DEVELOPING RESOURCES FOR TEACHING AND LEARNING MATHEMATICS WITH DIGITAL TECHNOLOGIES: AN ENACTIVIST APPROACH

MARIA TRIGUEROS [1], MARIA DOLORES LOZANO

As collaborators on the national educational project *Enciclomedia*, we are involved in the design of digital resources for the teaching and learning of mathematics in primary schools in Mexico The materials are intended to help teachers in their teaching of different mathematical concepts with the use of a computer and an interactive whiteboard. We also investigate how these resources are used in the classrooms and reflect on the processes through which our resources are developed, discussing the ways in which our theoretical and methodological frameworks support both our thinking and our way of working in *Enciclomedia*.

Our approach has emerged through the practice of creating resources and investigating their use in the classrooms. The process includes the

- analysis of the textbooks used by the students
- identification of some of the difficulties students and teachers have with the mathematical concepts we are addressing, found both in the literature and in conversations with teachers
- actual design and production of resources and the investigation of the way the materials are used by teachers and students.

Ideas that stem from classroom-based research often introduce changes in our thinking about the teaching and learning of mathematics. This, in turn, is reflected in the resources we are producing Our work in *Enciclomedia* constitutes, therefore, a recursive process.

In this article, we begin by considering some ideas related both to the learning of mathematics with computers and to enactivism, a theory about learning which guides us both theoretically and methodologically Later, we discuss the different aspects of the process we are immersed in and which we outlined above. Finally, we talk about the way in which our way of working has helped us in refining both our ideas and the resources we are creating so that they can be used more effectively in the teaching and learning of mathematics.

Learning mathematics with computer tools: some ideas from enactivism

From an enactivist perspective, the use of computer tools is part of human living experience since

> such technologies are entwined in the practices used by humans to represent and negotiate cultural experience (Davis, Sumara and Luce-Kapler, 2000, p 170)

Tools, as material devices and/or symbolic systems, are considered to be mediators of human activity. They constitute an important part of learning, because their use shapes the processes of knowledge construction and of conceptualization (Rabardel, 2003). When tools are incorporated into students' activities they become instruments, which are mixed entities that include both tools and the ways these are used. They are not merely auxiliary components in the teaching of mathematics; they shape students' actions and therefore their learning (*ibid*, 1999)

Every tool generates a space for action, and at the same time it imposes on users certain restrictions. This makes possible the emergence of new kinds of actions. The influence that tools exercise on learning is not immediate. Actions are shaped gradually, in a complex process of interaction. In the classrooms, students construct meanings through the use of computer tools, in a process of social interaction and with the guide of the teacher (Mariotti, 2001).

The purpose in *Enciclomedia* is to develop initiatives that can help teachers to create contexts in which certain actions, related to the learning of different mathematical concepts, can be fostered through the use of computer tools. The kinds of actions that we want students to be engaged with are specified, in part, by the mandatory textbooks which are used in the classrooms. This is why part of our work, which we discuss in more detail later on, consists in analysing these textbooks.

Because we believe learning occurs in the process of interactions, when students gradually modify their behaviour, we create digital resources and teaching guides that promote the joint exploration of mathematical ideas and concepts. Computer programs in *Enciclomedia* are intended to broaden users' experiences with mathematics by providing spaces where the need for using mathematical procedures arises naturally. Our activities are meant to challenge and develop students' (and teachers') intuitive (and sometimes inadequate) thinking and to trigger, in them, ways of acting that we consider mathematical

Some ideas about methodology

The process of development of digital resources and teaching guides, along with the investigation of the way in which these are used in the classrooms is, from an enactivist perspective, a learning process in itself. In enactivism, learning is thought of as effective or adequate action in a certain environment (Maturana and Varela, 1992). As researchers and software developers, we are individuals immersed in a particular context. Our actions are being modified as we interact with each other and with teachers, students and other researchers, and they will be effective or ineffective in our environment. What we mean here by effective actions, are those that allow us to produce resources and initiatives that teachers and students find useful and with which they can develop their learning of mathematics

According to enactivism, people interpret the world in a particular way, influenced by their previous experiences. In addition, in the process of learning, individuals influence and shape the context in which they are immersed (Reid, 1996, p. 206). The interdependence of context and researchers makes the process of creation and investigation of initiatives for the teaching of mathematics a flexible and dynamic one. The development of resources in *Enciclomedia* does not occur in a linear fashion; rather, it is a recursive process of having conversations amongst ourselves and with teachers, asking questions and observing and analysing what happens in the classrooms. We think of our educational initiatives as dynamic suggestions which are under constant modification. Teachers are intended to work with our proposals in their practice which is inevitably an ever-changing process. The interactive materials related to specific concepts in the curriculum are used by students in a process which is also dynamic. Researchers' and developers' ideas will be modified as they interact with textbooks, teachers, students, and with each other.

Being aware that the production of software, along with the investigation of its use in the classroom, is a recursive process, we keep records of the different phases of our work in *Enciclomedia*. In that way, we are able to reflect on our thinking and we can trace back the changes our programs go through. We write reports after meetings and after conversations with teachers. We also use different methods of data collection during teachers' workshops and in classroom observations. In what follows we describe, in more detail, the different activities with which we are involved in the process of development of resources in *Enciclomedia*.

The process of development of resources

The process we follow in developing digital resources involves several activities including the analysis of textbooks, having conversations with teachers and producing interactive programs. We do not think of these activities as a series of sequential steps that we follow. For example, even though we often start by analysing the textbooks, what we see in the chapters is a result of our previous experiences, which include teaching, reading the mathematics education literature, and having informal conversations with teachers and other researchers. All the activities we undertake are intertwined and each affects the rest.

Analysing the textbooks

The resources we create in *Enciclomedia* are intended to complement the textbooks that are used in years 5 and 6 (students aged 10 to 12 years), in all primary schools in Mexico. For this reason, when we work with a particular mathematical concept or process, we usually start by looking at those chapters in the textbooks related to it and we try to identify areas where a multimedia resource could be of help. The analysis of the textbook chapters is carried out from multiple perspectives, which is another feature of the enactivist methodology (Reid, 1996, p. 207). We each look at the activities posed in the books separately. Even though we all have first degrees in mathematics, we have different experiences regarding the teaching and learning of mathematics and have carried out research in different areas within mathematics education. Our different backgrounds provide us with a variety of ideas about the mathematical concepts we work with, which we discuss in joint meetings. Through the comparison of the different views we each provide, we are able to develop richer interpretations

The approach taken by the Mexican *National Curriculum* and associated textbooks consists in

> taking to the classrooms those activities which elicit students' interest [in mathematics] and invite them to reflect, to find different ways of solving problems and to formulate arguments that validate their answers. (SEP, 2003, p. 7)

Results from our analysis of the textbooks show that most problems and activities posed in them are challenging and useful; however, we have also noticed limitations in the presentation of concepts. The problems in the textbooks frequently require mathematical knowledge that most students do not have. Complex ideas are often presented without providing the students with any strategies for dealing with them

We have found certain areas in mathematics to be particularly challenging. In this article, we exemplify our work through two of these aspects: rational numbers and the concept of area. We have chosen these areas because they are the ones we have been working on longer, and because we believe they can help us illustrate the process we follow as we develop our resources in *Enciclomedia*.

In the textbooks, we have found that sophisticated problems involving rational numbers are presented without providing students (and teachers) with activities that could assist them in their learning. The concept of area, even when introduced with tangible examples, presents students and teachers with several difficulties – there is not a clear relationship between the concrete activities in the textbooks and the abstract formulae students and teachers use when they deal with regular shapes.

Our purpose is to complement the textbooks, providing teachers and students with resources that can help them in working with the activities posed in them

Conversations with teachers

We work with teachers through the whole process of development of resources. Three teachers who use the resources in their classrooms work part-time in *Enciclomedia* They go through the materials we produce and make comments on them We visit schools regularly, and we work, in particular, with five teachers from three different schools. We have also worked in several workshops including 200 teachers in total. Our conversations with teachers provide us with an additional perspective, enabling us to re-consider our thinking and, often, to modify our initiatives.

From the data we have collected through our interactions with teachers, we can say that most of them find many of the activities in the textbooks difficult. They usually say that the textbooks do not provide students with enough exercises for them to learn the concepts. Most teachers have mentioned that they do not understand the goal of many of the chapters and that they often avoid teaching some concepts Fractions are particularly problematic, as shown in the following comments:

> T1: I am not able to solve the problems in the textbooks while kids sometimes can! I feel useless. I would like to have the answer key, I don't know whether what I am doing is fine or not.
>
> T3: Sometimes the problems [in the textbooks] are too difficult I simply can't do them, I am not good at maths and I get confused, I hate fractions!
>
> T5: Students have problems understanding fractions in this chapter. It is difficult for me too ... What I prefer to do is getting them to do more exercises so they know how to do it, they need them

We have also found that teachers' strategies often differ from the approach taken by the *National Curriculum* and the textbooks (reported to us by the teachers themselves, and confirmed through classroom observations) During the lessons, teachers often give definitions for concepts and work on repetitive exercises For example, regarding area and perimeter, even when teachers say that they have a good understanding of the concepts, they prefer to use conventional formulae instead of working with non-standard units as is suggested in the textbooks.

> T2: I give them the formulae first, all of them. I will explain each formula later, but they have to learn them all.

We have observed that, in the classrooms, students are frequently left to work on textbook problems on their own, verifying answers in the group without discussing the problems or the procedures they used for solving them. Collaborative work is seldom used and students often get distracted

Our conversations with teachers, along with the observations we have carried out in the classrooms, suggest that there is a gap between the activities suggested both in the *National Curricula* and in the textbooks and those that teachers and students engage with in the classrooms. One of our challenges, in *Enciclomedia*, is to support teachers in developing their teaching practices so that this gap is bridged.

Many teachers often feel insecure about their 'mathematical knowledge' and on many occasions we have observed, in the classrooms, that the solutions and explanations they give for mathematical problems are inadequate. Our preliminary research results have indicated that the digital resources in *Enciclomedia* should support both teachers and students in their learning processes, and should also help teachers overcome their fears and difficulties with mathematics. Learning mathematics, which from an enactivist perspective means acting effectively, in this context, entails acting in ways that can be characterised as mathematical. By providing teachers with digital resources that challenge their mathematical thinking we wish them to engage in the same kinds of mathematical actions we would like them to foster in their classrooms. In this way, mathematical learning can occur as teachers explore (on their own, with their students and ideally with other teachers) digital contexts rich in mathematical content and activities

Within enactivism, actions are embodied, and they are embedded in a biological and cultural context (Varela, 1999, p 12). Learning occurs when, as a result of interactions with each other and with the world, individuals act in a way that is effective in a certain domain. Social interaction can provoke changes in individual behaviour, such as when participants' thinking might get challenged by others' interventions. For this reason, we are also interested in assisting teachers in the development of their teaching practices, so that these include collaborative work and whole group discussions. In *Enciclomedia*, we have created a section where teachers can find (on the hard disk of their computer) written guides with ideas for teaching with digital resources. Teaching strategies that promote interactions between students are included in these guides. Additionally, many of the interactive programs we design promote joint activities, for example, games in which work in small groups is required

We do not provide teachers with materials in which mathematical concepts and procedures are explained explicitly. Neither do we present them with teaching guides which are meant to be prescriptive. Rather, we try to trigger, in teachers, actions that we think can be effective in the context of mathematics teaching by providing them with resources where they can work with mathematical concepts and where they can find different ideas for their teaching practices (see below for more details about the teaching guides)

Looking at the literature

Finding out what is said in the literature about the teaching and learning of the mathematical concepts we are addressing in our project provides us with valuable ideas for our work. We are each familiar with different areas of the mathematics education literature – some members of our group have experience in doing research on the teaching of mathematics with the use of digital technologies and each of us has looked at research studies on different mathematical concepts

Common misconceptions and difficulties, related to the mathematical concepts we address through our work, are pointed out in numerous research reports. Additionally, activities that might help children in their learning of mathematical concepts identified as being particularly difficult are suggested in several studies. We explore what is said in the literature and we contrast reported findings with our own experience and with ideas we obtain from teachers.

For example, we have found that, in relation to the learning of rational numbers, the use of multiple concrete models is recommended (*e.g.*, Cramer, Post and del Mas, 2002). We have, therefore, designed several resources in which different representations of fractions are used. Concerning the learning of the concept of area, research shows that many students do not see it as a measure of the spread of a surface, even when they can use formulae to calculate it (*e.g.*, Baturo and Nason, 1996). This corroborates what we have found from our conversations with teachers. Students often confuse perimeter with area, and researchers advise teachers to work with this distinction from the beginning. Early introduction of formulae is seen as a contributor to this problem and therefore the use of non-standard units and of ways of obtaining areas and perimeters is suggested. Our intention is to help teachers with strategies for working with area and perimeter in the classroom so that, eventually, the use of formulae becomes more meaningful.

From the analysis of the textbooks, the conversations we have with teachers and the findings from the literature, many questions and ideas arise. We ask ourselves what kinds of experiences can help teachers and students in their learning of the mathematical concepts we are thinking about and whether digital technologies can be of particular help. For example, we start thinking about how multiple representations and computer simulations can help teachers and students reconsider their ideas about these concepts, and how teachers can benefit from the use of those resources in their day-to-day activities. We then start the production phase, in which we develop interactive programs and teaching guides.

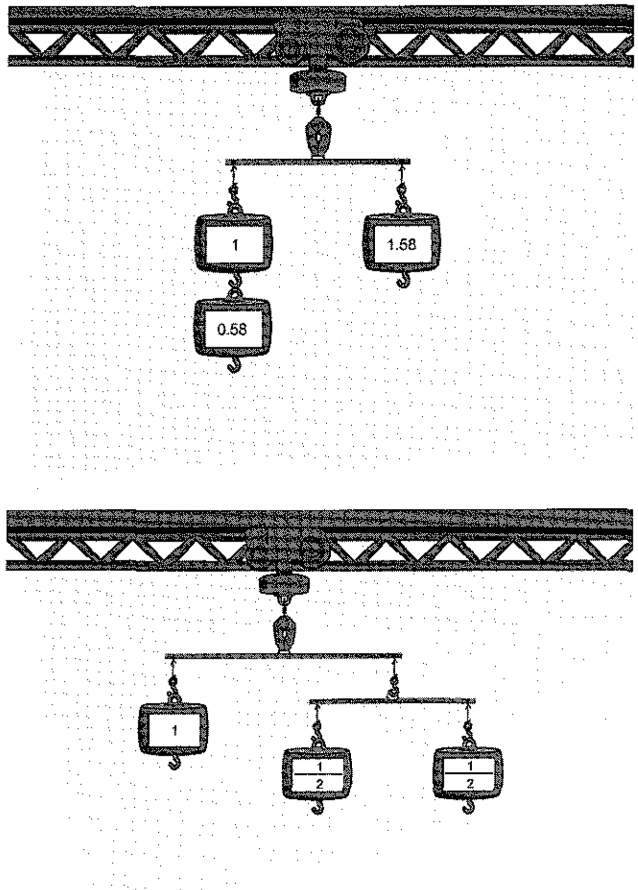

Figure 1: The balance.

Producing the computer programs

We have developed different types of programs. They vary, for example, in the kinds of interactivity they promote and in the types of problems they pose to the users. The programs are closely related to the activities in the students' textbooks, but they are mostly thought of as spaces for mathematical exploration. Resources usually provide users with something they would not get if they used the textbook only. Programs give the students immediate feedback on their actions on the computer and they often simulate situations that are difficult to recreate or experience in the classroom.

A first example is the interactive program *The balance*, which reproduces a problem situation from the year 6 textbook where scales need to be balanced by using fractions. *The balance* was created as a space in which students and teachers could explore some of their ideas about numbers. In particular, it was intended to help them work with equivalent fractions. The computer program provides the users with automatic feedback that helps them in identifying which parts of the mobile toy are balanced and which are not. In enactivist terms, the program is meant to trigger in students mathematical effective actions such as comparing rational numbers, and finding equivalent fractions.

Another example is the program *Perimarea*, which invites students to work with non-standard units and with different ways of calculating the area and perimeter of different shapes. For example, in one of the program's activities, students need to find the area of a shape by counting squared units using a grid. In other activities, they are also asked to draw different shapes that have the same area, or the same perimeter. The program gives visual feedback that indicates whether the student's answer is adequate or not. For example, if in a student's response there are some missing units, a darker colour indicates the missing squares on the grid, or when the student's answer is greater than expected, the squares flash intermittently.

Interactive programs such as *The balance* and *Perimarea* are designed taking into consideration that, by interacting with the computer, students and teachers can engage in mathematical activity. The visual representations that the computer provides the users with are intended to elicit particular kinds of actions. From an enactivist perspective, students will act according to what they already know and are prepared to do in a given context, according to their biological and social history. The computer programs, however, contribute to determining the effectiveness of students' and teachers' actions, therefore *shaping* their learning.

The actual production of digital resources is, in itself, an iterative process in which programs are continually being modified Starting with preliminary thoughts, which stem from the above mentioned sources, we have weekly meetings where we have discussions within ourselves and with programmers and graphic designers. In these discussions, valuable ideas emerge, as participants come from very different perspectives As we go along in the cycle of discussion and production, we continually review the development of the programs together with the development of our own ideas of what the resources are supposed to do and how. During meetings, we propose ideas for the design of new resources, and we analyse and try out the programs that are being created. The following are extracts from conversations in which the *Enciclomedia* mathematics group discusses the production of resources:

MT: It seems, from our observations and from what the teachers have told us, that *The balance* is a useful tool for working with equivalent fractions. I think, however, that we need to develop more resources to complement *The balance* and to cover other aspects of operations with fractions . ..

EJ: It is a good idea. I think that we need to have a sequence of activities to help students and teachers discuss the mathematical ideas involved in operations with fractions

MD: Perhaps .. but we should be careful about its design, we want students to engage in productive discussions and in exploring their ideas about those operations.

AL: I agree, we want students to be engaged in the activity we propose ... I also consider the use of multiple representations in this case is very important

The programs are modified until a first version is found acceptable, by the group, for its use in the classrooms.

Writing the teaching guides

The teaching guides we produce include suggestions how to use the computer programs with the interactive whiteboards in the classrooms As we have mentioned before, we recommend teachers to promote collaborative work in their classrooms, including discussions and small-group work. We also advise teachers to encourage the manipulation of concrete objects before getting students to work with abstract ideas. We acknowledge the complexity of teaching and learning and the uniqueness of every classroom Our intention is to provide teachers with guidelines, and, when possible, to work with them in the development of their practice of the teaching of each particular concept The teaching guides are produced in collaboration with teachers. Together, we discuss different ways of working and addressing the concepts in the textbooks. Teachers read the guides and make comments on them:

T1: This part is really clear. The first problem I had when I used *The balance* without the guide was that in the lesson I never understood that the problem was to balance all the levels, my first reaction was that the program didn't work! But here ... the goal of the activity is clearly stated, my answer was not wrong, but it was not what I was asked to do on this particular exercise. ... Now here, I think we need more help with these activities.

ML: What about this section, where it is suggested that children work in groups to design their own mobile toys and challenge the others to solve them?

T2: I think it is a good idea Children like games The only problem I see is that they can build a very difficult problem that nobody will be able to complete ...

ML: What could we do about that? We can write something about that and add it to this part of the guide. What would you do if that happened in the classroom?

Teaching guides are continuously modified, according to the feedback we receive from teachers, and also after our visits to schools In the future, we intend to have more than one teaching guide for each chapter of the textbooks, so that teachers can find different options and decide which ideas they find useful according to their particular situation

Doing research in the classrooms: back to the beginning

After a first version of a program and/or teaching guide is completed, we carry out research in the classrooms in order to investigate how the resources are used by teachers and students. We make observations and ask questions to students and teachers in order to identify mathematical actions and to obtain more information on what their strategies and difficulties are when using the software We look at mathematics learning by observing patterns in effective behaviour (Maturana and Varela, 1992). We use audio- and video-recording as well as field-notes to collect data (for a more detailed account on the methods we have used see Lozano, Sandoval and Trigueros, 2006).

Research has shown, for example, that *The balance* helps teachers work with different aspects of rational numbers effectively and that, used in the classroom, the program invites students to act mathematically in a number of ways Initially, it was observed that students started out by using trial and error. However, because this strategy was not effective, they gradually refined it and produced more sophisticated methods for obtaining fractions that equilibrated the mobile toy. While using *The balance*, students and teachers worked with mathematical concepts such as equivalent fractions and they operated on mathematical symbols. They used different representations when dealing with fractions and they often asked mathematical questions such as "Why is 1/2 heavier than 1/4?". Drawings were frequently used and hypotheses were tested by using the program We also observed students giving explanations to justify their answers to the group Justifications were usually incomplete, although we have also recorded sophisticated explanations, such as using graphic representations (of pizzas) to show

how a fraction with odd numerator can be divided into two equal parts (see Lozano, *et al*, 2006).

Some students found systematic ways of solving problems with *The balance*, finding out when it was appropriate to add or subtract fractions. Additionally, we noticed that students modified their interpretations as they worked together and they interacted with the program, which provided them with visual feedback:

> Student1: You have to add this and this, see, 1/3 add 1/3, that is 2/6 (They try their answers in *The balance*) .. No, it doesn't work ... this is smaller ..
>
> Student2: No, it is 2/3!! 1/3 add 1/3 is 2/3, not 2/6 (06/12/05)

Additionally, we have detected some changes in classroom dynamics with the introduction of programs such as *The balance*. When we were in the classrooms, students and teachers looked motivated and immersed in mathematical activity. In addition, we have noticed collaborative work occurring spontaneously as a result of students and teachers interacting with each other when trying to solve the activities in the programs.

We have also observed students and teachers as they used the program *Perimarea*, where they needed to calculate the area for different shapes by counting the squares on a grid. We noticed, both during the lessons and on the video from those sessions, that students often gave the answers by trial and error and that this strategy proved to be effective. They got immediate feedback from the program, showing them graphically whether they were either missing or they gave too many square units in their answers. They continued writing numbers until the program told them their answer was correct, and they would go on to the next exercise. While most students wrote random numbers, a few others counted squares. In both cases, however, we found out that they did not relate these actions to the concept of area or the formulae they have used beforehand. Because of the nature of the activities, students were not seen working together when they used *Perimarea*. The program did not offer challenges that motivated students to look for collaboration.

The original purpose for which we developed *Perimarea* was not accomplished. It did not trigger in students and teachers actions that we considered mathematical and we found that students soon got bored while using it. From an enactivist perspective, producing *Perimarea* does not constitute effective behaviour for the mathematics group in *Enciclomedia*. After visiting the classrooms, we decided to modify the program so that students are not given automatic feedback and so that they can establish relationships between their counting actions and the concepts of area and perimeter:

> IS: We have observed that, in the classrooms, students randomly write an answer for the area and the perimeter of the shapes
>
> ML: Yes, they don't seem to be thinking about area and perimeter. The program gives them immediate feedback, but this does not seem to help in this case.
>
> EJ: I think feedback should not be automatic.
>
> MI: We can give the teacher and the students the option to get feedback when they need it, but we should give some control to the teacher here. We can have another button; they can click on it whenever they want feedback.

We have already made some changes to the program and others are in progress. Figure 2 shows the way in which the interface has changed. For example, a new button *Listo*, which teachers can use whenever they want the program to give feedback to the students, was incorporated.

Research on how this version of the program works in the classroom is currently being carried out. We have also decided that we need to develop more activities that can help students deepen their understanding of area. Revisiting the textbooks and the literature are places to start.

Classroom observations often make us reconsider our ideas about the concepts we are working with and about the programs we are developing. After we observe and analyse students' and teachers' activities in different classrooms, we often decide to go back to the textbooks and the literature, and to talk to teachers again about specific issues regarding students' learning. Even when we think digital programs

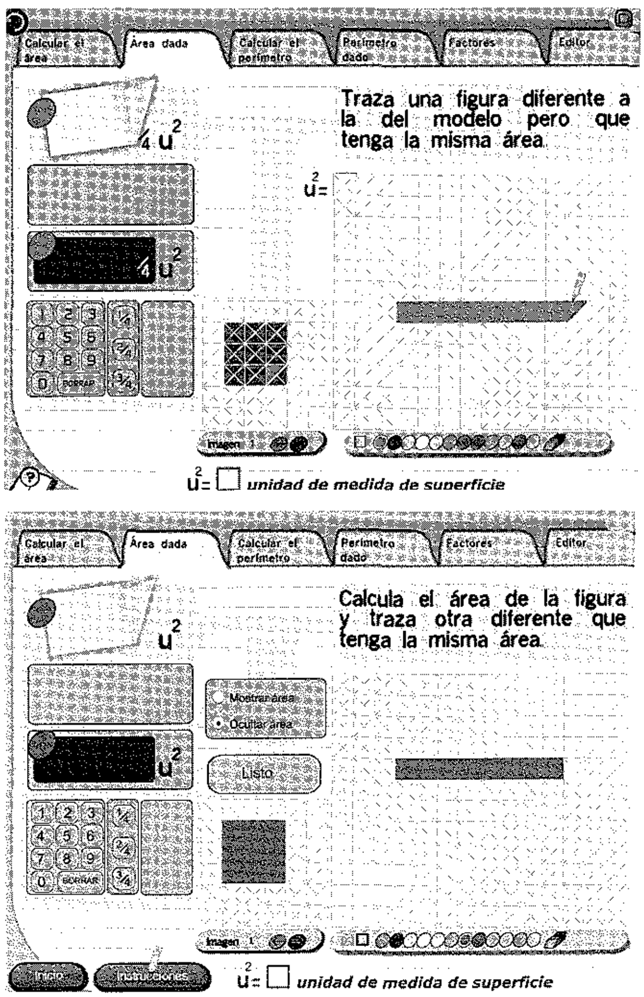

Figure 2: Changes in Perimarea.

are helping students and teachers, we jointly re-visit the intention of the materials; therefore going back to the beginning of the process.

Reflections

When we start the process of development of resources again, after having had conversations with teachers and having analysed classroom observations, we do it from a different place. Our thinking changes as a result of the process of production we go through, therefore our interpretations of the textbook activities, of the research reports we read, and of our interactions with teachers and students, will be different. Our own individual history, which in enactivism determines our actions in a given moment, is being shaped in particular ways by us being part of *Enciclomedia*. These individual histories, in turn, shape our collective work. As we interact with each other, through the development of resources for the teaching and learning of mathematics, we create our history as a group. We each bring different and unique perspectives to our work and this enriches what we produce greatly. As we work together, however, ways of collaborating emerge. The resources we create are the result of joint actions, and they would not exist, in their current form, if we worked on them individually.

The programs we produce do not always work in the way we would expect them to, in the classrooms. Teachers do not find all the resources helpful, and, as happened with the first version of *Perimarea*, sometimes students interact with the programs without being engaged in mathematical activity. Other programs, however, are frequently used in the classrooms and have been found helpful for the teaching and learning of mathematics. Through their use, students and teachers are able to solve the activities posed in the textbooks adequately. Creating resources or modifying existing ones so that teachers and students use them to develop their learning of mathematics, constitutes, for the mathematics group in *Enciclomedia*, effective or adequate behaviour in the context which we inhabit.

The questions that engage us, at present, are related to what makes our behaviour, as a group, effective or ineffective. We are interested in exploring what makes some of our digital resources useful for the learning of mathematics. Preliminary results indicate that programs such as *The Balance* give students and teachers freedom to explore mathematical situations while at the same time they provide them with interesting and challenging activities. The interactions these programs provoke enable users to modify their behaviour in a mathematically meaningful way. Other resources in *Enciclomedia* do not seem to enhance mathematical activity, either because the activities are too constrained, triggering in users behaviour that is limited, or because the activities are too complex and the programs do not provide students and teachers with enough support. In the future, part of the process of development of resources will include the comparative analysis of the different programs we have created, so that we can deepen our understanding of the way in which digital technologies can be of help for the teaching and learning of mathematics.

Notes

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