OBSERVING HOW A STUDENT BECOMES COHERENTLY AWARE IN MATHEMATICS

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Being aware of 'something' is generally used in different contexts: "I am aware of the colour of the traffic light", "I am aware of being punctual at my meeting", "I am aware of an event that happens in a classroom when I am teaching". Being aware of 'something' not only occurs for common activities, it also occurs for actions that involve studying a subject such as mathematics. Gattegno (1987) states "only awareness is educable" (p. 220), which points our attention to inculcating mathematical awareness in students. Mason (1998) proposes that the mathematics teacher should promote the development of a complex mechanism of awareness education through work on attention. These approaches, and others, share common characteristics of noting the role of the teacher in encouraging the students to be aware when doing mathematics; however, from the point of view of observation, as a research method, it is less clear how students become mathematically aware through a series of their own interactions.

By considering conversations about mathematics between students during a lesson within a classroom, this methodological study shows how a student becomes coherently aware. I present my specifications of my observations of three students while they performed a mathematical modelling task for the first time, discussing my position as an observer of the mathematical actions associated with the conversation in the classroom.

Enactivist perspective

From an enactivist perspective, during the course of interactions with others, cognition and therefore knowing occurs "on the basis of a history of the variety of actions that a being [for example a student] in the world performs" (Varela, Thompson & Rosch, 1991, p. 9). This variety in the history of actions between the participants brings about a set of possibilities in which the students are constantly involved in the decisions taken when they act; for example, a student can pose a question, provide an answer, describe a mathematical situation, write, and so forth.

To note the variety of actions in the history of actions between the participants, an observer forms a specification of the actions of the others according to the observer's observations. These specifications are configured by the observer, who is also part of the interactions that are happening by observing (or through other actions *i.e.,* writing notes), but they are also formed on the basis of the actions of the observed, in this study, the students. The observer and the students are mutually part of, without separation between them, an emerging history of interactions between the observer and the students observed. This enactivist concept is known as "structural coupling" (Maturana & Varela, 1992, p. 75).

Enactivism as a methodology

Reid (1996), noting that research is learning (p. 203) examines "enactivism, as a methodology, a theory for learning about learning" (p. 205). Enactivism, as a methodology, can work as a first-person approach and is a way for me to understand my learning as a researcher. Experiencing the mathematics classroom and its students, without division between the observer and the observed, offers the opportunity to perform an action, namely seeing, and thereby learn what is occurring.

The methodology works through multiple perspectives, which could be through the participation of multiple researchers, multiple revisitations of data, and the act of communicating our research to others (p. 207). In this process, I use multiple revisitations of data, one of which was through making an account of these observations, trying to demonstrate my own process of learning, as mathematical awareness emerges through the interaction of the students. I also communicated this research in five different instances to scholars and teachers, allowing me to observe and see multiple perspectives from the attenders, and also to note the growth of my understanding of becoming aware. I observed that at the beginning my attention was on observing the shift of actions of the students, while later it was about how a student becomes aware in their own shift of actions.

The data of this study was collected from observations during 2.5 months in a Chilean classroom with 23 8th-grade (13–14 years old) students and a mathematics teacher, as well as interviews with the participants. The observations of the students are based on an audio-video recording and in particular the students' conversations, which occurred in the context of 90-minute classroom observations.

Observing in the classroom: some specifications from my observations

Bearing in mind the enactivist perspective on the history of interaction, I will introduce in the next section some specifications from my own learning in my history of interactions that come from my observations during this study.

Observing a micro-historicity

Students and their experiences are part of a mathematical world emerging from them in which interactions are occurring. Ways of acting by the students can be observed, including how a student becomes aware. The emerging mathematical world happens because the structural coupling between the students and their surroundings creates a world of possibilities that produce changes in each student. Sometimes through these changes the students can come to share ways of behaving in the history of the variety of actions.

In the interactions of the participants, "knowledge is about situatedness; and that uniqueness of knowledge, its historicity and context" (Varela, 1999, p. 7). This is related to each interaction with the world, naturally including the students and the observer. In this history of interactions, I, as an observer, can note mathematical changes by the students, considering "the changes that result from the interaction between the living being and its environment are brought about by the disturbing agent [*e.g.,* a teacher] but *determined by the structure of the disturbed system* [*e.g.,* a student]" (Maturana & Varela, 1992, p. 96). One of these changes can be demonstrated through the decisions taken by the student at the moment of action.

Varela explains the word 'decision' in this way:

> C'est tout simplement le fait qu'on voit une espèce de multitude de possibilités qui émergent dans un moment donné et puis finalement, l'organisme [*e.g.,* a student] décide d'aller dans cette direction-là ou dans cette direction-là. On peut peut-être remplacer le mot 'décision' tout simplement par accomplissement d'un processus de choix […] On pourrait aussi remplacer le mot 'décision' par le mot 'sélectivité' [1].

Following my first specification of observations and identifying a moment of change in the transcript, I noted that there is a trajectory of each student, a history which is emerging through the interactions with the other students. I name this 'micro-historicity' due to the micro-moments that I observed by taking account of the decisions made in the variety the actions where the interaction is taking place. By micro-historicity, I mean a set of interactions that can be observed in a history of interactions, where the pathway of each student through the change of the actions (based on Varela, 1999) can be exposed to the observer through a transcript. This micro-historicity approach differs from what are known as micro-genetic methods, which have been applied by cognitive developmental researchers in an experimental context (Luwel, 2012). Although both approaches include detailed observations of the participants, a major difference between them is the experimental context.

Observing coherent doing in mathematics

In a classroom setting with a teacher, students and an observer, we can recognise a situation in which the seeing in the same classroom varies among the students; for example, if the students receive the instruction 'solve this equation', the observer may see how a student finds the unknown value in an equation or perhaps how the same student adds similar terms associated with the unknown value. In this example, there is a distinction made by the observer whether the students' actions are 'according to', 'coherent' or 'in tune' with the mathematical circumstances in which the students are operating. In this example there is a coherent behaviour 'expected' when solving the equation, related to adding similar terms or finding the unknown value within the context of a mathematics classroom. I use the phrase 'coherent behaviour' according to the description of Maturana, who states, "The life of a living system appears to us observers as coherent with its circumstances" (2000, p. 460). There is a coherent behaviour expected from the students according to what the observer is observing as coherent with the circumstances while solving an equation. Furthermore, this coherent behaviour depends on what the observer notes.

Naturally, what is expected by the observer is determined by the observer's own mathematics history of interactions around the doing of others. There is always a history of interaction that is emerging at the moment of observation, generating the observer's learning about what is happening. In other words, if somebody is solving an equation, I, as the observer, have 'expectations' about what actions that person will be perform according to my own learning about solving an equation. My history with solving equations allows me to observe if the actions performed by another are "coherent with its circumstances".

Distinctions in the coherent behaviour observed in mathematics

The distinctions that I present from my observations of the students' awareness of their coherent behaviour is derived from my constant interaction with them, by looking for changes in the students' conversation about mathematics through the decisions taken by the students, at the same time observing how they are still operating coherently in the observed micro-historicity. In this article, I show how I observe the coherent behaviour of the students, but at the same time I acknowledge that I am observing on the basis of my own learning of the learning of the students. As such, I was constantly adapting and learning throughout my observations.

In a similar way, in the eyes of another observer, I am also living my own coherent behaviour within the observations of the students observed. At this time, I will not elaborate more about this recursive process of the observation of my own coherence of the observations of others.

Observing a coherent mathematical doing in the transcript through micro-historicity

In this section, I present a transcript which I will then analyse in the rest of the article (performing an observation of the observations). The conversation of the three students, working in a group, took place in a lesson when the students were solving the following budgeting task to create a carnival game:

> A game manufacturer would like your team to create carnival game with 5 water bottles and 5 beanbags. Water bottles cost $1.00 each and beanbags cost $1.25 each. The game will be played at a carnival. The company expects 175 children to play. Small prizes cost $0.50 each, medium prizes cost $1.00 each and large prizes cost $3.25 each. The company plans to charge $250 for the game (including the prizes) and they want $100 profit, so you have $150 to spend on each game and the prizes. Plan how to spend your budget and use mathematics to show that your plan for the game will work for 175 children. [2]

This transcript is written in a linear way and this may affect the way of seeing and reading (see Towers & Martin, 2015); however, it is important to note the actions observed (movements and speaking) are happening in a real classroom situation (not an experimental or control group setting), including temporality. Therefore, the actions or behaviour of the students are never linear.

The transcript is translated from Spanish, and the names have been changed. Pauses lasting more than one second are marked, and italicised speech indicates emphasis in the speaker's voice.

1 *Sally* — We must have 175 *[children playing]*; if not I won't play *[she will not create the carnival game].*

2 *Hanna* — Okay, *[pause]* but it cannot be recurring *[decimal number].*

3 *Sophia* — Yeah, but if *[drawing out the word]*

4 *Hanna* — She has a recurring *[decimal]* number; this doesn't work like that. We have a better chance of making *[the game]* without a recurring *[decimal]* number.

5 *Sophia* — Shh. Do you remember when teacher told us what happens if? Shh.

6 *Sally* — But I have to buy 58 *[prizes]* because 58 plus 58 plus 58 is 174.

7 *Hanna* — And where is the number three? And where is the recurring *[decimal]* number?

8 *Sophia* — Shh. What happens if it is one hundred and seventy *[pause] listen to me, listen.*

9 *Sally* — That is 175 exactly. But I have 174, which is not exact. So, it is not 175 *[snapping her fingers].*

10 *Hanna* — So *[pause]* we cannot do it with a recurring *[decimal]* number.

11 *Sally* — But not because with 58.333 *[multiplied by 3]* you have 175 *[children],* but 58 times 3 *[pause]*

12 *Hanna* — 58 times 3 *[pause]*

13 *Sally* — is 174, and it is not a recurring *[decimal]* number.

14 *Sophia* — Look, what happens if all the children won the big prizes? We will *exceed the* budget *a lot.* Do you see it? *[pause]*

15 *Sally* — Let's have a look.

16 *Hanna* — So, we cannot do it *[dividing]* into 3 *[groups].*

17 *Sophia* — There is a chance that all the players could win the big prizes; so *[hitting desk with hand]* I suggest, the game *[that we are creating]* should have a trick.

18 *Sally* — As the way I said it? No, I think we *[must]* divide into 3 *[groups]* that are 58 *[players]* each and we don't exceed the budget because it is 174, not 175, and if we exceed *[the budget],* we'll sell tickets *[to play the game].* It's not so complicated.

At first glance, the transcript is a series of questions and answers among the students; that is, a type of conversation has taken place among them as they solved the budgeting problem.

Later, with the transcription of my observations, I heard and read their conversations again. Multiple revisitations of the data were performed according to enactivism as a methodology. As I did so, other observations began to catch my attention with regard to the coherent behaviour of each student, according to the definition of Maturana.

As the students worked on this mathematical problem, I started to follow the micro-historicity of a history of interactions of each student, noting how each one is engaging with mathematical features of what they are doing. As the different students participated in this interaction, different pathways of coherent behaviour could be observed through the micro-historicity of each one, as illustrated in the following description of my observations about the transcript.

I observed Sally's coherent behaviour about the distribution of the groups when she was working with the grouping of the children playing, starting with 175, "We must have 175" (Line 1). I noted she made a correspondence between the equal number of prizes (small, medium and large), dividing them into three groups in Line 6. She was assuming that dividing the group into three, she will have prizes for the 175 because she has the same number of prizes for each group, thus maintaining the budgeting. After that and the constant questions by Hanna about recurring decimal numbers, Sally explained why she does not take into account in her distribution the use of the recurring number in Lines 9, 11, and 13. There is a particular moment when she expresses, "Let's have a look" (Line 15) where she is aware that there is *something* mathematically between what she has done until now and what Sophia proposes, "What happens if all the children won the big prizes? We will *exceed the* budget *a lot.* Do you see it?" (Line 14). This can be illustrated by the expression "Let's have a look", where there is a change in the manner of her behaviour and her action.

From the coherent behaviour in the micro-historicity, Sophia was trying to mention a restriction of the budget and the potential winners (Lines 3, 5 and 8). I observed that Sally was habitually demonstrating what she had done to Hanna (after her recurrent perturbations expressing that it was not possible to work with a recurring decimal number). This is demonstrated in the transcript when she said, "But I have to buy 58 because 58 plus 58 plus 58 is 174" (Line 6), "That is 175 exactly. But I have 174, which is not exact. So, it is not 175" (Line 9) and "174, and it is not a recurring number' (Line 13). I also observed that the usual habits of Sally relate to dividing the 175 participants of the game into three groups, but after Line 13 there is a moment of moving from her usual habit (working to divide the players into three groups) and moving on to something new, leading to a redirecting of the mathematical situation, of becoming aware of what happens if one exceeds the budget in the problem that is not being solved under the conditions of the mathematical problem.

In this same conversation among the students, I observed that Sophia was following what Sally was doing, but at the same time generating her own awareness about the budget as a factor of the problem, as can be seen in Lines 14 "we will exceed the budget a lot" and 17 "there is a chance that all the players could win the big prizes". If that happens, they will exceed the budget, showing in Sally's micro-historicity the coherent behaviour in solving the budgeting problem by taking into account the budget and the distributions of the winners.

I noted that, at the same time this mathematical conversation was occurring, Hanna was observing Sally, who was splitting the potential 175 children into three groups. The dividing into three groups by Sally may be happening because the budgeting in the mathematical modelling problem should consider the cost of the small prizes, medium prizes and large prizes and the cost is part of the budgeting that they need to work out.

I noted Hanna was aware of the use of a recurring decimal by Sally, as can be seen in Line 2, when she said, "But it cannot be recurring" and in Lines 4, 7 and 10. She maintains her coherent behaviour about the recurring decimal, and still operates mathematically in solving the mathematical problem about the budgeting while taking into account why not to work with the recurring decimal. Later in Line 16, she seems to find a 'support' in her awareness of the division in a group of three based on what Sophia suggests in Line 14 (about winning the big prize with only one group). However, this support is not the same awareness that she was referring to before (if you divide by three you will get a decimal recurring number in each group). This support is another way to complement her idea, completing Sophia's idea of "so, we cannot do it [dividing] into 3 groups" (Line 16).

In addition, although the structural coupling is present in the actions of the participants all the time, an example in the actions of these students can be observed from Line 14 to Line 16 where there is an engagement between them provoking similar behaviours in a history of interactions where mathematical knowing is occurring based on an enactivist perspective.

Moving towards becoming aware of something mathematically in my observations

Taking account of the changes of the actions and their decisions in the micro-historicity of each student, I observed each one's coherent behaviour. Eventually, in the structural coupling, I noted, there is *something* that becomes part of the interaction between the students, something mathematical that each one had not seen before—for example, their work with the distribution of players (as illustrated in the transcript)—and suddenly each student starts to see it, becoming aware of *something* mathematically.

I recognise there are other moments of awareness that occurred between Sally, Hanna and Sophia. For example, in the observations of the micro-historicity, Sophia seems to have had a 'silent' awareness generating about exceeding the budget, that she is trying to mention and suddenly the comments that she makes in Lines 14–17 shows her awareness to the others.

I understand that the observation of a feature of the moment of becoming aware in the micro-historicity of interactions between the students is necessarily limited by my observations as an observer of one micro-historicity of one student (here, Sally). Nevertheless, I consider the perturbations received by the others and therefore explore explicitly and in detail in my analysis her own path in the micro-historicity observed of becoming aware mathematically.

I chose Sally's micro-historicity because in the structural coupling that is happening in the actions that is demonstrated in the transcript between Sally, Hanna and Sophia, there is a significant change that I noted from the usual habits in their micro-historicity.

Presenting Sally's micro-historicity of becoming aware

In order to state clearly what I observed in Sally's micro-historicity in the interaction with her peers, her path of actions in this moment, I will restructure the interaction. On one hand I want to show Sally's micro-historicity and on the other, the way I learned to read the micro-historicity by following a history for each student.

I have marked Sally's interactions in bold to help the reader follow Sally's micro-historicity through my observations.

1 *Sally* **We must have 175 *[children playing]*; if not I won't play *[she will not create the carnival game].***

2 *Hanna* Okay, *[pause]* but it cannot be recurring *[decimal number].*

3 *Sophia* Yeah, but if *[drawing out the word]*

4 *Hanna* She has a recurring *[decimal]* number; this doesn't work like that. We have a better chance of making *[the game]* without a recurring *[decimal]* number.

5 *Sophia* Shh. Do you remember when teacher told us what happens if? Shh.

6 *Sally* **But I have to buy 58 *[prizes]* because 58 plus 58 plus 58 is 174.**

7 *Hanna* And where is the number three? And where is the recurring *[decimal]* number?

8 *Sophia* Shh. What happens if it is one hundred and seventy *[pause] listen to me, listen.*

9 *Sally* **That is 175 exactly. But I have 174, which is not exact. So, it is not 175 *[snapping her fingers].***

10 *Hanna* So *[pause]* we cannot do it with a recurring *[decimal]* number.

11 *Sally* **But not because with 58.333 *[multiplied by 3]* you have 175 *[children]*, but 58 times 3 *[pause]***

12 *Hanna* 58 times 3 *[pause]*

13 *Sally* **is 174, and it is not a recurring *[decimal]* number.**

14 *Sophia* Look, what happens if all the children won the big prizes? We will *exceed the* budget *a lot.* Do you see it? *[pause]*

15 *Sally* **Let's have a look.**

16 *Hanna* So, we cannot do it *[dividing]* into 3 *[groups].*

17 *Sophia* There is a chance that all the players could win the big prizes; so *[hitting desk with hand]* I suggest, the game *[that we are creating]* should have a trick.

18 ***Sally*** **As the way I said it? No, I think we *[must]* divide into 3 *[groups]* that are 58 *[players]* each and we don't exceed the budget because it is 174, not 175, and if we exceed *[the budget]*, we'll sell tickets *[to play the game]*. It's not so complicated.**

The moment of becoming aware for Sally, I noted, after Line 14 and before Line 16, is when she becomes aware of the budgeting and what happens if all the children win the big prizes. This is prompted by Sophia, who says "what happens if all the children won the big prizes? We will exceed the budget a lot" (Line 14). This leads to Sally's redirection, saying "let's have a look" (Line 15), thus moving away from what she was doing in mathematics previously, to see the new mathematics.

The expression "let's have a look" demonstrates how Sally is moving to something new, and I, as an observer, noted this new seeing as part of her observed micro-historicity. Her coherent behaviour through the change of actions and decisions made by her, perceiving this new awareness of what happens if the budget is exceeded after the perturbation received from Sophia (Line 14), opened the possibility of seeing other mathematical aspects in her work, such as the restriction what happens if all win the big prizes. However, in Line 18, Sally stays where the interaction started for her, dividing the players into three groups, without moving her actions in this interaction into a new action, which can be demonstrated by her expression: "No, I think we divide by 3 that is 58 each". She recognises what she has done before habitually. At the same time, I can observe that Sally notes the suggestion made by Sophia because she says, "We don't exceed the budget because it is 174", noting that *something* mathematically new is there, and this has been prompted by the question posed by Sophia. However, the way that Sally is approaching may not be the same mathematical way as Sophia suggests.

Sally's action reveals that although she has become aware, acting differently in the micro-historicity observed, she has decided (a change in her action) to retain her normal mathematical habits, for example, staying with the action of dividing the players into three groups.

Discussion

I started from the position of the observer, to observe how *a* student becomes coherently aware in mathematics, with emphasis on the 'a'. In the history of interactions of the participants there is a history that can be shown through the micro-historicity, a method that allowed me to observe in more detail the changes and decisions made by each student. I noted the richness in the details of the mathematical conversations between them, in particular how a student is becoming mathematically aware.

As exhibited in the transcript, observing being aware of *something* mathematically does not only say what the student is aware of, but rather there is series of actions that occur in the history of actions that lead to the observation of what the student is aware of. However, not all awarenesses are reachable by the observer. Some awarenesses change silently. In a 'silent' awareness, what happens if the observed students share the same awareness that never becomes visible to the observer? How can a teacher who is trying to promote mathematical awareness work with this silent awareness?

The coherent mathematical actions bring me, as an observer, to know the distinctions made by the participants, in terms of how they are operating mathematically in their own actions. However, I also recognise that there is a coherent behaviour that is attached to my own history of observation. In this case, there is a mathematical history when the observation is carried out. From here, if multiple distinctions in mathematical terms can be made in that history, then are we taking account of the coherence of the observer in the observation? If the observer is a teacher, student or researcher, how does the distinction of the observer, in their own history of interactions made in the observation, shape the method of observation? Are we aware of our own coherent behaviour in the observations? Would that matter?

Finally, if "knowing is doing" (Maturana & Varela, 1992, p. 27), how are we observing the knowing of the students? What are the considerations as an observer we are taking? Are we observing the mathematical doing in detail? Is it enough to note the mathematical conversations, gestures, and body movements to know what the students are doing?

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Notes

[1] From a 1994 interview with Varela entitled 'Ne pour creer du sens.' Online at https://www.youtube.com/watch?v=9qIWCMssyTk. The passage quoted occurs between 21:10 and 21:50. It can be translated as "It is merely the fact we are seeing a multitude of possibilities which emerge at a given moment and then, finally, the organism [*e.g.,* a student] decides to go in one direction or in another. We can simply replace the word 'decision' with achievement of a process of choice […] we can also replace the word 'decision' with the word 'selectivity'."

[2] This mathematical task is part of a collection of mathematical modelling problems from the book *Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME)* (2016), publsihed by the Consortium for Mathematics and Its Application (COMAP) and the Society for Industrial and Applied Mathematics (SIAM), p. 129.

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