

# Physics Word Problems as Exemplars for Enculturation

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Recent ethnographic studies have begun to explore the roles of physics word problems in physics education (Hasse, 1997; Nespor, 1990, 1994; Traweek, 1988). Noting a developing conversation in this journal about word problems in mathematics education (Gerofsky, 1996; Gerofsky, Thomas, 1997), I felt the need to consider the role of word problems in physics education and propose that word problems serve as exemplars of prototype categories that define physics. These exemplar problems play a key role in enculturating students into the community of the physics literate. Word problems are central and necessary in physics education, but they are insufficient and frequently hegemonic in high school curricula. Word problems seemingly play a somewhat different role in enculturating students into the community of the mathematically literate. However, these comments may be helpful to mathematics educators grappling with the structure and presentation of these forms of pedagogical rhetoric.

## Word problems in physics education

On the surface, physics word problems are functionally isomorphic to mathematics word problems. We can see this isomorphism in the common assumptions required to understand each genre. Susan Gerofsky (1996, p. 39; slightly paraphrased) identified the following implicit assumptions within mathematics word problems:

1. that 'this' is solvable;
2. that 'X' can be found;
3. that the word problem itself contains all the information needed to do this task;
4. that no information extraneous to the problem may be sought;
5. that the task can be achieved using the mathematics that the student has access to;
6. that the problem has been provided to get the student to practice using a principle recently presented in class;
7. that there is a single correct interpretation of the problem;
8. that there is one right answer;
9. that the teacher can judge an answer to be correct or incorrect;
10. that the problem can be reduced to a mathematical form, and that the solution can be transformed back into a meaningful answer for the word problem.

The same assumptions seem to apply to physics word problems, as can be seen in their structure, presentation and

pedagogical use. These end-of-chapter questions - two from senior high school, one from freshman and one from sophomore university level - are examples drawn from standard texts.

(a) A ramp 5.00 m long is used to accelerate rolling objects. Calculate the acceleration produced by rolling an object from each of the following ramp heights. (i) 50 cm (ii) 25 cm (iii) 12.5 cm (iv) 2.5 m (Heath *et al.*, 1981, p. 72, from the fourth chapter, entitled 'The Acceleration of Gravity')

(b) A 20.0 kg toboggan is pulled along by a force of 30.0 N. What is the force of gravity on the toboggan? What is the coefficient of friction? How much force is needed to pull the toboggan if two 60.0 kg girls are sitting on it? (Heath *et al.*, 1981, p. 92, from the fifth chapter entitled 'Forces')

(c) A block slides down a smooth plane having an inclination of  $\theta = 15^\circ$ . If the block starts from rest at the top and the length of the incline is 2 m, find (i) the acceleration of the block and (ii) its speed when it reaches the bottom of the incline. (Serway, 1986, p. 101, from the chapter entitled 'The Laws of Motion')

(d) A particle of mass  $m$  slides down an inclined plane under the influence of gravity. If the motion is resisted by a force  $f = kmv^2$ , show that the time required to move a distance  $d$  after starting from rest is  $t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg\sin\theta}}$ .

(Marion, 1965, p. 92, from the chapter entitled 'Fundamentals of Newtonian Mechanics')

These questions are fairly typical of physics word problems in many respects. The questions are provided for students to practice the concepts from the preceding chapter. There is a single accepted means of answering the questions, and one acceptable solution for each. Students can assume the questions are solvable; indeed, the 'answers' are often given at the end of the book or in the chapter or even in the problem itself. Students also learn that their instructor is more concerned with appropriate explication than right answers. Finally, students must assume that their 'everyday' knowledge of physical phenomena is extraneous.

As students work through the questions within a single course, they are expected to use increasingly sophisticated techniques, as we see from questions (a) and (b). In (a), the student is expected to ignore the possibility of friction, but in (b), the student is expected to account for friction. As students move on to more advanced courses, they revisit similar

questions time and again, as we see in examples (c) and (d). For question (a), students are expected to use the equation provided in the text,  $a = g \frac{h}{d}$  (Heath *et al.*, p. 64), which is adequate for small angles of inclination, as students at this level are not expected to be able to use trigonometry. However, in question (c), students are expected to use trigonometry. Also, whereas question (b) introduces friction as a constant vector independent of motion, question (d) presents a frictional force as proportional to the square of velocity, requiring the use of differential equations. As one of Nespor's (1994) informants discussing his experience of revisiting the same phenomena in his progression of physics courses remarked:

By the time you've gotten into classical dynamics or classical electrodynamics the math is so powerful - it's just amazing to be able to solve these problems that you had to slave over in earlier courses just in one line. [...] In graduate school I'll take exactly the same thing [e.g. mechanics], except at a higher level of mathematics. (p. 75)

These examples demonstrate that physics word problems require all the assumptions Gerofsky identifies. There are some trivial and some important differences, however. In slight variance to number 3 in Gerofsky's list, a physics educator expects students to know certain constants such as the rate of gravitational acceleration near the Earth's surface. In this way, the questions implicitly refer students to a question's physicality. The solutions are constrained, and not entirely arbitrary as with some of Gerofsky's examples of mathematics word problems. In contrast to numbers 5 and 6 from Gerofsky's list, physics educators are not principally concerned with students' ability to use mathematical methods *per se*. Rather, physics teachers are concerned to enable students to practise the use of physics equations or sets of equations that embody physical phenomena (I write more on this notion of embodiment below).

While physics teachers must frequently review or introduce mathematical methods or concepts, the central purpose is to help students understand the correct physical principles needed to solve problems. Along with number 7 from Gerofsky's list, the physics educator is especially concerned that the students develop the correct *physical* interpretation of the word problem. For this reason, physics teachers spend a lot of class time teaching students to diagram word problems, especially 'free body' diagrams (Anzai, 1991). Finally, with assumption number 10 from Gerofsky's list, the emphasis is on representing a physical principle or set of physical principles in mathematical form. This requires, like mathematics word problems, the decontextualization and recontextualization of the problem.

By examining these subtle differences, we begin to see one major difference between word problems in mathematics and in physics education. Gerofsky argues quite convincingly that the 'truth value' of word problems is questionable. This is a useful means of understanding the concerns of many mathematics educators worried about a lack of connections between 'school math' (as reified by these word problems) and 'real math'. However, a categorical purpose of physics problems is to teach the

generalisability of the physical principles involved. Certainly, students often balk at point particles, massless ropes and frictionless pulleys used in physics questions, arguing that these problems are not representative of the 'real world'. This is precisely their purpose. The physics curriculum is an attempt to recapitulate the progression of sophisticated approximations that have slowly allowed physicists to make sense of physical phenomena. In doing so, physics educators hope that students will develop certain habits of thought when approaching physical phenomena.

The remainder of this article explores how physics educators try to help students develop these habits of thought: that is, how mathematical practices are enrolled in the development of physics literate students, and how the physics curriculum can be understood to be entrenched in canonical physics problems. By examining these functional questions, I believe that we can better understand the 'ontological' questions Gerofsky investigates, answering in part what these questions 'are'. I suggest that both mathematics and physics problems are part of a broader genre of 'pedagogical rhetoric' which enculturates students into ways of knowing. To demonstrate this, I use expert-novice studies indicating what 'successful' physics education accomplishes. I then use Polanyi's notion of *tacit knowledge* to explore how successful students embody physics knowledge through solving physics word problems. Polanyi's notion of tacit knowledge helps us to understand better how students become members of a culture. I then examine more closely aspects of the culture into which students are being enculturated, and how these map onto aspects of physics word problems.

### Expertise in solving physics problems

Gerofsky (1996) has observed that certain word problems are hundreds of years old and that very similar problems can be found in most contemporary mathematics textbooks. Like mathematics, a standard canon of problems can be found in most contemporary physics texts (AIP, 1988a, 1988b). Nespor (1994) demonstrates that:

[the] standardization of curriculum is accomplished through the use of textbooks that are highly standardized and demodalised in form and content. In this sense, textbooks define the undergraduate physics curriculum. (p. 55)

The same is true for high school physics, and a major means of standardization is the use of canonical questions. In commenting on the role of paradigm formation and perpetuation, Kuhn (1970) commented on the role of the canon of physics problems. He argued that these problems constitute 'exemplars' that define 'prototype' categories of solvable physical phenomena. The physics literate, the 'experts', demonstrate tacit knowledge of these categories and use them in solving problems.

Like similar studies in mathematics, expert-novice studies of physics problem solving provide a useful means of demonstrating the development of paradigmatic understanding in physics (Chi *et al.*, 1981; Hagerty, 1991; Larkin *et al.*, 1980). An expert problem solver is considered 'expert', in this literature, when able to solve canonical and quasi-novel problems in the standard prototype categories of

problems. When novices are asked to categorize a set of physics problems, they place problems together according to their manifest similarities, for example, tending to place all inclined plane problems in one category and all problems with pulleys in another. Expert problem solvers, on the other hand, categorize problems according to the physical principle involved in their solution. Someone who is literate in physics will recognize that examples (a), (b) and (d) are all solvable using Newton's laws of motion, while example (c) is solvable using the law of conservation of energy. Further, someone who is physics literate will also recognize that examples (a), (b) and (d) require quite different mathematical methods: simple algebraic substitution, vector arithmetic and differential calculus, respectively.

How do expert physics problem solvers learn to do this? In learning physics, students work through a very standardized cannon of increasingly sophisticated problems. Having solved earlier problems helps to solve later problems. In this way, physics students develop a 'vocabulary' of problems. Physics curricula are structured so that each solved problem expands this vocabulary. In time, these exemplars form poorly definable categories of types of problems that can be called *prototype* categories (Rosch, 1973, 1978). I say 'poorly definable', because the very process of recognizing the category 'problem' is in large part being able to solve the problem. That is, correctly categorizing a 'problem' determines its solution - except for solving the equation or set of equations. Once the category of a problem is correctly recognized, it becomes an 'exercise'. Somewhat novel questions are solvable by analogy to and comparison with known problem types.

Doing problems is learning consequential things about nature. In the absence of such exemplars, the laws and theories he [the student] has previously learned would be of little empirical impact. [...] The student discovers, with or without the assistance of the instructor, a way to see his problem as like a problem he has already encountered. Having seen the resemblance, grasped the analogy between the two or more distinct problems, he can interrelate symbols and attach them to nature in the ways that have proved effective before. The law-sketch, say  $f = ma$ , has functioned as a tool, informing the student what similarities to look for, signaling a *gestalt* in which the situation is to be seen. The resultant ability to see a variety of situations as like each other, as subjects for  $f = ma$  or some other symbolic generalization is, I think, the main thing a student acquires by doing exemplary problems, whether with a paper and pencil or in a well-designed laboratory. (Kuhn, 1970, pp. 188-189)

While purely cognitive explanations, such as Kuhn's and most expert-novice studies, are inadequate to explain the robust nature of canonical physics problems, they do provide a view into the roles that word problems play in physics education. To understand their roles more completely, we must examine both the embodied and social aspects of becoming physics literate.

### Solving physics problems as tacit integration

It is worth dwelling on the relationship between the physical principles to be learned and the exemplar questions used to teach them. While expert problem solvers are likely able to verbalize the principles of physics that they use in solving problems, they do not use these principles *per se* in solving problems. Rather, they transfer solutions from exemplar problems (Bassok and Holyoak, 1996), and the principles of physics are embodied in these exemplars. By 'working the problems', students in turn embody the physics principles. Polanyi's (1961) notion of *tacit knowledge* helps to understand this process.

For Polanyi (a physical chemist by training), consciousness is divided between the objects of focal and subsidiary knowledge, the latter making the former meaningful. More specifically, he writes of *focal* and *subsidiary* awareness. Subsidiary awareness should not be confused with subconscious or preconscious awareness.

Focal and subsidiary awareness are definitely not *two degrees of attention* but *two kinds of attention* given to the same particulars. (1961, p. 462, italics in original).

For example, as you read this passage you are likely focally aware of the meaning of my words and subsidiarily aware of the font used to print them, the paper they are printed on and the place where you are reading. Similarly, as a senior high school student works example (a), she will likely need quite consciously to check which formulas to use and that the algebra is correct. Later, as a freshman, this student doing example (c) might tacitly understand how to diagram the problem, the trigonometry and the equations for calculating conservation of energy problems. Her focal attention will likely shift to determining which physical principles to use. Polanyi claims that one cannot be both focally and subsidiarily aware of an object at the same time. However, one can shift one's attention from focal to subsidiary awareness, and back again.

When we are relying on our awareness of something (A) for attention to something else (B), we are but subsidiarily aware of A. The thing B, to which we are thus focally attending, is then the meaning of A. The focal object B is always identifiable, while things like A, of which we are subsidiarily aware, may be unidentifiable. The two kinds of awareness are mutually exclusive; when we switch our attention to something of which we have hitherto been subsidiarily aware, it loses its previous meaning. (1964, preface)

Polanyi also distinguishes kinds of human activity. All human activity lies somewhere between the poles of bodily activity and conceptual activity. Purely bodily activity is essentially non-verbal, while purely conceptual activity is essentially non-bodily. Most activity lies between these poles. Again, to take the example of you reading this passage, your eyes are tracing the page (a learned physical skill), and perhaps you are highlighting or making marginal notes to help you to understand the conceptual content of these words. This interplay of activity underlies Polanyi's

notion that human judgement is never purely conceptual, but relies on both conceptual and bodily knowledge. While we may tend to think of physics as purely cognitive activity, the body is involved not least in the physical act of solving the problems, but also in developing a 'physical intuition'. Physics laboratories play a crucial role in physics education by decontextualizing physical phenomena, enabling students to 'get a feel' for physical phenomena. Albert Einstein claimed that he came to his understanding of relativity through embodied visualization, and Richard Feynman was frequently discovered rolling about in his office, trying to 'get a feel' for a phenomenon (Mehra, 1994). Manipulatives seem to play a similar role in mathematics education, enabling students to embody mathematical concepts better.

Understanding this division of awareness and activity helps us to understand Polanyi's theory of cognition. Human knowledge can either be *explicit* or *tacit*. Explicit knowledge is that which can be articulated conceptually or proportionally. For example, any physicist worth her salt can articulate the law of conservation of energy. Tacit knowledge, on the other hand, involves knowledge that we cannot fully articulate. For Polanyi, 'we always know more than we can tell'. For this reason, the same physicist will likely be unable to explain *how* it is she knows that question (c), part (ii) above is most easily solved using the law of conservation of energy. So the explicit knowledge of a skill is neither sufficient nor necessary for tacit knowledge; one need not be able to explain how to do something to be able to do it. Tacit knowledge is:

indeterminate in the sense that its content cannot be explicitly stated. (1969, p. 141)

Cognition constitutes the continua between the two dimensions of awareness and activity. Tacit knowing lies in the domain of subsidiary awareness of bodily knowledge, while explicit knowledge results from focal awareness of conceptual knowledge. Since it is the subsidiary awareness that makes the focal awareness meaningful, tacit knowledge is prior to and more fundamental than explicit knowledge. It is not meaningful, in Polanyi's sense, to say 'I know' something explicitly unless there is tacit knowledge providing an undergirding. And while much of our tacit knowledge can be made explicit through reflection on experience and concepts, not all tacit knowledge can be made explicit. Polanyi explains it this way:

Things of which we are focally aware can be explicitly identified; but no knowledge can be made wholly explicit. For one thing, the meaning of language, when in use, lies in its tacit component; for another, to use language involves action for our body of which we have only a subsidiary awareness. Hence, tacit knowing is more fundamental than explicit knowing; we can know more than we can tell and we can tell nothing without relying on our awareness of things we may not be able to tell. (1964, preface)

Tacit learning arises from what Polanyi calls 'indwelling', the active engagement of the body with the factors comprising our subsidiary awareness. For this reason, one must

empathetically participate in whatever is being learned. The inarticulate rules required to gain a tacit skill within a domain such as physics often requires that one submit to an authority.

The hidden rules can be assimilated only by a person who surrenders himself to that extent uncritically to the imitation of another. (1964, p. 53)

The 'successful' student of physics learns to understand the hidden rules of physical phenomena by engaging in the processes of solving the canon of physics problems, learning to perceive and behave like a physicist, covering the intellectual ground of the physicists who came before (albeit in the reconstructed logic of textbooks). The learner gains knowledge of her environment (the domain of physics problems) by actively engaging in it, first by mimicking an expert and then by slowly gaining competence within that domain. She indwells aspects of her environment, especially canonical problems (but also the decontextualised physical phenomena of standard labs), as extensions of her body such that she is eventually able to shift her focal attention to increasingly novel and sophisticated phenomena.

### Physics problems as a tool of enculturation

As a student indwells the various aspects involved in a given skill, she or he goes through a series of 'integrative acts'. The learner integrates piece-wise, shifting focal awareness from one facet to another until he or she grasps the full meaning of the task. One might not be able to say exactly what has been learned or how it was learned, but one recognizes that it has been learned. For this reason, practice is crucial to being able to say meaningfully that one 'knows'. This practice is provided by the problems. The frequently inexplicable nature of tacit knowledge leads to an idolatry of intuition. This focus on intuition represents some people as 'born problem solvers', ignoring the experiences that have shaped who they are, and the skills and knowledge they have developed as a result.

Rather than relying on notions of intuitive ability, we need to see that 'successful' students have more successfully enculturated themselves into the community of the physics literate. Just as I have learned the dominant entrenched values, norms and ideologies of Canadian culture by unconsciously engaging in it, 'successful' physics students unconsciously learn physics' norms, values and ideologies by engaging in the recurrent actions involved in solving canonical physics problems. These ways of knowing and understanding that are valued by a community become tacit. To understand this more concretely, we need to be more specific about what is being subsidiarily embodied. Denning and Dargan's (1996, pp. 116-117) articulation of the 'domain of action' is a useful means of seeing what a student learns by engaging with the standard canon of physics problems. This 'domain of action' includes:

(a) the *linguistic distinctions* people in physics use to organize their action (for example, distinguishing mass from weight, speed from velocity, and energy from momentum);

(b) the speech acts that the physics literate use to declare and report the state of affairs, initiate action, signify completion and co-ordinate with other people (for example, in example (a) a student will learn that by stating the length of the ramp as '5.00 m', the text intends the student to give an answer to two decimal places; similarly, by using the phrase 'smooth plane' in question (c), a student will understand that friction can be ignored);

(c) the standard practices (recurrent actions, organizational processes, roles, standards of assessment) performed by the physics literate (for example, learning what would constitute an appropriate solution for each of the four example questions);

(d) a set of ready-to-hand tools and equipment [1] that people use to perform actions (for example, the particular textbooks assume that students have trigonometry and differential calculus ready-to-hand to solve examples (c) and (d), respectively);

(e) the breakdowns, which are interruptions to standard practice and progress caused by the limits of standard practices or tools (for example, students learn the limits of problems solvable with standard concepts and mathematical tools);

(f) the set of on-going concerns of the physics literate – common missions, fears and interests (for example, students learn the aesthetic in physics that seeks elegant solutions and has a fetish for increasingly 'fundamental' theories).

These aspects provide a conceptual framework for interpreting the world in terms of recurrent actions within the domain of physics students. Many of the linguistic distinctions, speech acts, standard practices, ready-to-hand tools, breakdowns and on-going concerns of the physics literate are embodied in these canonical problems. Becoming a member of a community means that one has indwelt the tacit knowledge of how to *be* a member of that community. In this way, a 'successful' student progresses from senior secondary school through an undergraduate degree in physics or an affiliated discipline. The canon of physics problems that defines the curriculum of physics programs throughout the world serves as a common vocabulary in the culture of practicing physicists and members of allied professions that uses physics. This is a process of enculturation into the community of the physics literate.

### Physics problems as rhetorical agents

I speak of 'literacy' in physics quite purposefully. Using the preceding articulation of physics problems and math word problems I argue that they belong to a category of pedagogical rhetoric that seeks to engage students in solving problems. By solving successions of problems such as the examples above, students learn ways of knowing and understanding that are valued within a particular discourse community. These problems share the assumptions that

Gerofsky indicated were particular to mathematical word problems. These assumptions indicate their social function. This category of pedagogical rhetoric also includes:

- koans of Zen Buddhist practice;
- biblical and secular parables;
- Socratic questions;
- Greek moral plays;
- dilemmas of contemporary ethics;
- problems of Talmudic study;
- citizenship examinations;
- business school case studies;
- 'cookbook' labs in science education.

Common features of these problems include uncertain literal truth value, an often implicit problem to be solved, an implied solution to that problem that only initiates are able to access, an implied injunction preventing the learner from bringing additional information to the solution of the problem. These features define the function and explain the utility of these problems: that is, all of these problems enable students to develop a 'privileged' way of looking at phenomena. It is 'privileged' in the sense that the ways of knowing and understanding embodied in these questions are beyond the scope of plain language and frequently require intensive study to make the knowledge and understand tacit. But also 'privileged' in the sense that initiates may gain special status within a community.

This confusion of form and function is evident when Gerofsky discusses Pimm's (1995) suggestion that word problems are like parables. Gerofsky dismisses the possibility because, she argues, word problems do not engage people the way parables do. She misses the observation that parables have force because everyone is inculcated into 'ongoing (moral) concerns' from an early age. Mathematics word problems only gain emotive force when a learner adopts the ongoing concerns of the mathematical community. By looking just at single instances of word problems out of context of the canon of word problems, Gerofsky missed what might be viewed as a larger rhetorical form. Just as it is difficult to understand a Talmudic question outside its pedagogical function, so word problems seem odd outside the broader context of learning to understand mathematically. Physics problems, like math word problems, are idealized representations, but taken together they bring the students from grossly decontextualized, idealized representations to rather useful ways of seeing. Trying to understand them in comparison with other *literary* genres leads her to miss their place in *pedagogical* canon.

By successfully 'mobilizing' (Nespor, 1994) problems in the genre of pedagogical rhetoric, students learn exemplars through which they will interpret future experience within a particular domain of action. Embodied in these exemplars are the linguistic distinctions, speech acts, standard practices, tools, breakdowns, and on-going concerns of a domain. At first, these aspects of the domain will be focal and conceptual, but gradually they will become subsidiary and more bodily. In other words, the students come to indwell the culture of the domain.

From his ethnographic study of undergraduate physics, Nespors (1994) also argued that:

Mathematical formalization became the necessary bridge from the pre-physics notions of physicality that the students brought with them to the program to more 'abstract' disciplinary notions of 'concrete' reality. (p. 76)

More specifically, I have argued that it is canonical problems that embody physics knowledge, and that being literate in physics means having access to the standardized body of knowledge that these canonical problems manifest.

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### Note

[1] A tool is *ready-to-hand* if the person using it does so with skill and without conscious thought. To use Polanyi's language, we could say one has tacit knowledge of a ready-to-hand tool.

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