BRICOLAGE IN MIDDLE YEARS SCHOOL MATHEMATICS

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> Students' conceptualizations were not perfect, but good-enough to engage, and keep moving forward in relation to the tasks at hand. Thinking about it, is this not actually happening *all the time*? How far would we go if *all* students had to *perfectly* understand everything before moving on to the next idea? Students "make do" with sort-of-right understandings and, according to Zack and Reid, in this way, they come up with the most interesting insights and ideas. (Maheux, 2016, p. 24).

Maheux wonders, "What if sort-of-right was right enough?" in school mathematics and builds on the concept of 'good-enough' mathematics described by Zack and Reid (2003). These ideas dovetail nicely with what I have observed during my study of small groups of middle school students working on mathematical tasks in their classrooms. Students at this level have limited experience with mathematics; they have been exposed to concepts but not necessarily given the time and space to fully explore and understand them. Perhaps as a result, students may demonstrate 'sort-of-right understandings' in the mathematics they use. As well, student conversation during mathematical tasks can be quite divergent. A problem about the minimum and maximum number of ways to cut up a pizza may bring up topics such as which is the best shape of pizza, what is the biggest pizza you've ever seen, and what do you feed someone who does not like pizza? As Herbel-Eisenmann and Wagner write, mathematisation "moves between the personal and impersonal, between context and abstraction. Mathematics lives in this tension" (2007, p. 13). For middle school students, who have limited experience in performing mathematics, and who do not have a large variety of algorithms at their fingertips, nor exposure to formal proofs, it is natural to bring in whatever personal and concrete experiences they may have as they engage in working with abstract concepts in order to develop their solution paths.

I argue that mathematisation is evident through how students perform 'bricolage' in doing their school mathematics, and this includes the use of sort-of-right mathematics. Originating from the French verb *bricoler*, 'bricolage' means to tinker with, or to 'do it yourself,' working with whatever you happen to have on hand. I seek to characterize bricolage in the mathematical thinking of middle school age students by presenting a case study of a small group of Grade 8 students working with what they have as they investigate a mathematical task. Group work is necessarily public, with members striving to share their understandings and ideas with each other. Observation of the collective discourse of small groups can provide an indication of how the process of bricolage proceeds. The data analysis moves between levels in order to consider both the group itself as a learning agent and the group as four members who are each learning agents. I observe bricolage as an emergent, self-structuring process through which the group defines the boundaries of its understanding of the task and then pushes these boundaries in order to explore and develop possible solutions, sometimes through the use of good-enough mathematics.

Characteristics of bricolage

The term bricolage first appears in Claude Lévi-Strauss's anthropological work *The Savage Mind* (1966), where the behaviors of the 'engineer' and the 'bricoleur' are compared. The engineer uses a scientific, abstract way of thinking to plan and prepare for an assigned task. The bricoleur's method, in comparison, is more concrete, more hands-on, making do with whatever materials happen to be at hand.

> Unlike the engineer, he does not subordinate each of them to the availability of raw materials and tools conceived and procured for the purpose of the project. His universe of instruments is closed and the rules of his game are always to make do with 'whatever is at hand,' that is to say with a set of tools and materials which is always finite and is also heterogeneous because what it contains bears no relation to the current project, or indeed to any particular project, but is the contingent result of all the occasions there have been to renew or enrich the stock or to maintain it with the remains of previous constructions or destructions. (pp. 17-18)

In a study of both grade school and college age computer programming students, Turkle and Papert define bricolage as "a style of organizing work that invites descriptions such as negotiational rather than planned in advance" (1990, p. 144). The authors compare the 'planner' (their equivalent of Lévi-Strauss's engineer) who has a 'formal' method to the 'bricoleur' who uses a 'concrete' method: "The bricoleur resembles the painter who stands back between brushstrokes, looks at the canvas, and only after this contemplation, decides what to do next. For planners, mistakes are missteps; for bricoleurs they are the essence of a navigation by mid-course corrections" (p. 136).

It is noteworthy, but perhaps not surprising, that Turkle and Papert describe the planner as having a 'mathematical' style. Traditionally the learning of mathematics has been considered to be sequential, where foundational concepts must be mastered before newer concepts can be introduced. Yet there are researchers who describe how students may perform mathematics through negotiation. Bauersfeld calls the messy "pragmatic adaptations" involved in mathematical thinking 'tinkering' and 'bricolage' (1994, p. 144).

Hershkowitz, Schwarz and Dreyfus term the process of combining familiar objects as components to resolve a problem 'building-with' (2001, p. 214), arguing that this kind of reconstruction is central to the process of abstraction. And finally, of students who work with example spaces (for instance, within a set amount of time developing as many examples as they can of what '2' is) Watson and Mason write, "In our experience, the bricolage of example construction can yield surprising results, because the knowledge and resources being brought to the task are different for different learners" (2005, p. 80).

What drives bricolage

There is a sense of motion in bricolage—thinking on one's feet, viewing situations from a variety of angles to see other possibilities, adapting, and reinventing. I believe what drives this motion is an awareness of the gaps that exist between what is and what could be, attempts to bridge these gaps (Mäkitalo, Jakobsson & Säljö, 2009), followed by the subsequent awareness of further gaps and the attempt to bridge them in turn. These gaps of understanding may be characterized as problems, and the act of trying to bridge them as an attempt to solve them. Further, student awareness of these gaps may be demonstrated through the act of problem posing. Problem posing may be defined as "the creation of questions in a mathematical context and […] the reformulation, for solution, of ill structured existing problems" (Pirie, 2002). However, problem posing is not necessarily expressed through questions (although, grammatically speaking, questions are the language device used to point to problems), but may be manifested through a statement, an aside, or perhaps even as a joke, all depending on the context and the personalities involved. I will revise Pirie's definition, then, to read "the creation of *problems* in a mathematical context and the reformulation, for solution, of ill structured existing problems."

Bricolage as a process

Although bricolage is very much a reaction to a situation in the present, it is also an act that both is anchored to the past and projects into the future. Lévi-Strauss suggests that when the bricoleur is following the "rules of his game" he is seeking to challenge himself. The heterogeneous tools and materials with which he works are not blank but are marked by "all the occasions" in which they have been previously used. Not only is original meaning sedimented in the artifact being used, but as artifacts are recombined with other ones through bricolage, the resulting connections develop into new codes of meaning (Barker, 2004). I suggest that the bricolage that occurs through problem posing works in a similar fashion. Students pose problems, and use the mathematics they are able to, in order to bridge these gaps in understanding. This mathematics is good-enough to keep moving towards a possible solution, the kind that Zack and Reid found in their own study to be "incomplete, tentative, and sometimes inconsistent. Yet at the same time the students [are] able to continue to work on the problem, to explore, and even to explain things to each other that they seemed not to fully understand themselves" (2004, p. 28). The act of bricolage can be observed through the emergence

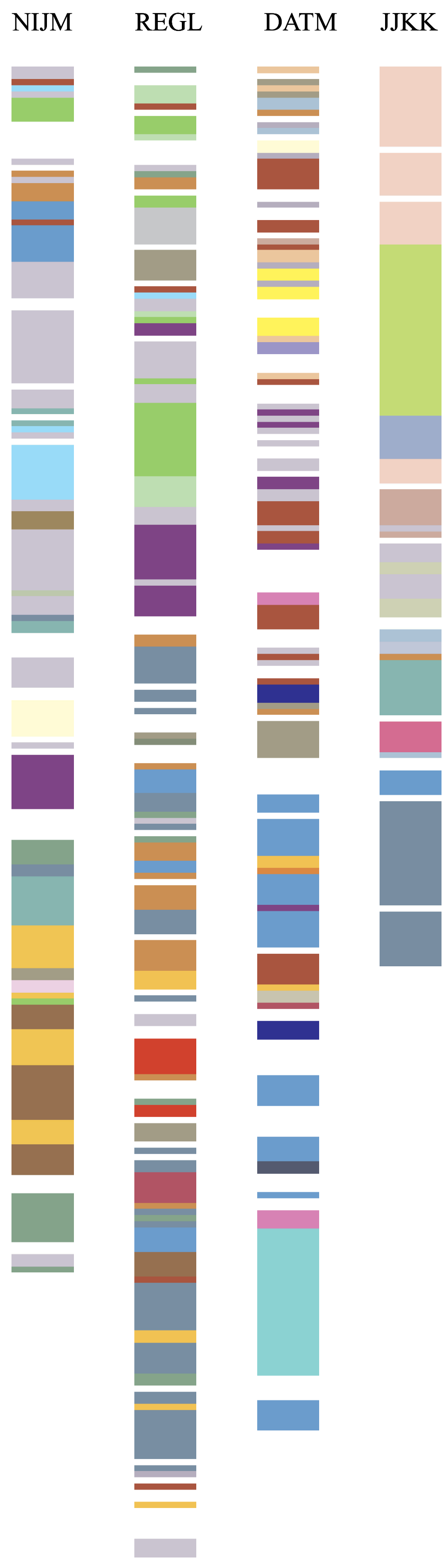

Figure 1. Tapestries for four groups engaged with the Bill Nye task

Colour	Problem posed (generalized)	JJKK	DATM	NIJM	REGL
Lavender	Do we use time and divide by five?	X	X	X	X
Medium blue	What about if everyone brings x gifts each?	X	X	X	X
Purple	Is there an extra five minutes? (because last gift is opened starting at 5:35)	X	X	X	X
Deep red	How many people are there?	X	X	X	X
Slate blue	What are the factors of x?	X		X	X
Lime green	What is meant by an interval?	X		X	X
Olive green	Do all members give to everyone?		X	X	X
Goldenrod	Do they also bring gifts for themselves?		X	X	X
Orange	Does everyone bring the same amount of gifts?		X	X	X
Sky blue	How many gifts are there?			X	X
Brown	What if there are x people?			X	X
Green	How do we think outside the box?			X	X
Teal	Is it a square root?	X		X	
Fuschia	Why did we get x?	X	X		
Dark pink	How long does it take to open all the gifts?	X	X		
Light purple	Can they take breaks in between opening gifts?	X	X		
Pale yellow	Does it start at one o'clock?		X	X	
Gray	What is a tournament?				X
Red	What if it's an exchange?				X
Light green	How long does it take to open one gift?				X
Forest green	Can't we just count how many people?				X
Lilac	How many gifts does each person bring?	X			
Coral	How many gifts are opened in an hour?	X			
Gold	Is another group's answer right?			X	
Sage	Can they bring partial gifts?			X	
Pink	What if someone doesn't get a gift?			X	
Dark blue	How do we know if we're right?		X		
Blue	What if there are x people and gifts?		X		
Peach	Does it take five minutes to open one gift or five minutes to open all the gifts that one person brings?		X		
Light blue	How can we use the 24 hour clock?		X		
Yellow	Can they open gifts at the same time?		X		

Table 1: Colour coding of problems posed, ordered by frequency

and re-emergence of problems posed by the group as a solution pathway develops.

Documenting bricolage

The larger study on which I draw considers the unique solution pathways developed by four small groups of students while engaged in non-routine mathematics tasks (Armstrong, 2013). The participants were students in Grade 8 classes at a suburban public middle school in the spring of the school year. Small groups of four were videotaped for five class sessions of 20-25 minutes in length working on 'Problem of the Day' tasks, before being drawn back together for full class discussions of their ideas and solutions led by their classroom teacher.

To consider a collective as an individual learning agent in and of itself necessitates a change in focus on the part of the researcher. As group members must make their ideas public to one another in order to be understood, the group's discourse may be considered to represent its thinking (Stahl, 2006), but how does one to define the unit of analysis used in considering this level of data? Bakhtin defined utterances as "not a conventional unit, but a real unit, clearly delimited by the change of speaking subjects" (1986, pp. 71-72), *i.e.,* individuals. Moving the focus up a level in order to consider the group as a whole rather than the individuals who make up the group, means moving from considering who is speaking to considering the ideas being spoken about. I employed the unit of the 'collective utterance'—a unit delimited by the change of posed problems—in the creation of what I call 'tapestries' (See Figure 1). After transcribing the conversation of each group, I identified the problems posed by the four groups, assigned a unique colour to each problem, and then colour-coded the transcripts according to the collective utterances discussing these particular problems. The tapestries that result from the colour-coding provide visual traces of the process of bricolage in which each of the groups engages. This coding helps to bring to the surface how the fabric of a group's mathematical conversations is woven from multiple threads, emerging and reemerging, and how the math is never fully, finally, established—it is just 'good-enough' to keep the weaving of threads going on as the task is discussed.

Comparing bricolage of groups

One task assigned to the groups reads as follows:

> **The Bill Nye Fan Club Party**
> The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party. If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there? [1]

Experienced educators are aware that groups of students may take difference approaches to the same problem task, even if they have been in the same mathematics classroom with the same teacher since the beginning of the school year, and thus have shared lessons and materials for several months. What the tapestries do, through the distinct colour patterns that emerge for different groups, is provide visual evidence of how each group performs bricolage in dealing with the task, choosing to draw on particular experiences, ideas and tools. In comparing the tapestries we can see how certain colours (problems) emerge and (sometimes) re-emerge in varying locations, showing how the same problems may be posed at different times during different groups' conversations. As well, the bands of colour themselves vary in width, indicating how some problems only emerge very briefly before another problem is posed, while other problems are taken up at length by the group. Table 1 offers another representation of how the problem posing of each group varies while pursuing the same task. Only four of the total of 31 posed problems are posed by all of the groups. What is even more telling is that almost half of the posed problems—14 out of the total 31—are unique to particular groups. For instance, only one group (NIJM) discusses the (im)possibility of party-goers bringing partial gifts. The presence of unique problems like this suggests that each group has its own gaps of understanding to bridge.

Bricolage through problem posing

For the purpose of characterizing the process of bricolage, I will focus my discussion on NIJM (Nitara, Ian, James and Michael) a group who works very efficiently on the Bill Nye task, rarely becoming sidetracked in its discussion. The first half of the discussion is of "Do we use time and divide by 5?", the problem that the group reposed the most often (see Table 2). What is interesting about this reposing process is how this particular problem takes on different roles as the context of the discussion changes and other problems are posed.

NIJM begins working together as soon as it receives the task sheet, even before the class discussion introducing the task begins, and the group immediately poses "Do we use time and divide by 5?" This suggests a strategy with which to start—counting out the number of intervals in the given time period—and is followed in short order by two other posed problems that consider the original task. In the course of the discussion that follows, "Do we use time and divide by 5?" is reposed as a means to consider what kind of strategy would be best (and easiest) to pursue, to confirm that the group has decided that it will use the counting strategy, and then to actually get going with the counting.

Once the group embarks on its strategy, the role of "Do we use time and divide by 5?" evolves. As the group counts the intervals, the posing of the problem at first works as a suggestion that this particular problem might lead NIJM to determine the number of gifts each party-goer will bring. The group agrees to continue with the counting method and that, if the number of intervals for one hour can be determined, the group can "keep doing it" from there. The problem is then raised to narrate the ongoing calculations and then again to point to predictions the group is making as to what the final answer will be. The next posing of "Do we use time and divide by 5?" occurs when the counting is completed and the group is considering what to do next. This takes place approximately half way through the session, and is followed by much discussion about other posed problems. The final time the problem reemerges is at the very end of the session, when the group is checking its solution, and assigning different members of the group to perform a recount. This leads to a discussion of whether or not there is another way to determine a solution.

The dominance of the problem "Do we use time and divide by 5?" in the first half of the discussion, the fact it alternates with other posed problems and that these problems are addressed quite quickly while "Do we use time and divide by 5?" persists, suggests that "Do we use time and divide by 5?" is a problem that helps to structure the discussion through its frequent reemergence. As well, it may play a more complex role. When a collective works together, ideas may need to be reiterated to ensure everyone is still in agreement. However, even though a problem is still pointing to the same gap of understanding when it re-emerges, the role it plays in the group's discourse may change, just in the same way that a specific tool/resource can be used by a bricoleur in a different way on a different occasion, depending on what is required in the moment. It may be challenging for researchers who are observing a group whose members have a history of working together; for instance, "some long-standing groups generate catchphrases which for them carry implications which are closed to everyone else" (Barnes & Todd, 1995, p. 144). In a similar manner, as a session goes on, the posed problem itself evolves, becoming sedimented with meaning for the group based on the discussion of other posed problems and the attempts to use various strategies. And as it evolves, and the group's history evolves, the role of the posed problem changes.

Problem	**Occurrence**
Do we use time and divide by five?	1
How many people are there?	1
How many gifts are there?	1
Do we use time and divide by five?	2
What is meant by an interval?	1
[*brief non-mathematical discussion*]	
What is meant by an interval?	2
Do we use time and divide by five?	3
Does everyone bring the same amount of gifts?	1
Do we use time and divide by five?	4
Does everyone bring the same amount of gifts?	2
What if everyone brings x gifts each?	1
How many people are there?	2
What if everyone brings x gifts each?	2
Do we use time and divide by five?	5
Is it a square root?	1
How many gifts are there?	2
Do we use time and divide by five?	6
How many gifts are there?	3
Do we use time and divide by five?	7
Is another group's answer right?	1
Do we use time and divide by five?	8
Can they bring partial gifts?	1
Do we use time and divide by five?	9
What are the factors of x?	1
Is it a square root?	2
Do we use time and divide by five?	10
Does it start at one o'clock?	1
How many gifts are there?	4
Is there an extra 5 minutes?	1
How do we think outside the box?	1
What are the factors of x?	2
Is it a square root?	3
Do they also bring gifts for themselves?	1
Do all members give to everyone?	1
What if someone doesn't get a gift?	1
Do they also bring gifts for themselves?	2
What is meant by an interval?	3
What if there are x people?	1
Is it a square root?	1
Do they bring gifts for themselves?	3
What if there are x people?	2
Do they bring gifts for themselves?	4
What if there are x people?	3
How do we think outside the box?	2
Do we use time and divide by five?	11
How do we think outside the box?	3

Table 2. Chronological listing of NIJM's posed problems

Posed problems and good-enough mathematics

I now change my level of focus to the regular transcript in order to discuss how NIJM performs good-enough mathematics to work towards a solution. In their regular lessons, the class had been studying square roots, in particular how to estimate square roots for numbers that are not perfect squares by using number lines and other strategies. Although the Nye task is not directly related to these lessons, the concept of square roots is a part of the group's bricolage repertoire.

As this episode opens, the group is counting intervals and Ian makes a prediction.

Ian This is square roots. It's square roots guys.

Michael	Oh… I see, I see what they're doing *[watching group members who are counting]*
Ian	Since we're learning about square roots, I'm pretty sure it's about square roots

The counting continues a little longer and then the problem of "Is it a square root?" re-emerges.

After some discussion, the group agrees that the answer required must be a whole number, so the square root of 56 would not work, and continues its counting. The group then considers whether the number of intervals is 55 or 56, and the "Is it a square root?" problem emerges again.

Nitara	I just want to find the square root of fifty-six. It will be point something
Ian	Yeah, it's going to be
Nitara	Fifty-six. Seven point four eight three three
Ian	Seven point five
Nitara	Yeah *[Nitara puts her calculator back in the pencil case]*
Ian	We can't have seven and a half people going to the party though!
	[Laughter]
Michael	It could be like the show *Two and a Half Men*

Once again the group seems to have ruled out the idea of the answer being the square root of 56. Yet, a little while later, the problem emerges one more time while the group is discussing the logistics of the present exchange in order to confirm how many people attended the party. They now appear to consider a new way to use the concept of a square root. Because for this task there needs to be a difference of one between the two numbers comprising the factor pair for 56, since each guest is bringing a gift for everyone in the group except herself, a possible method could be to find the square root of the number of intervals and then round up the answer to find one factor and round down to find the other.

Ian	Yeah, I think it was actually eight people. Watch, the closest number up to sixty-four but maybe you go down.
James	Eight people with seven presents.

By the end of the discussion, NIJM has noticed that if you take the square root of 56, which the group earlier calculated to be 7.4833, and round it up you will get 8, a number that will square to 64. However, if you round the square root of 56 down you will get 7, which would be the number of people at the party. The group is using a concept that the students are somewhat familiar with, and through posing and reposing the problem "Is it a square root?" the group is able to reach an insight about how else the concept of square roots might be applied— a case of how good-enough mathematics may lead to a deeper understanding of mathematics.

Concluding remarks

One could argue that we all act as bricoleurs every day. Although there are times that we may plan and prepare to perform a certain task, our lives often involve reacting to the current situation and using what we have on hand, living in the moment. In mathematics, we use not only physical tools, but also reified ideas and concepts, so we too may work in the style of the bricoleur. Rather than using the preconceived purpose of a tool to determine the boundaries of our task, we may let the boundaries evolve with the mathematical situation as we explore. Sometimes we do this purposefully, and sometimes we do this because we are in new intellectual territory, still working out where we are and what is possible, as is very much the case with middle years students.

In this paper, I've sought to offer a way to visualize the paths that small groups of students develop when engaged in a mathematical task. The tapestry method provides a physical tracing of problems groups pose and pursue, showing how the gaps of understanding they are attempting to address are not dealt with in a linear fashion, being resolved before moving on to the next gap, but emerge and re-emerge in various patterns, seemingly triggering (re)awareness of other gaps. Even a group such as NIJM, which is very efficient and focused in its approach to the task, performs bricolage.

This paper also shows how posed problems do more than bridge gaps. When a problem is posed and discussed, this discussion becomes part of the group's history, informing the development of further problems which are posed and discussed, and so forth. In this way, bricolage is a self-structuring process: not only does a group perform bricolage with the tools and resources on hand, but it performs bricolage using its own ideas and the experiences, and it continues to perform bricolage with its ideas, understandings and new experiences *as they evolve.* As a result, the available resources are also evolving along with the situation, allowing students to push boundaries and move to deeper levels of understanding, as NIJM does with the concept of square roots. At its best, mathematical bricolage has the potential to be generative and creative.

One of the goals of school mathematics should be the ability to apply knowledge in the moment, not just for predictable situations (such as tests). As experienced classroom teachers know, what students will do is unpredictable—no matter how much a teacher may front-load a class with certain tools and strategies, there is no guarantee that students will use them as the teacher intends, if they use them at all. The emphasis should not be just on using tools 'properly', but being able to adapt their use to the task at hand and use them effectively, on being more open to and encouraging of "What else can we use this for?" Learning mathematics, doing mathematics, is a process of evolving ideas and reinvention, of making connections. Like light energy which can be considered both as a particle and a wave, mathematics can be both a reified product, as it has been traditionally taught, and a process. The notion of bricolage offers us a way to talk about math that lets us move between these two interpretations.

Note

[1] Implicit in this task, and something that surfaced during class and group discussions, is the expectation that each club member brings one gift for each of the other members who are at the party.

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