approaching a limit; or jumping from one value to another, we are engaging in fictive actions which lie at the heart of doing mathematics

- how physical gestures penetrate mathematical explanations and arguments at least in their living forms (illustrated with numerous quick-time movie examples)

- reports of research that showed synchronicity in brain activity related to such gesturing and the related uttering.

For me the two ideas of fictive and gestural actions illustrate well what is meant by mathematics knowing as fully embodied. In particular, they show that mathematics is a communicating form of knowing and that in both forms of activity we are trying to engage others with and in our knowing actions in ways that seem to me to embody both the actor and the other (such as, listener, watcher, evaluator) in a larger body of mathematics. Further, both these two features, fictive and gestural actions and their detailed study, may provide us with a means for looking at and using the tension of awarenesses, raised by Terezinha Nunes and Daniel Ansari, in our theories and practices as we try to serve the communities represented by those who so cordially and thoughtfully greeted us as ICME-10 participants.

References

Johnson, M. (1987) *The body in the mind: the bodily basis of meaning, imagination, and reason*, Chicago, IL, University of Chicago Press.

Lakoff, G. and Núñez, R. (2000) *Where mathematics comes from: how the embodied mind brings mathematics into being*, New York, NY, Basic Books.

Lost and found in ICME-10

FULVIA FURINGHETTI

Traditionally ICME conferences are places where you meet people or lose people – the definition varies if you are an optimist or a pessimist. At an ICME conference you meet a lot of interesting people, but they are not necessarily those that you planned to meet! The number of participants, after the heights of ICME-8, Spain (2762 participants) is decreasing (2074 in Japan, ICME-9; 2161 in Denmark, ICME-10), but there are still a lot of people.

For instance, as was the same for many participants, my accommodation was a long way away from the conference venue. After an initial moment of dejection (when I realised that breakfast would be at six thirty all week), I saw the happy side – having a long, relaxed trip with time for talking with my colleagues, at the same time as enjoying the wonderful Danish public net of transportation. Whenever else have I had, or ever will have again, a half hour of conversation with David Tall, without interruptions from other colleagues asking him something?

As chair of the group *History and Pedagogy of Mathematics* (HPM), affiliated to ICMI, my main interest has been to go to events in the programme that combine history and mathematics education. The tradition of having a substantial activity in this field, which dates back to the working group on *History and pedagogy of mathematics* held at ICME-2, was continued at ICME-10 through Topic Study Group 17 (TSG 17): *The role of the history of mathematics in mathematics education*, chaired by Man-Keung Siu and Constantinos Tzanakis.

TSG 17 was the place where the three souls of the HPM Group (mathematics, history and education) met together. Thus we attended presentations by mathematicians looking at history to find materials for their teaching, historians providing suitable excerpts and historical information, researchers in education who attempt to investigate if and how the cultural artefact 'history' might become a mediator of mathematical knowledge in the classroom.

The programme offers a wide range of events that can be confusing. As well as TSG17 there were other forms of activity that explored the role of history in mathematics education for students and teachers:

- a session in the form of 'poster round tables' labelled *History of mathematics and mathematics education* allowed a better understanding of the concise content of posters through discussion in an informal, fruitful atmosphere

- lectures from Victor Katz, Luis Puig and Evgeny Shchepin

- workshops from Victor Katz with Karen Michalowicz and from Avikam Gazit.

All these activities evidenced that that spirit of the HPM group is much more than the use of history in mathematics education – it is the conception of mathematics as a living science, a science with a long history, a vivid present and an as yet unforeseen future, together with the conviction that this conception of mathematics should not only be the core of the teaching of mathematics, but it should also be the image of mathematics spread to the outside world. This spirit of the group was evidenced by the telling of its history, since its foundation in 1976, told by Florence Fasanelli in one of the meetings of the group during ICME-10.

This brings me to another issue in which I'm particularly involved, that is the history of mathematics education itself. I applaud the scientific committee, which for the first time has established a new topic group (TSG 29 chaired by Gert Schubring and Yasuhiro Sekiguchi): *The history of the teaching and the learning of mathematics*. I'm strongly convinced that researchers in mathematics education and teachers need to know about this history to feel themselves to be part of the big project of modern civilization. This topic was also developed through regular lectures, such as that of Anosov on the development of Russian mathematics education in the past 300 years and in National presentations, such as those from Romania and Russia.

In his regular lecture, Geoffrey Howson illustrated the role of two main characters of the past (Klein and Freudenthal) in the development of mathematics education. His talk was a first-hand witness, by a person who lived through years of the ferment of initiatives after the second world war.

The drop in numbers of participants that I mentioned in the first paragraph could have many different explanations: the proliferation of conferences, the cuts in funding in many countries, changed research policies, the loss of impetus in curricular reforms, and new forms of communication. The first ICME in Lyon (1969) was held in the wake, and on the eve, of great events in mathematics education: the modern mathematics movement, the foundation of the journals of mathematics education that are now the backbone of our community, and the societal changes after the second world war. In short, the first ICME was the concrete acknowledgement of the birth of a new community of researchers (in mathematics education) in a new educational setting. The context, however, has now changed.

So, finally, I look to the future. The affiliations of the various contributors to ICME-10 show that there is a lack of the voices of teachers. I am tempted to say that the format of ICME needs re-styling from its current planetary and ecumenical dimension to a more local and focused dimension. On the other hand, I cannot forget that ICME is organised under the auspices of ICMI, whose birth and aims are rooted in the ideals of communication, solidarity, and internationalism claimed in the journal *L'Enseignement Mathématique*. This journal (founded 1899) was the cradle of ICMI and has been its official organ since the beginning (Furinghetti, 2003).

Thus, after having complained about the too-rich program and the confusion provoked by too many people, I am ready to face it all again to experience whatever is going to happen at ICME-11. I like being part of the long-lasting, wonderful project underlying ICMI activities too much!

References

Furinghetti, F. (2003) 'Mathematical instruction in an international perspective: the contribution of the journal *L'Enseignement Mathématique*', in Coray, D., Furinghetti, F., Gispert, H., Hodgson, B. and Schubring, G. (eds), *One hundred years of L'Enseignement Mathématique, Monographie 39 de L'Enseignement Mathématique*, 19-46.

*The following three communications are comments inspired by two articles about algebra, particularly solving linear equations, by Brizuela and Schliemann (pp. 33-41) and by Dickinson and Eade (pp. 41-47), which appeared in FLM **24**(2). (ed.)*

Solving linear equations – why, how and when?

ABRAHAM ARCAVI

I would like to organize the thoughts these two articles provoked in me as the answers to three questions, which are not directly addressed by them, but are certainly implicit in most of what they describe and claim.

Solving linear equations – why?

> It was with the number puzzle that algebra seems to have taken its start. The desire of the early philosophers to unravel some simple numerical enigma was similar to the child's desire to find the answer to some question in the puzzle column of a newspaper [...] Such was the first algebraic problem (Number 24, in *Peet, Rhind Papyrus*, with slightly different translation) of Ahmes (*c.* 1550, BC), "Mass, its whole, its seventh, makes 19" [1]. (Smith, 1958, **2**, p. 582)

1. Many would disagree with the above claim about the origins of algebra. Sfard (1995) summarizes some of the arguments and counter-arguments about the origins of algebra, and points to the heart of this controversy as stemming:

> [...] not so much from different historical information as from the fact that they obviously have their own interpretation of the term *algebra*. (Sfard, 1995, p. 17)

We may attribute to Smith (based on the quotation above) an implicit view of algebra in which equations and their solutions are at its core. This would contrast with the view that seems consensual in mathematics education nowadays:

> Students in the middle grades should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations. (NCTM, 2000, p. 223)

Thus, according to the National Council of Teaching of Mathematics (NCTM), solving linear equations is not at the core of algebra, it is just one of the competencies listed – only at the end – of the NCTM's opening paragraph on algebra:

> In the middle grades, students should also learn to recognize equivalent expressions, solve linear equations, and use simple formulas.

So, a first negative answer to the question, 'why learn the solving of linear equations?', may be, certainly not because they are considered to be at the core of algebra (even if this is implicit in history treatises or school textbooks).

2. Regardless of the epistemological or historiosophical controversies about algebra and its origins, there is a rich history of methods for solving linear equations. This history tells us about methods such as 'false position', 'double false position', 'method of the scales' and *regula infusa* (see, for example, Smith, 1958; Boyer, 1985; Charbonneau and Radford, 2002). Students also generate methods of solution (*e.g.*, counting techniques, cover-up and undoing as described, for example, in Kieran, 1992, p. 400).

The creation of solution methods both in history and by students leads me to consider further parallels, in an attempt to see in the evolution of this topic an example of the assertion that "ontogeny recapitulates phylogeny". This phrase was coined in 1866 by the biologist and philosopher Ernst Haeckel (1834-1919), and it summarizes the disputed claim that the development of a single embryo of a species retraces the evolutionary development of that species. In our case, it would mean that the development of a mathematical idea for individual students retraces the evolution of that idea through the history of mathematics. Such parallels would indeed constitute an answer to our question, "Why?". A possible point of contact between historical methods such as 'the rule of false' and 'trial and error', which are natural for