of the lesson, the teacher finished the lesson. She treated the difference-finding problem also in the next lesson by modifying the problem situation. She started the second lesson to review a remainder-finding problem that was the same problem treated at the beginning of the previous lesson. Then she presented a difference-finding problem:

Nine children came to buy cream buns to a bakery but there were only six cream buns. Talking through the story, she presented drawings of nine children's faces and six cream buns on the blackboard (see Figure 1), and a large card on which a problem was written: "How many children are there who will not eat a bun?"

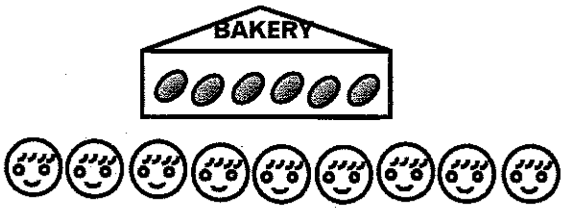

Figure 1 Drawings of children and buns

Some students said the answer was three, then the teacher prompted them to think with marbles and represent it by a number sentence. After a while, a student put nine and six marbles in two rows on the blackboard, drew six lines to indicate the one-to-one correspondence and enclosed three marbles that did not match (see Figure 2).

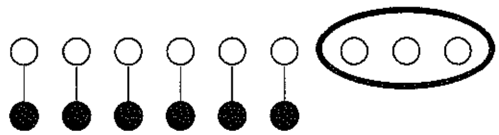

Figure 2 One-to-one correspondence between marbles

However, when the teacher told the class to write the number sentence, four students wrote 6 - 9 = 3, 6 + 4 = 9, 6 - 9 = 0 and 9 - 6 = 3 on the blackboard.

T: There are addition and subtraction. Can you use addition? You use addition in the cases of joining and increase, don't you? Let's read the problem again! [All read the problem.]

Na: There are six buns, so six can eat. So, three can't eat.

The teacher asked whether addition was suitable for the problem and, if addition was unsuitable, whether subtraction was. A student went in front of the blackboard and distributed drawings of six buns to drawings of six children on the blackboard (see Figure 3).

Figure 3 Matching of children and buns

T: The first problem [the remainder-finding problem] was subtraction because you took away three buns. Why can you use subtraction in this case?

Ka: At first, there were six buns. If more buns are made, it is addition. But children bought buns, so it is subtraction.

T: It is not addition. Why subtraction? In the case of that problem [the first remainder-finding problem], you took away three buns, so it was subtraction. In this case, do you take away, too?

Ha: Three can't get.

T: Three can't buy. What do these six do?

Na: Only children who can buy buns eat them, so there is no bun. So, it is subtraction.

T: The problem is: How many children are there who will not eat a bun?

Sh: If they divide a bun between two children, all can eat.

Mo: Sharing is a good idea! But three had better buy other buns

T: Only cream buns we think!

Sh: [In relation to Figure 3] These three can't eat, so it is subtraction

Na: Children who couldn't eat go out, so it is subtraction. The sentence is 9 - 3 = 6

T: Let's read the problem again! [All read it.] How many children won't eat? [All uttered 'three'.]

Ya: I have an idea! We take away children who ate buns, like this way (Figure 4). Then the sentence is 9 - 6 = 3.

T: What do you think of his idea? [Every student made a circle with both of his or her hands to indicate agreement.] Try with your own marbles!

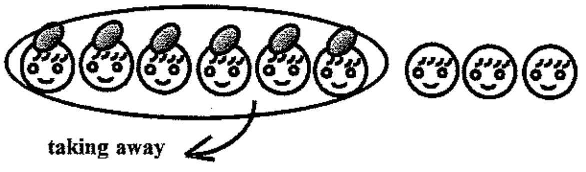

Figure 4 Taking away six children with buns

The teacher made students match marbles and remove them, and write the number sentence on their worksheets. Then she took up the difference-finding problem treated in the first lesson again, advised them that it was also represented by the subtraction symbolism and finished the lesson.

When she had made the teaching plan of the second lesson she had designed, in the last part of the lesson, she intended to give six buns (drawings) to six out of nine students, one by one, at the front of the classroom, and then make the six go back to their seats. This action corresponds to the scene where six children in the problem go out of the bakery. However, this action was omitted in the actual lesson, since there was insufficient time to execute it. Moreover, since the lesson was the last opportunity for her in the teaching practice term, she preferred to resolve the problem left unsolved from the first lesson. If the action was really executed, students could understand thoroughly why the story of taking away children who got buns was more reliable and consequently why the symbolism of subtraction was adequate.

The lesson proceeded mainly as the chain of the teacher's initiative, a student's reply and the teacher's evaluation. However, the evaluation was not simply the provision of teacher approval or disapproval, but included a variety of questioning and prompting of the students to remember the problem and its answer. Many students also spoke not just a few words but told stories about the problem. It is to be desired that such kinds of questioning (and answering) appear among students, but it is difficult for first-graders who have entered school not long before.

Though the problem situation was modified in the second lesson, its representation by symbolism was not an easy task for students. Some students presented the one-to-one correspondence between two groups of objects (Figures 2 and 3). This could indicate the answer to the problem, but it did not bring the correct number sentence with it. Students proposed various number sentences, which might be produced from their intention of representing the problem situation by symbolism.

Halving a bun and getting other sorts of buns were realistic solutions to the problem. At the same time, such student assertions showed their perplexity in not knowing how to resolve the deadlock. The first problem presented did not clearly prohibit halving a bun or getting other buns. However, at the stage of representing the problem situation by marbles, the implicit assumption of not halving or getting others came clear. Though it was not stated definitively, students seemed to accept it naturally. Really, there was no student who halved a marble, to which its physical constraint might have contributed to some extent. Also, students lined up nine and six marbles to represent nine children and six buns according to the problem.

At the stage of representing the situation by symbolism, although some students proposed various number sentences, the teacher did not accept them, while additionally asking why addition was not adequate and subtraction was adequate. The problem situation of 'children and buns' might be more familiar for students in the class than the problem of 'birds and worms'. It is probable that the problem situation of children and buns evoked their sympathy and made them express their ideas of halving and/or getting others At nearly the end of the period, finally, removing the drawings of children with buns was proposed and "the teacher's research for the one correct answer to her question" (Mehan, 1979, p. 293) came to an end.

Discussion

2.1 Transformation of difference-finding problems

In Case 3, under the premise that a subtraction number sentence be introduced based on the remainder-finding situation, the difference-finding problem is transformed into the remainder-finding type, from where its number sentence is obtained The following illustrates the process of its transformation.

(a) *Initial comparing situation*: There are two disjoint sets, children and cream buns, whose relation is static A question like "How many children are there who won't get a cream bun?" is asked about the situation

(b) *Construction of one-to-one correspondence:* In order to distribute a bun to each child, the small set (the set of buns) is embedded in the big set (the set of children). The subset isomorphic to the smaller set is formed in the bigger set by virtue of the one-to-one correspondence.

(c) *Taking away the part from the whole*: Subtraction of remainder-finding type is applied as taking away 'children who got cream buns' from the 'whole set of children', which can be realised as children who got buns going out of the bakery Then the difference-finding problem is represented by the same subtraction number sentence as the remainder-finding problem that the students already know. In the difference-finding problem, the small set is not simply taken away from the big set because it is disjoint to the small set. We cannot remove red flowers from white flowers (in case 2) or cream buns from children (in case 3). Instead, we take away white flowers that corresponded to red flowers from white flowers and children who got cream buns from the whole set of children

Taking these steps into account, at the introductory teaching to the difference-finding problem, we must prepare an adequate problem situation in which students think out one-to-one correspondence naturally (Thoyama and Ginbayashi, 1976; Hudson, 1983; Fan, Mueller and Marini, 1994). For example, birds and worms, children and caps, and children and buns facilitate their matching On the other hand, white flowers and red flowers or red cars and blue cars are more difficult, since there is no cue for matching.

In the case of naturally-paired objects, it is crucial which type we adopt, either the 'won't get' problem or the 'more' problem. Moreover, in order to take away the subset isomorphic to the smaller set from the larger set, the initial problem situation must contain (or at least suggest) the removal when we teach the number sentence of difference-finding problems. "Birds which got worms fly away" and "Children who got buns go out of the bakery" provide such examples. In a classroom, as in case 3, it is one way to present the problem of the 'more' type of naturally-paired objects that suggest removal and then to encourage students to construct the 'won't get' type of story. At any rate, only the presentation of one-to-one correspondence that displays the static relationship does not produce students' spontaneous conception of its number sentence.

Stories students told in the classroom

An arithmetic word problem, for example, "There were eight bean-jam buns and Taro ate three. Now how many buns are there?", indicates the beginning of a story. When it is presented to students in a mathematics classroom, they are usually asked to represent it by a number sentence. At the same time, they are asked to explain and give reasons why the sentence is correct. In many cases, it is implemented by making up a sequel to the initial story:

> Let's look at these eight marbles as buns. Taro ate three buns, so you take away three marbles from eight Then the remainder is five So the equation is subtraction and is 8 - 3 = 5

If the story is acceptable to other students and the teacher, it is legitimised in the classroom. If not, students may make distinct stories or may revise a previous story until an acceptable story is proposed.

In the three classrooms discussed, too, students made up various stories. In particular, in each of the classes discussed in cases 1 and 2, students made up subtraction word problems prior to solving the difference-finding problem, which might have led them to talk about a sequel to the main problem. They did not always ignore the initial problem, but instead took into account some parts of the problem. They recast the initial problem to accord with their knowledge of remainder-finding subtraction they had previously learned while keeping invariant the numerals and objects (morning glory flowers or buns) in the problem. The stories the students gave were their attempts to harmonise the problem they faced, the correct solution they already knew and their previously-acquired knowledge. However, their stories in these cases could not provide the missing link between difference-finding problems and remainder-finding problems

In a mathematics classroom, students learn not only mathematical content but also what a lesson is and how to participate in one (Lampert, 1990) Lampert worked on two teaching agendas simultaneously in her class. One was what she called 'knowledge of mathematics or mathematical content' and the other was 'knowledge about mathematics or mathematical practice'. Generally speaking, students learn about these kinds of knowledge in their mathematics classes They may acquire knowledge *of* mathematics by rote or a mechanical manner and may construct knowledge *about* mathematics where mathematical practice simply means to find key words and numerals necessary to solve a problem. How to gain knowledge of these kinds contributes to the construction of students' mathematical worldviews in general and their problem-view of mathematics in particular, which may be called 'knowledge for mathematics'.

As I mentioned earlier, when students solve a word problem, they are encouraged to represent the problem situation by a number sentence and to make up a story relevant to the problem. Conversely, what is relevant is decided according to what extent other students and the teacher accept the story When the story is irrelevant to the problem, at least if the teacher regards it to be so, it may be excluded by a manner of questioning that suggests the speaker's misunderstanding or their lack of or insufficient comprehension of the problem ("Let's read the problem again!" or "Did Taro eat?", for example) or by a more direct indication of its error.

Moreover, no matter how realistic the story, if it is irrelevant to the problem, it is excluded: for example, students' suggestions to halve a bun and to buy other sorts of buns in case 3 are not adopted as adequate solutions Through these conversations, students learn how to express a story in a mathematics classroom, what stories they can come up with and the limits of recasting the original story

The contribution of constraints on what students talk about in a mathematics classroom is twofold. On the one hand, it serves to help students construct knowledge for mathematics and how they must infer in the class. Which conditions are changeable and which are not in a problem, what kind of knowledge they have previously learned both in and out of school is exploitable and available to solve the problem they are confronting and what kind is not, suggests to students what mathematics is, how it differs from other subjects and from their realistic steps to solve daily problems.

On the other hand, it may separate a mathematical problem they have just solved from their experiences, practices and daily cognition, since the constraints may reinforce a view that students are only permitted to use and exploit the conditions described explicitly in the problem. However, a problem setting always contains somewhat implicit and tacit information: for example, the problem adopted in the second lesson in case 3 includes the assumptions that there is no one who dislikes buns or who prefers a bean-jam bun to a cream bun and that sharing a bun half and half is not permitted. These alternatives set up distinct problem situations which differ from the teachers' initial objectives However, it is permitted to interpret that children in the problem who got cream buns go out of the bakery, which is a customary action after buying something at a store The boundary of what is 'permitted' and what is not may well seem ambiguous and fluid for the first-graders.

Students' confusion and their offering irrelevant stories show that they cannot solve, infer about and understand the problem fully. It suggests the problem itself is inadequate for them and that we must contrive and improve the problem situation in order to connect the problem naturally with their daily cognition and previously-learned knowledge of mathematics

Conclusion

Even students who do not receive formal instruction spontaneously develop various strategies to solve word problems. It is necessary to investigate students' spontaneous concepts while taking into consideration the interdependence of spontaneous concepts and scientific concepts (Vygotsky, 1962)

At the same time, theoretical and practical study of how to teach the symbolism of difference-finding problems is indispensable. The difference-finding problems range from the 'more' type which is difficult to match to the 'won't get' type. In this article, I examined only the introductory part to the difference-finding problem. When starting to learn the symbolism based on the 'won't get' type, students must learn the difficult-to-match 'more' type treated in cases 1 and 2 and in the first lesson of case 3.

According to Greeno, Smith and Moore (1993), transfer can occur if a learner can perceive the invariant structure of activity across changes in the situation. Symbolical representation of a difference-finding problem does not always come under the case of transfer, or it may be classified as 'near-transfer' However, it gives us a suggestion that we must arrange various difference-finding problems so as to indicate or prompt student activity to make up suitable stories to solve the problem and understand its symbolism. We must investigate the whole course of instruction on the difference-finding problems and, at the same time, we must examine whether the learning of the symbolism of difference-finding problems in the early stages of first grade is suitable

In this article, I presented students offering various stories. It reminds me of a panicked or impulsive action when a bird or butterfly inadvertently flying into a room pushes ahead to a windowpane to go out into the open air again when seeing the view outside. In the case of students, as opposed to a bird or a butterfly, we must try to suggest the open position or encourage them to find out for themselves how to open it before we as teachers open the window too quickly It is not to say that we need to design and position suitable windows.

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