ETHNOMODELLING AS THE ART OF TRANSLATING MATHEMATICAL PRACTICES

MILTON ROSA, DANIEL CLARK OREY

Etic (global) is related to an outsiders' view on the beliefs, customs, and scientific and mathematical knowledge developed by the members of diverse cultures. According to Sue and Sue (2003), this approach is known as *culturally-universal.*

Emic (local) is related to insiders' view on how they do, in this case mathematical tasks. It focuses on *this is how we do it.* It respects cultural practices, social understandings, customs, religion, gender, and beliefs. According to Sue and Sue (2003), this approach is known as *culturally-specific.*

Emic and etic concepts were first introduced by the linguist Pike (1967) who drew on an analogy with two linguistic terms. Phon*emic*, which are the sounds used in a particular language and phon*etic,* which are the general aspects of vocal sounds and sound production in languages. All possible sounds human beings can make constitute *phonetics*. However, when people learn to speak a particular language over time, they do not use or hear all possible sounds. In this regard, as modelled by linguists, not all sounds *make a difference* because they are locally significant. This means that they are the *phonemics* of that language (Pike, 1967).

If we make a similar analogy to modelling, it is possible to state that the emic approach is about differences that make mathematical practices unique from an *insider's* point of view. *Emic* models are grounded in what matters in the world of those being modelled by investigating mathematical phenomena by means of their interrelationships and structures through the eyes of the people native to a specific cultural group.

However, many models are *etic* in the sense that they are built on an outsider's view of the world being modelled. In this context, etic models represent how the modeller thinks the world works in the context of the person or group under study, through systems taken from their reality while emic models represent how people who live in such contexts think these systems work in their own reality.

Emic-etic represents a continuous interaction between globalization and localization. It offers a perspective that both approaches have elements of, or valuable perspectives related to, the same phenomenon. It is a blending, mixing, and adaption of the two approaches in which one component must address, indeed involves, the voices of the local culture, system of values and practices. By focusing on local (emic) knowledge and then building on it to integrate global (etic) influences, it is possible to develop a mathematics curriculum that is rooted in local traditions and contexts, but also equipped with a global knowledge that creates a sort of localized globalization.

Pike (1967) argues that a more influential trend in cross-cultural investigations privileges the etic approach based on outsiders' accounts of *other* cultures. This is a sensitive issue because such privileging blinds us to the importance of emic knowledge that is related to an insiders' perspective providing insights into cultural nuances and complexities. The danger in this process is that the emic could be relegated to the *quaint*, *primitivist*, or *exotic*, not seen as a unique application of a powerful mathematics. This despite the fact that many emic uses of mathematics have a far longer history than modern academic mathematics has.

This is why we feel it is necessary to incorporate emic knowledge into the existing etic framework of the mathematics curriculum to develop a holistic understanding of cultures. In this process, the elaboration of curricular activities emerges from creative and dynamic encounters of local and global knowledge. Consequently, it is necessary to develop an anthropological perspective in relation to teaching processes in order to help students to sort out their own learning challenges (François, Pinxten & Mesquita, 2013). We propose *ethnomodelling*, which combines ethnomathematics and mathematical modelling, as an approach that promotes emic knowledge as complementary to etic knowledge.

The aims of this article are related to important emic-etic (dialogic) perspectives using ethnomodelling by discussing its anthropological approaches when developing investigations that connect ethnomathematics and mathematical modelling. We argue that ethnomodelling creates a firm foundation that allows us to integrate emic, etic, and dialogic approaches in exploring mathematical knowledge developed by the members of distinct cultural groups.

Ethnomodelling

Figure 1 (overleaf) shows how ethnomodelling can be described as the intersection between cultural anthropology, ethnomathematics, and mathematical modelling. It can help members of distinct cultural groups to translate mathematical ideas, procedures, and practices found in their own communities among diverse mathematical knowledge systems.

In the ethnomodelling process, the intersection between mathematical modelling and ethnomathematics relates to respect and valuing of the tacit knowledge of members of cultural groups, and which enables us to access, translate, and assess problem-situations faced daily as we elaborate mathematical models in different contexts.

It is a socioculturally bound construct, and allows us to define ethnomodelling as the study of mathematical phenomena that adds cultural components to the modelling

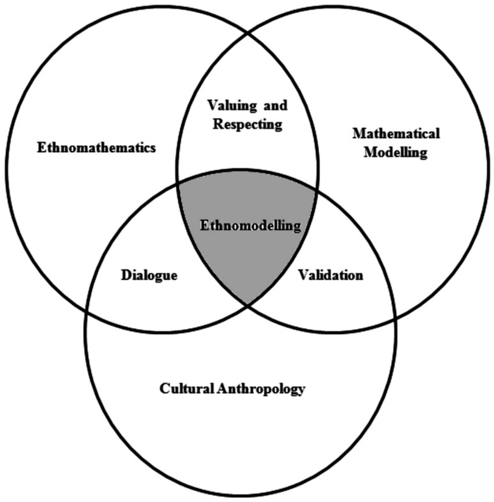

Figure 1. The intersection of three research fields described by ethnomodelling (from Rosa & Orey, 2010).

process. This perspective forms a basis for significant contributions of an ethnomathematical perspective in re-conceiving mathematics through innovative pedagogical actions for modelling processes in classrooms.

When we look at how members of cultural groups use their own mathematical knowledge and traditions to translate and solve problems faced in their own environments, local (emic) knowledge serves as an intersection between ethnomathematics and cultural anthropology. For example, Eglash, Bennett, O'Donnell, Jennings and Cintorino (2006) state that cultural anthropology has always depended on acts of translation between emic and etic knowledge, which are addressed to help the members of distinct cultural groups to understand specific mathematical practices acquired in diverse contexts.

This translational process is used to describe the process of modelling local (emic) cultural systems that may have Western (etic) mathematical representations because the mathematical knowledge, found in this act of ethnomodelling, arises from emic rather than etic origins (Rosa & Orey, 2010).

In this context, Eglash *et al.* (2006) argue that it is reasonable to expect that an ethnomathematical perspective applies modelling procedures to establish relations between local conceptual frameworks and the mathematical ideas embedded in global designs through translations.

On the other hand, it is important to state that the epistemological basis of ethnomodelling is not restricted to methods of direct and/or literal translations used in non-Western mathematical practices to the Western tradition because it consists of historical studies that emphasize cultural procedures and practices such as found in Chinese, Hindu, and Islamic contexts.

The emic and etic constructs of ethnomodelling

The emphasis of ethnomodelling tends to privilege the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups in order to enable its communication and transmission through generations. The models that describe these systems are representations that help the members of these groups to understand and comprehend the world. Ethnomodels link cultural heritage with the development of mathematical ideas, procedures, and practices.

During ethnomodelling investigations, emic constructs represent the accounts, descriptions, and analyses in terms of conceptual schemes and categories that are meaningful and appropriate to the members of the cultural group under study. These emic constructs are essential for the development of an intuitive and empathic understanding of mathematical ideas, procedures, and practices rooted in a culture and they are essential for conducting effective ethnographic fieldwork.

Ethnomodelling investigations also involve creating etic constructs, which are accounts, descriptions, and analyses of mathematical ideas, concepts, procedures, and practices expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers and investigators (Lett, 1996).

Because such comparisons demand the use of standard units and categories, etic constructs must be precise, logical, comprehensive, replicable, and observer independent. These constructs are essential for translation, understanding, and cross-cultural communication, which are important components of ethnology.

Ethnomodels of the Landless Peoples' Movement in Brazil

In the struggle for agrarian reform, access to a plot of land, and living and producing on it, make measuring the land a central activity of the members of the *Landless Peoples' Movement (Movimento dos Sem Terra – MST)* in Southern Brazil, mainly because of the importance placed on sustainability and planning of agricultural production.

The elaboration of curricular mathematics activities related to the demarcation of land with participants of this movement is proposed in the study conducted by Knijnik (1993). These activities are related to the method of *cubação* of land, which is a traditional mathematical practice applied by participants of this specific cultural group to measure and determine the area of the land in their settlements (occupation sites).

The daily necessities of the *MST* members caused them to capture the procedures of these techniques, showing that, despite their low level of schooling, they were able to apprehend and apply knowledge related to the methods of *cubação* of land, which is one of the tools used to solve problems related to the measurement of land with irregular shapes by applying distinct methods to determine this area.

According to Knijnik (1996), this method met the specific needs of the members of this movement because they applied it to determine land areas related to the delimitation of planting sectors as well to demarcate the plot of land of each family in the settlement.

In this case, they established production goals related to their own logistic possibilities such as storage and drying, bagging, transport, and selling products at local markets. The land worked by *MST* members was prepared according to the type of farm and quantity of the product these members harvested and commercialized.

It is important to state here that the emic knowledge related to the development of these methods was orally transmitted and diffused to *MST* family members by their ancestors across generations. Thus, the mathematical knowledge involved in these local methods is also related to productive activities that members of this specific cultural group performed in their daily routines. For example, the need for the development of *cubação* of land with irregular shapes was in accordance to its accessibility depending on its topology and the quality of desired agricultural products.

This method is used to calculate the total area of a region, after its occupation, in order to calculate the amount of money needed to be paid or received for the cleaning work of the property or for the preparation of the land for planting as well the demarcation of areas to be cultivated, to plan and to delimitate areas for the construction of houses and shelters for animals.

This method is also used to make payments for work done by the members of a settlement in the state of Bahia, in Brazil, according to the land *frames* or shapes. For example, there are jobs related to land with *two corners, three corners,* or *four corners* in accordance to the shape of the cultivated area (Silva, 2012).

According to D'Ambrosio (1999), the validation of these methods within agricultural communities and settlements results from the development of informal agreements of signification that results from a long cumulative process of generation, intellectual organization, social organization, and diffusion of this knowledge.

Cubação with quadrilateral shapes

Mathematical practices investigated in the study conducted by Knijnik (1993) consisted of two methods that were called by her students in the classroom *Adão's Method* and *Jorge's Method.*

These two students who were members of *MST* presented, explained, and taught these specific ways of measuring their land in their settlements to the other learners in the classroom.

The investigation of these two methods reveals the interrelations between local (emic) and academic (etic) mathematical knowledge concerning the upper bound estimation of the area of a tract of land with irregular shapes.

The first method is called the *Adão's Method* that transforms the shape of an irregular quadrilateral into a rectangle. In this context, Adão explained how to determine his method, which we consider as an emic ethnomodel:

> Well folks, this is the most common formula that is used on the countryside, up there on the farm, right? And, let's assume that I am the owner of a crop and I lent this *frame* here to a friend *to mow* and I told him that I will pay three thousand by the *fourth*. Then, he mowed this land and he even *passed the rope* himself to find its area. Then, he measured this *wall* here, 90 meters, the other, 152 meters, 114 meters, 124 meters *[See Figure 2].* Did you notice that there is no *wall*, no base, and no height that has the same measure, right? The two landmarks that are *lying down* are the *bases* and the *heights* are those that are *standing up*. Ok. So,

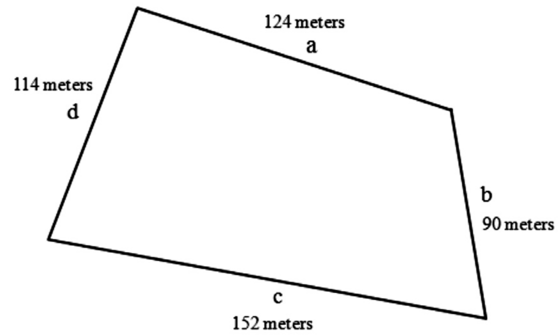

Figure 2. Adão's frame.

> I did the following here, right: I added the two bases and divided the sum by 2. I found 138. So, the base is 138 here and 138 there, understood? So, I have here the two heights, 114 plus 90. I found 204 and divided it by 2, 102, right? So, now we just need to multiply the base times height, Ok? I think the answer is 14076 square meters, right? This is the area that he *mowed.*

It is important to state that, during his narrative, Adão used expressions such as:

a) *Walls* (paredes) that mean the landmarks of the land.
b) *Frame* (quadro) that means the area of a land with a quadrilateral shape.
c) *To mow* (carpir) means to clean or to prepare the land for planting.
d) *Fourth* (quarta) that means an area measurement used in the Brazilian rural context that is equivalent to a quarter of a Paulista bushel that is used in the state of São Paulo, Brazil, which measures 24200 square meters.
e) *Pass the rope* (passar a corda) means to measure the land by using a rope.

These terms are the vocabularies (jargons) used by the members of this distinct cultural group to describe the procedures of the development of their local mathematical practices.

Table 1 shows Adão's method of estimating an area of a land with irregular shape.

This emic mathematical knowledge can be represented by an etic ethnomodel that transforms the shape of the given land into a rectangle of 138 meters x 102 meters with an area of 14076 square meters. It encompasses the following etic ethnomodel procedures:

a) transform the shape of the irregular quadrilateral in a rectangle whose area can be determined through the application of the area formula,
b) determine the dimensions of the rectangle by calculating the average of the two opposite sides of the irregular quadrilateral, and
c) determine the area of the rectangle by applying the formula: $A = b \times h$.

It is important, indeed relevant here to state that there is historical evidence that the method of *cubação* in which a quadrilateral is transformed into a rectangle was used with the purpose of land taxation in Ptolemaic and Roman periods, as well in ancient Egypt (Peet, 1970). This method is also used in Chile and Nepal, and in the Brazilian states of Bahia, Pernambuco, Rio Grande do Norte, Rio Grande do Sul, São Paulo e Sergipe (Silva, 2012).

The second approach is called *Jorge's Method* that is related to *how to square the land,* which means to transform the initial quadrilateral into a square with the same perimeter. Table 2 shows Jorge's method of estimating an area of a land with irregular shape.

To measure land shaped like Adão's *frame* (see Figure 2) Jorge explained that "Since the land has four different sides *[irregular shape],* I added all four sides: 90, 124, 114, and 152 and the result is 480. Now, I divide this result by 4, which gives 120. Then, I multiply 120 by 120, which gives me 14400". Thus, the quadrilateral is transformed into a square whose side is the fourth part of the perimeter of the original polygon.

Thus, their emic mathematical knowledge can be represented by the following etic ethnomodel that transforms the shape of the given land into a square of 120 meters each side:

a) transform the shape of the irregular quadrilateral in a rectangle whose area can be easily determined through the application of the area formula,

b) determine the dimensions of the rectangle by calculating the average of the two opposite sides of the irregular quadrilateral, and

c) determine the area of the square by applying the formula $A = a \times a = a^2$.

Table 1. Adão's method of estimating an area of a land with irregular shape (from Knijnik, 1993, p. 24).

Adão's explanation (Emic knowledge)	**Academic explanation (Etic knowledge)**
This is a piece of land with four walls	This is a convex quadrilateral
First, we add two of the opposite walls and divide them by two	First, we find the average of two opposite sides
Second, we add the other two opposite sides and also divide them by two	Second, we find the average of the other two opposite sides
Third, we multiply the first obtained number by the second one	Third, we determine the product of the two average numbers previously determined
That is the *cubação* of the land	This is the area of the rectangle whose sides are the average of the two pairs of opposite sides of the convex quadrilateral

In this context, Knijnik (1996) has affirmed that the methods used by Adão and Jorge are mathematical practices that rural workers in southern Brazil employ in order to transform irregular figures into regular ones. For example, in the *Adão's Method,* there is a reduction of the area of the land in a rectangular shape while in the *Jorge's Method* this area is reduced into a square shape.

Nevertheless, it is important to state that the method applied by Jorge shows that there is an increase in area in relation to the method used by Adão because among all the quadrilaterals with the same perimeter, the square has the largest area. However, when it comes to determine the area of any quadrilateral, the results of squaring the land are superior to those that effectively correspond to the original surface.

Knijnik (1996) states that by considering the application of these techniques, the members of this movement disregard any kind of internal angulation between two consecutive sides of the land by using right angles during the conversion process. These two methods are procedures that rural workers in this distinct cultural group employ in order to transform figures with irregular shapes that represent their land into squares and rectangles, which are well known geometric figures.

Knijnik (1996) argues that the choice of the quadrilateral geometric shape by these members is due to the fact that it is the one that is similar to the configurations of the agricultural areas in southern Brazil.

Cubação with other shapes

It is important to state that the *frames* are not always quadrilaterals. They can also be triangular or curved. It is interesting to show how to determine the area of land that has a triangular shape (see Figure 3). First, the members of this movement

Table 2. Jorge's method of estimating an area of a land with irregular shape (from Knijnik, 1993, p. 24).

Jorge's explanation (Emic knowledge)	**Academic explanation (Etic knowledge)**
Here is a piece of land with four walls	This is a convex quadrilateral
First, we add all the walls	First, we determine the perimeter of this convex quadrilateral
Second, we divide the sum by four	Second, we divide the perimeter by four
Third, we multiply the obtained number by itself	Third, we determine the area of the square whose side is given by diving its perimeter by four
This is the *cubação* of this land	This is the area of the square obtained from the perimeter of the convex quadrilateral

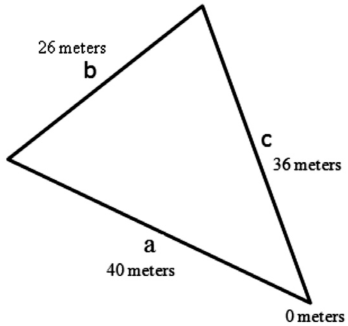

Figure 3. A triangular piece of land.

start from one of the bases of the triangle and then they place a zero in its opposite vertex. In order to calculate the area of this triangle, they apply rules that constitute the methods of Adão or Jorge. For example, Knijnik (1996) affirmed that this method is a particular case of a rectangle in which one of the sides of the triangle measures zero.

Using an etic model, it is possible to determine the area of a land with a triangular shape by applying the formula:

$$Area = \left(\frac{a+c}{2}\right) \times \left(\frac{b+0}{2}\right)$$

The results of the study conducted by Silva (2012) show that the *cubação* is also used to determine the area of land with rounded shapes. For example, one of the participants in his study, a worker in a pineapple plantation from a rural settlement in the interior of the state of Bahia, in Brazil, explained how to calculate the area of rounded land:

> I calculated the area in this way because it does not have two sides. See, there are only two headboards here *[sides]*. So, I had to add the sum of these two sides and then divide it by four. I add everything and divide by four and then I multiply two sides.

The area of the region shown in Figure 4 is obtained by applying an etic ethnomodel:

$$Area = \frac{A+B}{4}$$

Even for rounded figures, members of distinct cultural groups in Brazil use the term *frame*, which is a jargon related to land with quadrilateral shapes. In this case, they transform the rounded land into a square by squaring it. When the sum is divided by four, they obtain a figure with four equal sides (square) that is a particular case of the Jorge's Method.

The area of more complex curvilinear *frames* is found by

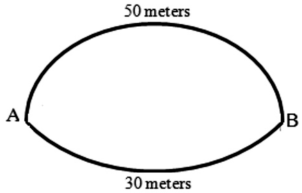

Figure 4. A piece of land with curved borders.

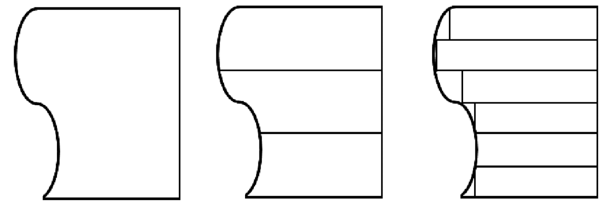

Figure 5. Curvilinear land divided approximately into quadrilaterals.

dividing its total area into roughly quadrilateral subareas, as many as necessary, in order to approximate the curvilinear *frame* and then to find the sum of their areas. This method depends exclusively on the shape of the curvilinear land. In the example in Figure 5, the land was divided in several subareas that form individual quadrilaterals in which its curved side approximates to a rectilinear shape.
After this procedure, Adão or Jorge's methods can be used to calculate each subarea of the land. Similarly, when the surface of the land is very rugged or mountainous it is subdivided with the purpose of enabling its measurement.

Some considerations about Brazilian land *cubação*

The emic approach in these examples may be considered as an attempt to discover and describe a mathematical system of this specific cultural group in its own terms by identifying its units and structural procedures whereas the etic approach is primarily concerned with characteristics pertaining to academic mathematics. This particular type of mathematical knowledge developed by MST members consists of socially learned and transmitted mathematical practices, which are represented in the elaboration of ethnomodels taken from sociocultural systems.

This process aims to translate procedures used in this mathematical practice for the understanding of those who have different cultural backgrounds, so that they can understand and explain this practice from the perspective of outsiders. This is similar to translating or communicating an idea from one language to another. In our point of view, the emic approach helps clarify intrinsic distinctions of cultural procedures while etic approaches seek to show the objectivity of external observations of these procedures.

In these examples, in the dialogic approach, the emic observation sought to understand the mathematical practice of *cubação* of land from the perspective of the internal cultural dynamics and relations of this movement with the environment in which they lived. On the other hand, an etic approach provided a cross-cultural contrast, employing comparative perspectives with the use of academic mathematical concepts.

In effect, there are two ways in which we came to recognize, represent, and make sense of mathematical phenomena that we encounter:

> 1) There are general common forms of logic and cognition that we all share, to varying degrees, with members of our own and other cultural groups. This is often attributed to a common worldview, paradigm, or cosmology. As well, it may be attributed to the common

human condition that includes cognitive models that we may elaborate at a non-conscious level, which serves to provide an internal organization of external mathematical phenomena and to provide the basis upon which mathematical ideas, procedures and practices takes place. For example, the members of all cultures count, order, measure, and classify, *etc*.

2) There are culturally constructed representations of external mathematical phenomena that also provide an internal organization for this phenomena; but where the form of the representation arises through formulating an abstract, conceptual structure that provides both form and organization for external phenomena in a manner that needs not be consistent with forms and patterns of those phenomena as external phenomena; that is, the cultural construct provides a constructed reality.

The implications for mathematical modelling of systems taken from reality are that models of a cultural construct may be considered a symbol system organized by internal logic of members of distinct cultural groups. A model built without a first-hand sense for the world being modelled should be viewed with suspicion (Eglash *et al.,* 2006).

The emic-etic dilemma in ethnomodelling investigations

Like all human beings, both educators and investigators have been enculturated to some particular cultural worldview. As Ascher and Ascher (1997) put it:

> Ethnomathematics is not a part of the history of Western mathematics although we will, of necessity, need to use Western terminology in discussing it. As Western, we are confined in what we can see and what we can express to ideas in some ways analogous to own (p. 43-44).

While emic and etic are often thought to create a conflicting dichotomy, Pike (1967) originally conceptualized them as complementary viewpoints. We argue that rather than posing a dilemma, the use of both approaches deepens our understanding of important issues about cultures. The etic approach can be a way of penetrating, discovering, and elucidating emic systems developed by the members of distinct cultural groups, which is in accordance with the work of Pike (1967). Hence, an important goal of ethnomodelling investigations is the acquisition of both emic and etic knowledge.

An emic-etic (dialogic) approach includes the recognition of other epistemologies, as well the holistic and integrated natures of mathematical knowledge found in many urban centres through ethnomodelling by combining ethnomathematics and mathematical modelling. The etic knowledge has no necessary priority over its complementing emic claims and vice versa in the dialogic approach of ethnomodelling. In this regard, cultural specificities may be better understood from the background of communality and the universality of theories and methods.

Final considerations

Currently, numerous and diverse mathematical knowledge systems are at risk of becoming extinct because of economic, social, environmental, and political changes occurring on a global scale. For example, many local mathematical procedures and practices are disappearing because of the intrusion of top-down curricula and foreign technologies, or as a result of the development of techniques and strategies that promise short-term gains or solutions to problems faced by the members of distinct cultural groups. This is done without relating them to or respecting local contexts.

Ethnomodelling supports the development of profound mathematical ideas, procedures, and practices that connect to how mathematics originated in diverse cultural contexts is used to help us to understand and share how members from distinct cultures can decolonize educational frameworks, and, most importantly, this allows us to unpack ways in which mathematical knowledge has been used across places and time.

By applying the pedagogical action of ethnomodelling through ethnomathematics and mathematical modelling, students learn how to find and work with real-life problems and daily phenomena. This context allows ethnomodelling to take into consideration processes that help students to construct and develop their own mathematical knowledge, which includes collectivity, creativity, and inventivity. In this regard, ethnomodelling opens new possibilities for the development of anthropological investigations.

References

Ascher, M. & Ascher, R. (1997) Ethnomathematics. In Powell, A.B. & Frankenstein, M. (Eds.), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education,* pp. 25-50. New York: State University of New York Press.

D'Ambrosio, U. (1999) Literacy, matheracy, and technoracy: a trivium for today. *Mathematical Thinking and Learning* **1**(2), 131-153.

Eglash R., Bennett, A., O'Donnell, C., Jennings, S. & Cintorino, M. (2006) Culturally situated designed tools: ethnocomputing from field site to classroom. *American Anthropologist* **108**(2), 347-362.

François, K., Pinxten, R. & Mesquita, M. (2013) How anthropology can contribute to mathematics education. *Revista Latinoamericana de Etnomatemática* **6**(1), 20-39.

Knijnik, G. (1993) An ethnomathematical approach in mathematical education: a matter of political power. *For the Learning of Mathematics* **13**(2), 23-25.

Knijnik, G. (1996) *Exclusão e Resistência: Educação Matemática e Legitimidade Cultural* [Exclusion and resistance: mathematics education and cultural legitimacy]. Porto Alegre, Rio Grande do Sul, Brazil: Artes Médicas, 1996.

Lett, J. (1996) Emic-etic distinctions. In Levinson, D. & Ember, M. (Eds.), *Encyclopedia of cultural anthropology,* pp. 382-383. New York: Henry Holt and Company.

Peet, T.E. (1970) *The Rhind mathematical papyrus. British Museum 10057 and 10058.* Nendeln, Liechtenstein: Kraus Reprint.

Pike, K.L. (1967) *Language in Relation to a Unified Theory of the Structure of Human Behaviour.* The Hague: Mouton.

Rosa, M, & Orey, D.C. (2010) Ethnomodelling: a pedagogical action for uncovering ethnomathematical practices. *Journal of Mathematical Modelling and Application* **1**(3), 58-67.

Silva, G.R. (2012) Cultura e etnomatemática: apresentação de algumas estratégias matemáticas dos lavradores do assentamento rural Natur de Assis (BA) [Culture and ethnomathematics: presenting some mathematical strategies of peasants in the Natur de Assis (BA) settlement]. *Anais do 4o Congresso Brasileiro de Etnomatemática – CBEm4,* pp. 1-12. Belém, Pará, Brazil: UFBA/ABEM.

Sue, D.W. & Sue, D. (2003) *Counseling the Culturally Diverse: Theory and Practice.* New York: John Wiley & Sons.