A Theory of Intellectual Development

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PART II*

SOCIO-CULTURAL PERSPECTIVE

In the past decade, Vygotsky's work has gained widespread attention and commendation. Indisputably the work is worthy of careful study. And according to its own fundamental principles, the theory itself is a product of its historical-cultural situation, and in examining it, one needs to first understand it from within that time period and then consider how it might be modified in the light of current cultural and historical times. I will try to demonstrate in this paper that Vygotskian theory can support two opposing interpretations, one which fundamentally supports reform and one which can, in fact, undermine it.

To begin, a summary of Vygotskian theory is offered. Vygotsky proposed to develop a theory of intellectual development that would: 1) recognize a central role for social and cultural influences; 2) build upon the characteristics that separate humans from other animals; 3) create a Marxist psychology [Wertsch, 1985]; and 4) contribute to the social program of making literacy accessible to all. It should be kept in mind that Vygotsky's contributions were made at a time when literacy was seen as a positive accomplishment, entailing only gains and no losses, and literacy assumption was to be accomplished in a relatively uniform manner across all peoples [John-Steiner, 1985]. My discussion of the Vygotskian perspective is organized in six major categories: 1) socio-cultural perspective; 2) Marxist influences of historical analysis and the role of labor; 3) semiotics and psychological tools; and 4) the dialectic of thought and language; 5) conceptual development; and 6) learning and development.

A framework for Vygotskian theory

1. **Socio-cultural perspective**

Vygotsky's central tenet was that socio-cultural factors were essential in the development of mind. In fact, for Vygotsky, the individual emerges from a socio-cultural context. All intellectual development, including meaning, memory, attention, thinking, perception, and consciousness, evolves from the interpersonal (social) to the intrapersonal (individual). For Vygotsky, "the social dimension of consciousness is primary in time and fact. The individual dimension is derivative and secondary" [Wertsch, 1985, p 58]. "The very mechanism underlying higher mental functions is a copy from social interaction. All higher mental functions are internalized social relationships" [p 66]. Vygotsky sought to examine "how the collective creates higher cognitive activity in the child" [Vygotsky, 1981, p 165].

This view entails a paradigmatic shift in the conceptualization of the learning child. The learning child is not viewed as an autopoietic (self-regulating) system who forms connections by modeling others as other self-regulating individuals. His/her very identity emerges from social/cultural relations. Vygotsky distinguished two lines of development, the natural and the socio-cultural, and for him it was the socio-cultural that distinguished humans from other animals. Theoretically Vygotsky argued that the two lines of change (the natural and the cultural) interpenetrate one another and essentially form a single line of sociobiological formation of the child's personality" [ibid., p 41]. Methodologically Vygotsky's emphasis was on the impact of the socio-cultural on the natural. As Wertsch described this, Vygotsky's empirical work concentrated on "the natural line as providing the "raw materials" that are then transformed by cultural forces" [ibid., p 43].

2. **Marxist influences: Genetic analysis and the role of labor**

A second central tenet of Vygotskian theory was Vygotsky's commitment to the creation of a Marxist psychology. Although this quality is being increasingly de-emphasized by Russian psychologists in the light of the disintegration of the Soviet Union, its influence on the development of Vygotsky's theory was profound. In this paper I will stress two components of Marxist theory: dialectical and historical materialism. As described by Cole and Scribner, "A psychologically relevant application of dialectical and historical materialism would be one accurate summary of Vygotsky's sociocultural theory of high mental processes" [Vygotsky, 1978, p 6].

**Historical materialism**

To examine the development of cognitive thought one must undertake an historical analysis. According to this theory one examines the conditions and the trajectory that produced the current state of an object in order to know what that current state is. Vygotsky wrote that a positive picture of the child, as opposed to a negative one, specifying only what the child is lacking, "becomes possible only if we change our idea of child development in a fundamental way and if we take into consideration the fact that it represents a complex, dialectical process characterized by a multifaceted, periodic timetable, by disproportion in the development of various functions, by metamorphoses or qualitative conversion of one set of forms into others, by complex combinations of the process of evolution and involution, by complex mixing of external and internal fac-
tors, and by the process of adaptation and surmounting difficulties” [ibid., p 151] “This commitment to “genetic analysis,” like Piaget’s to “genetic epistemology,” suggests that one must examine the genesis of the higher mental functions to understand them.

Dialectical materialism

Marx and Engels emphasized the central role of labor in cultural development. In their work they argued that it was through the act of production that the truth of an idea was revealed. Davydov took this same position when he wrote, “Productive activity that concerns practical objects—labor—is the basis of all human cognition” [Davydov, 1990, p 232]. Engels himself wrote, “The most highly essential and immediate basis for human thought is precisely man’s modification of nature, rather than nature alone as such, and man’s reason has developed according to how man has learned to modify nature” [ibid., p 232].

Davydov, a follower of Vygotsky and a mathematics educator, stresses that it is through the process of labor, through the transformations of objects using tools, that an object's incidental conditions are factored out, its invariances can be glimpsed and “their internal, essential properties—the necessary forms of their motion,” are revealed [ibid., p 234]. The internal or essential characteristics of an object, objectivity in Davydov’s meaning, “[I]n contrast to the external, has existence only in a relationship, has a reflected rather than an immediate being, a being mediated in itself” [ibid., p 234].

From these quotes, one gets the sense of the centrality of the activity of labor on cognition for Marx, Engels, and subsequently Vygotsky. Also, one learns that the internal character of the object is not a direct perceptual thing but a mediated relational meaning. Finally, tools, as the means of transformation of labor, possess a central role as both means of cultural transmission and as intimately associated with the results of labor. Vygotsky wrote, “If one changes the tools of thinking available to a child, his mind will have a radically different structure” [Vygotsky, 1978, p 126]. Built from this central emphasis on tool use in two ways: 1) he argued that linguists who try to understand language development without looking at the use of tools fail to recognize the interplay between the two systems of practical activity (as evidenced by the mastery of tools) and speech; and 2) he proposed that language is itself a type of psychological tool. He wrote, “The most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity, two previously completely independent lines of development, converge” [Vygotsky, 1978, p 24].

3. Semiotics and psychological tools

Vygotsky extended the mediational role of tools to psychological tools such as sign systems (language, writing, number systems). Vygotsky saw languages as playing a special role in the development of thought.

The specifically human capacity for language enables children to provide for auxiliary tools in the solution of difficult tasks, to overcome impulsive action, to plan a solution to a problem prior to its execution, and to master their own behavior. Signs and words serve children first and foremost as a means of social contact with other people. The cognitive and communicative functions of language then become the basis for a new and superior form of activity in children, distinguishing them from animals [Vygotsky, 1978, p 28-29].

4. The dialectic of thought and language

Briefly, Vygotsky’s argument on thought and language developed as follows. Thought and language, argued Vygotsky, have separate roots. Speech, the basis for language, evolves out of gestures and affective responses. It is developed within the context of communication and social interaction. For instance, Vygotsky discussed the development of pointing, explaining it as a movement from grasping, an action, to pointing, a communicative effort to achieve the same end as grasping [Vygotsky, 1981, p 160-161]. “We consider this transitional gesture a most important step from unadulterated affective expression toward objective language” [Vygotsky, 1978, p 35]. For Vygotsky a child’s babbling, crying, even his first words, are quite clearly stages of speech development that have less to do with the development of thinking than they are a means of social contact.

Thought, for Vygotsky, especially the development of logical thought, evolves from the child’s activity—“the child’s experience with the physical properties of his own body and the objects around him, the application of this experience to the use of tools: the first exercise of his budding practical intelligence” [Vygotsky, 1962, p 46]. Thus two lines of development—of thought which is non-verbal and speech which is non-intellectual—occur and then merge around age two. Although current research on infant capacities suggests much earlier development than was posited by Vygotsky, the underlying conceptualization remains robust. At this time, we see the development of verbal thought and inner speech, and this marks the transition whereby, “The nature of the development changes, from biological to sociohistorical” [ibid., p 51].

To understand Vygotsky’s claim, one needs to recognize his intellectual ties to Hegel. Vygotsky was a dialectician—he believed that by pulling apart the distinct roots of thought and language, one could begin to understand how it is that they mutually transform each other. [4] However, in the end, he describes “the alloy of speech and action” [Vygotsky, 1978, p 30], suggesting, as does Hegel, that the dialectic results eventually in a synthesis. Thus, ultimately, Vygotsky postulated their unity.
Although practical intelligence and sign use can operate independently of each other in young children, the dialectical unity of these systems in the human adult is the very essence of complex human behavior. Our analysis accords symbolic activity a specific organizing function that penetrates the process of tool use and produces fundamentally new forms of behavior [ibid., p. 24].

Thus, in Vygotskian theory, languages, sign systems, possess a central role in the development of higher cognitive thought. In summary, he wrote that a law of development was “A sign is always originally used as a means of influencing others, and only later becomes a means of influencing oneself” [in Wertsch, 1985, p. 92]

5. Conceptual development

Vygotsky’s theory of conceptual development is probably the arena of his research program most in need of modification in current perspectives. His empirical work was based on classification tasks using what we call attribute blocks (combinations of shapes, color and size) which were classified into categories and assigned a one-syllable label. The interviewer would turn up one block and reveal its label and ask the child to put the other blocks with the same label in a pile without looking at their labels. If there were blocks that did not belong to that label category, the interviewer would turn one of those over and ask the child to continue trying. As in many similar studies, in this study the interviewer plays an active role of challenging the child, and words play a key role in the investigation.

From studies such as these, Vygotsky postulated that concepts develop via heaps (unorganized categories) to complexes (family resemblance, collections, chain complexes, and pseudo concepts) to true concepts. He warns that a pseudo concept can appear to be the same as a true concept; however, it does not hold up to intense scrutiny. For Vygotsky, the movement from pseudo concepts to true concepts requires the assistance of an adult. The process of developing a true concept comes, for Vygotsky, when the child begins to abstract out the concept.

Vygotsky argued that the way children outside the experimental setting pick up the adult use of language and use it correctly syntactically before a more complete conceptual development has been achieved is an instance of the use of a pseudo concept. This process is an essential stage of development and is based on imitation. And of imitation he wrote:

Children can imitate a variety of actions that go well beyond the limits of their own capacities. Using imitation, children are capable of doing much more in collective activity or under the guidance of adults. This fact, which seems of little significance in itself, is of fundamental importance in that it demands a radical alteration of the entire doctrine concerning the relationship between learning and development in children [Vygotsky, 1978, p. 88].

6. Learning and development

The changes Vygotsky anticipated in the relationship of learning to development included postulating an interaction between the two. In Mind and society, he soundly criticized methods of “concrete, look-and-do methods” that were used with retarded students and reinforced their handicaps by “accustoming the children exclusively to concrete thinking and thus suppressing the rudiments of abstract thought” [ibid., p. 89]. He criticized any instruction that lags behind development and argued, “The only good kind of instruction is that which matches ahead of development and leads it; it must aim not so much at the ripe as at the ripening functions” [Vygotsky, 1962, p. 104].

Zone of proximal development

For Vygotsky, imitation is not a simple or mechanical process. He claimed, “To imitate, it is necessary to possess the means of stepping from something one knows to something new. With assistance every child can do more than he can by himself” [ibid., p. 103]. This view of a central role for imitation led Vygotsky to argue for “a zone of proximal development.”

As is well-known, the zone of proximal development is the territory between those tasks which a student can undertake successfully independently and those which require the assistance of an adult. Built from Montessori’s notion of a “sensitive period,” Vygotsky’s conception stressed the importance of adult-child interactions to propel development along.

Brown and Ferrera [1985] offer a description of the interactions between adult and child in the zone of proximal development:

Vygotsky’s theory of cognitive development rests heavily on the key concept of internalization. Vygotsky argues that all higher psychological processes are originally social processes, shared between people, particularly between children and adults. The child first experiences active problem-solving activities in the presence of others but gradually comes to perform these functions independently. The process of internalization is gradual; first the adult or knowledgeable peer controls and guides the child’s activity, but gradually the adult and the child come to share problem-solving functions, with the child taking the initiative and the adult correcting and guiding when she falters. Finally, the adult cedes control to the child and functions primarily as a supportive and sympathetic audience. Teachers, tutors, and master craftsmen in traditional apprenticeship situations all function ideally as promoters of self-regulation by nurturing the emergence of personal planning as they gradually cede their own direction [p. 282].

Extensional work by Wood, Bruner and Ross [1976] has argued that “Tutorial interactions are, in short, a crucial feature of infancy and childhood” [p. 89], and has described the concept of a scaffolding process “that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts” [p. 90]. They stress, however, that scaffolding requires that a learner must be able to recognize a solution to a particular class of problems before he is himself able to produce the steps leading to it without assistance” [p. 90]. It is also important to point out that the majority of
the tasks in which a scaffolding process has been examined entail such activities as puzzle solving, pattern completion, or blocks construction.

**Scientific and spontaneous concepts**

According to Vygotsky [1962], schooling is where children are introduced to scientific knowledge. Scientific knowledge in Vygotsky is not so much knowledge of science as it is systematic or taxonomical knowledge. This knowledge is contrasted with spontaneous knowledge which is the knowledge a child develops in “face-to-face” meetings with a concrete situation. Vygotsky writes that “the development of the child’s spontaneous concepts proceeds upwards and the development of the scientific concepts downwards” [p 108], although he recognizes:

... [In working its slow way upward, an everyday concept clears a path for the scientific concept and its downward development. It creates a series of structures necessary for the evolution of a concept’s more primitive, elementary aspects, which give it body and vitality. Scientific concepts in turn supply structures for the upward development of the child’s spontaneous concepts toward consciousness and deliberate use; and spontaneous concepts grow upward through scientific concepts” [ibid., p 109]

He suggests that the development of scientific concepts “usually begins with its verbal definition and its use in nonspontaneous operations—working with the concept itself.” With the spontaneous concept “he knows the object to which the concept refers. but is not conscious of his own act of thought” [ibid., p 108]

**The contributions of the Vygotskian perspective**

Vygotskian theory makes a number of significant contributions to our understanding of intellectual development and mathematics education. In particular it draws our attention to the larger social structures in which educating is embedded. For instance, with mathematics, a socio-cultural perspective encourages us to consider the role of tools in mediating knowledge is a clear focus of Vygotskian theory. He writes:

...human psychology is concerned with the activity of concrete individuals, which takes place either in a collective—i.e., jointly with other people—or in a situation in which the subject deals directly with the surrounding world of objects—e.g., at the potter’s wheel or the writer’s desk. ... if we removed human activity from the system of social relationships and social life, it would not exist. the human individual’s activity is a system in the system of social relations. It does not exist without these relations [Leont’ ev, 1981, cited in Cole, 1985 p 151]
son and interplay between the two characterizations. For instance, in a classroom of third graders, the two problems were given: “How would you share 696 jelly beans among three children?” and “How would you share 174 jelly beans between two children?” In formal mathematics the two problems are considered equivalent, and yet in the school setting, where the children were just learning about division, the problems worked out very differently. Some children chose to solve the problem with Dienes blocks, and in this case most did the first problem by splitting up among three: 6 flats, nine longs and six individual blocks. In the second case they proceeded similarly, representing 174 with one flat, seven longs, and four blocks. However they faced a problem about how to share the flat (10 by 10) among three people. Some did this by segmenting 99 into 33 3’s, and adding the one to make 75 blocks, which they shared by dividing six longs into groups of two and trading in the final long for 15 blocks shared into piles of 5. A few did the problem by finding 3 50’s in 174 (not so easily seen with Dienes blocks) and then sharing 24 among three. Some did the problem by switching the flat to 10 longs and then sharing seventeen longs and four individual blocks among three. In the group discussion, a term, “nice parts”, emerged as a useful description of how the children chose to segment the whole quantity for easier distribution.

Another group of students tried these problems using short division (no division algorithm had been taught to them in school, but some parents had introduced short division to the students for an earlier problem). The first problem, 3 divided into 696, yielded easily to the method, but the second problem, 2 divided into 174, did not. Many of these students, having abandoned the Dienes blocks on the first problem, were not able then to use them successfully to solve the second problem.

On reflection, it seemed to me that a tool-language distinction and dialectic is useful in interpreting this. The students who used the Dienes blocks were working in ways that were more immediately connected to their use of physical tools. The distance between their solution strategy and the problem interpretation was minimal, as evidenced by their easy explanations (“nice parts”) and demonstrations of the methods. The students who used the division algorithm were working with a symbol system whose manipulation and use was more consistent with formal language use. The method was taught to them and they were imitating it. When imitation failed they had few options to resort to, and a return to the materials was not easy for them because the material methods did not map easily onto the method of short division.

Vygotskian theory helps one to: 1) recognize the legitimacy of both methods; 2) draw distinctions between the methods based on their character as psychological or as physical tools and; 3) consider how the symbolic (and semiotic) method drew on an adult perspective and created a “pseudo concept” that was useful in certain situations, but could impede progress in others. However, contrary to constructivist interpretations, the socio-cultural perspective asserts a value in having students in the class who could introduce the expert method into the classroom discussion, because it created for all the children a challenge to progress towards long division, a method they had heard about from older siblings and parents. And, it asserts the value of imitation. “Can we figure out a way to do the problem 2-divided-into-174 without the blocks?” It seemed that this was probably an example of a situation in which learning was leading development in a very significant and appropriate way towards long division.

Limitations of Vygotskian theory
As with constructivism, in discussing the limitations of Vygotskian theory, it is difficult to distinguish when the criticisms are of the theory and when they indicate a limited interpretation of the theory. And especially in considering a theory which, by its own principles, establishes cultural and historical precedents, one expects to have to reconsider a theory’s interpretation in the light of the current circumstances. In mathematics education Vygotskian theory has the following limitations:

1. **Vygotskian theory may encourage the neglect or devaluation of concrete activity**

Vygotsky argued for two roots of conceptual development, thought and language. These two strands interweave to lead to mature human development. Neither thought nor language is proposed as superior to the other, except that Vygotsky asserts that it is the introduction of the social-language component that creates the possibility of higher cognitive thought and differentiates humans from other animals. And since Vygotskian theory introduced the role of language in guiding intellectual development, empirically it focused on language. In fact, Vygotsky [1978] argued for a predominance of speech over activity in the more developed forms:

> Initially speech follows action, is provoked and dominated by activity. At a later stage, however, speech is moved to the starting point of an activity, a new relation between word and action emerges. Now speech guides, determines and dominates the course of action [p. 28].

In developing the dialectic between thought and language, Vygotsky also introduces the idea of language as a psychological tool: one can use it to influence another. One can interpret Vygotsky as valuing both kinds of tools, but his description can also communicate that it is symbolic activity as a psychological tool which penetrates its use as a physical tool and not vice-versa. That this results in a privileging of abstract sign use over functional practical intelligence can be seen in Vygotsky’s original work and can be seen in his work with Luria where cross cultural studies were investigated. In his categorization scheme, where subjects were given a hammer, a saw, a log, and a hatchet and asked to say which three belonged together, he and Luria devalued the response linking a saw, a log, and a hatchet.

Wertsch indicated that non-literate populations tended to resist the experimenters’ “suggestions grounded in decontextualized word meanings an hierarchical relationships among them” [Wertsch, 1985, p 34]. Again, when given a glass, a saucepan, spectacles, and a bottle, the grouping...
together of the glass, the spectacles, and the bottle was valued because they are made of glass, whereas putting the glass, the saucepan, and the bottle together on the basis of practical experiences of these as containers was devalued [5]

Later Vygotsky [1978] rejected any ways of teaching retarded children which relied exclusively on concrete methods. While rejecting an approach which eliminated abstract approaches in their entirety as inconsistent with the dialectic, ultimately he argued for only a weak inclusion of the concrete: "Concreteness is now seen as necessary and unavoidable only as a stepping stone for developing abstract thinking—as a means and not an end in itself" [p 88]

It appears that within Vygotsky there is an inherent tension between his dialectic Hegelian approach and his Marxist roots. As a Hegelian idealist, Vygotsky was raised as an intellectual, and a preferential valuing of intellectualism emerges intermittently.

2 Advocates of Vygotskian theory may focus on and privilege language to the detriment of other forms of intellectual interaction and inquiry.

In mathematics, however, and in the sciences, educationally, we see a tendency to give definitions as though definitions were a sufficient guide to intellectual development. As Vinner [1983] has demonstrated, such a basis for guiding mathematical development has proven inadequate. In research on 10th and 11th grade students in Jerusalem, 88% of the students could state the definition of function but only 34% of those students acted accordingly.

However, Vygotsky clearly doesn’t suppose words to be weakly connected to concepts, given statements of his such as:

There is every reason to suppose that the qualitative distinction between sensation and thought is the presence in the latter of a generalized reflection of reality, which is also the essence of word meaning; and consequently that meaning is an act of thought in its full sense of the term. But at the same time, meaning is an inalienable part of word as such, and thus it belongs in the realm of language as much as in the realm of thought. A word without meaning is an empty sound, no longer a part of human speech. Since word meaning is both thought and speech, we find in it the unit of verbal thought we are looking for [1962, p 5].

The implications of such a quote are that, in interpreting Vygotsky, we must insist on the injection of meaning into our discussion of words and recall that the meaning comes from the dialectic between thought and language, and not as some have argued from language alone.

Social constructionism, not to be confused with radical constructivism or social constructivism [Bauersfeld, in press] or with constructionism [Harel, 1990] is a theoretical perspective expressed by Gergen [in press] and Shotter [in press] which is closely allied with Vygotskian perspective but which rejects the psychological dimension of Vygotsky. Vygotsky's interest, according to Gergen, is described as being "mental processes of abstraction, generalization, comparison, differentiation, volition, consciousness, maturation, association, attention, representation, judgment, sign-mediated operations, and so on" [p 10] whereas Gergen is described as being interested in "negotiation, cooperation, conflict, rhetoric, ritual, roles, social scenarios, and the like" [p 10]. In social constructionism, language and language games are presumed to make up the whole of knowledge, as witnessed by the following quote from Gergen [in press]:

... there is nothing about the nature of the world that demands, requires, or necessitates any particular linguistic representations. In principle, then, we are free to use whatever configuration of sounds and marking we please on any particular occasion. In principle, this is no more a table before me than it is Gouda cheese or a griffin. In practice, of course, we are not free. By virtue of negotiated agreements, widely shared within the culture, we agree to speak of it—dully, perhaps—as a desk. Or to put the conclusion more bluntly, all that we take to be the case—our propositional representations of everything from physics to psychology, geography to government—gain their legitimacy not by virtue of their capacities to map or picture the world, but through processes of social interchange [p 9].

If Vygotskian theory is used to draw attention exclusively to the verbal or written language of social interactions, then its influence on mathematical development will be detrimental. Furthermore, even if "languaging" is used more broadly to include figures and drawings, and symbolic notational systems, and computer software which creates semiotic microworlds, the absence of connections between these uses of language and physical activity will serve to diminish children's mathematical development.

The contributions of each part to the dialectic must be made truly equal, with both acting as guides to each other.

3. The socio-cultural perspective has the potential to reintroduce formalism into mathematics.

This is because Vygotskian theory posits a discontinuity between spontaneous and scientific concepts. This discontinuity is argued to be due to the systematicity of scientific concepts, and to their suspension in an orderly, logical set of relations. Bridging that discontinuity to bring students into alignment with scientific perspective is assumed to be the responsibility of a more expert "other", who does this by appropriating the student's goals until they mirror the goals of the adult. This description is arguably close to mathematics education as it it taught at the university. There a student is presented with theorem-proof sequences ad nauseam until the student loses his/her initial perplexity about the value of the enterprise and appropriates the goals of the teacher and becomes a producer of proof by him/herself. Without a more explicit theory of learning, the Vygotskian perspective cannot distinguish between teacher-student interactions that will lead to pseudo concepts and those which will lead to conceptual development.
4. The Vygotskian perspective avoids critical examination of mathematics itself

Many studies in the Vygotskian tradition focus on how the more capable other assists the novice to become able to independently to carry out a task. Examples from mother-child interactions include puzzle-building [Wertsch, 1985] peek-a-boo, and picture book reading [Forman and Cazden, 1985] In these cases the goals of instruction are typically well-defined and agreed upon. Imitation is viewed as an appropriate means to accomplishing the ends. There is little assumption that the ends themselves might need revision. In mathematics education we have argued, however, for the importance of reconsidering the outcomes of instruction. From close listening to students we have revised our understanding of mathematics. There is little or no provision for such activity within the Vygotskian framework.

5. The Vygotskian perspective can limit rather than promote and protect diversity in a classroom

In discussing tutoring from a constructivist perspective, we find authors such as Arcavi and Schoenfeld [in press] warning of the dangers of pursuing student methods. In this paper they document a student’s method for finding the equation of a linear function from a table of values in a way that deviates from standard procedure but which is considered legitimate by a number of mathematicians. Because the student is initially introduced to equations of the form $y = x + b$, the student establishes an approach involving subtracting the $x$ and $y$ values. However when faced with an equation of the form $y = mx$, the student’s method leads to a linear pattern in the differences, rather than a constant difference. The authors recognize the legitimacy of the student’s approach, but they warn about placing undue stress on tutors’ knowledge and judgment, and of “keeping the student on shaky or superficial ground.” As a result, they question whether or not one should accept and pursue a student’s method. I would argue, however, that their analysis of the student’s method was unduly complex, relying on algebraic notation to reach the equation. A graphical approach makes it easier to see why $y = x + b$ gives a constant difference between $y = x$ and $y = x + b$, whereas the multiplicative term $y = mx$ gives an arithmetic progression as a difference.

Not all Vygotskian perspective leads in the direction of the suppression of diversity. The work on multiculturalism [Vera John-Steiner, 1985, 1991] and literacy has been used for exactly the opposite purpose. In mathematics education, however, the tendency to use Vygotsky to reinforce rather than to challenge the uniform presentation and development of mathematics points to a limitation in the theory. Perhaps it is because so many people ascribe a universality to the language of mathematics that it is quickly challenged from a multi-cultural perspective.

FORGING A REVISED THEORY

When one considers the theories of Piaget and Vygotsky, one sees places in which their views conflict and where they complement each other. Drawing on their complementarity while trying to consider their conflicts, I will propose an outline of a revised theory of intellectual development. As will become evident, I draw upon feminist scholarship in proposing these revisions.

Genetic epistemology

Both theories share a commitment to the evolution of thought—and the need to look at the process of development to understand a current state of affairs. Both theorists possessed a keen interest in phylogeny (historical development) as well as ontogeny (lifespan development), and although neither explicitly argues for a recapitulation theory, they both use historical analysis to enlighten their examination of development.

The implications of applying a genetic epistemological approach to mathematics and/or mathematics education are profound. One no longer accepts formal logical relations as sufficient warrant for an idea, but instead traces the route of its development over time and over place. Genetic epistemology brings one squarely into the Lakatosian territory of proofs and refutations, where the logic of discovery, the process of acceptance or rejection by the discipline and the logic of justification are all equally explored. Knowledge is not established by examining the immediate facts of the case, but by examining the position of a claim within a theory, by considering its contextual roots and its path of transformation.

For example, an introduction to $pi$ might not only include calculations of its value and examination of its use in calculating the area and circumference of a circle, but could explore the genesis of $pi$. In the current introduction, students are taught that $pi$ is a constant and, for all circles, the circumference equals $2\pi r$. Quickly the discussion turns to numerical approximations for $pi$. Little or no explanation of the identification and development of $pi$ is offered.

An approach through genetic epistemology might begin with the recognition that all circles are similar. Consider the claim that all squares are similar; we find little surprise in an argument asserting that the ratio of the perimeter to a side is 4:1 for all squares. Would not an important step in a genetic epistemological presentation of the ratio of the circumference to the diameter (a ratio we label $pi$) be to generate the expectation that this ratio should be constant for all circles?

Not all assertions of genetic epistemology are compatible, however, and the distinctions need examination. For instance, Lakatos’s rational reconstruction has inherent in it some remnants of Hegelian idealism: Lakatos (1976) writes:

Mathematics activity produces mathematics. Mathematics, this product of human activity, "alienates itself" from the human activity which has been producing it. It becomes a living, growing organism, that acquires a certain autonomy from the activity which has produced it, it develops its own autonomous laws of growth, its own dialectic. The genuine creative mathematician is just a personification, an incarnation of these laws which can only realize themselves in human action. Their incarnation, however, is rarely perfect. The activity of human mathematicians, as it appears in history, is only a fumbling realization of the wonderful dialectic of mathematics ideas [p 146].
Lakatos' editors include a footnote suggesting that Lakatos might have modified this statement somewhat had he the opportunity to edit the section himself; however they also acknowledge that Lakatos did want to admit an "existence" to problems which is independent of human recognition.

Likewise, in discussions on radical constructivism, there is occasionally an ambiguity about whether what is evolving and subject to adaptation and equilibration is a system of ideas or a person's conceptual structures. "Viability" as a quality of endurance, adaptation, comprehension, and survival is applied sometimes to the idea itself, as a product of human activity, and at other times, it becomes a component of intelligence as evidenced in human action. This is particularly an issue in mathematics where research often focuses on the evolution of a mathematical concept as an isolated entity.

To illustrate this, let's return to the example of pi. Our analysis has led us to explain, first, why one would believe that pi as a ratio applies to all circles. However, it still ignores the question of why one would seek out pi. And it neglects the fact that, for the Greeks, pi was not a number. The history of number and the history of geometry evolved separately, so that pi was originally a ratio that expressed the invariance across proportions such as: C:R and C^2:R^2 (where C symbolizes circumference and R radius). Finally, our analysis avoids the discussion of why one might wish to find a way to describe that ratio differently than as the ratio of circumference to diameter. The answer lies in two types of cultural activities. The Greeks sought to rectify arc length; that is, to find a straight line equivalent to a given arc; that is, to measure circumference using straight lines. Not only could one thereby be able to predict the circumference given a radius, but one could predict the length of the path of a rolling object. If one creates a mechanical device by hooking up a linkage to a wheel, one needs to be able to move easily between measures (or movements) of curvature and measures (movements) of line segments. One sees in this discussion an even deeper application of genetic epistemology, one that places the investigation squarely within the productive activities of a culture.

The socio-cultural perspective avoids detaching conceptual development from the knower and eradicates the vestiges of Platonic idealism by rejecting the idea of universal concepts. It can be used to legitimate the idea of mathematics as differing across cultures and across time. So the genetic epistemology one seeks to include in a revised theory needs to allow for competing, independent strands of development, as well as long-term trends that are well-integrated into the cultural activities of the time.

In any view of genetic epistemology, one must decide one's position on what constitutes progress. In this revised theory I would argue for a coherent view of progress that is local (both geographically and temporally) but not universal or eternal. To understand conceptual change over longer time periods, one must posit a view of paradigmatic shifts and examine how they come to degenerate, to compete, and to replace or be replaced among communities of scholars.

**Paradigmatic metaphors**

Margaret Masterman [1970] has argued that one way to capture the sense of a paradigm is to examine its metaphorical commitments. Metaphors have the charming quality of drawing upon elements of tools and physical interactions and on elements of language and the turn of a phrase. The metaphors used by Piaget and Vygotsky present an interesting contrast. Piaget, being a biologist (as well as a developmental psychologist, a philosopher etc.) chooses to rely on the metaphor of evolutionary biology to explain the development of knowledge—and his processes of assimilation, accommodation, equilibration, etc., are woven into this metaphor. The human being is viewed as a self-organizing organism who regulates her/his behavior by acts of problem solving to restore equilibrium. Von Glasersfeld has also committed himself to this metaphor by emphasizing the role of viability in explaining the durability of knowledge. Humans seek to gain prediction and control of their environment, and those schemes that produce these outcomes become stable; those that do not, fall away.

This is a powerful metaphor for it removes one quickly from the assumption that what is known exists independently from the knower, and that knowledge remains eternal. It allows for a pragmatic view of knowledge as establishing specific structural and functional relationships, and limits the relativism by placing it in a larger system of evolution. The metaphor creates powerful ties between one's interactions with one's environment seen as a system of constraints. Finally, the metaphor of evolutionary biology encourages one to treat development as a metamorphic process of changes and hence to observe carefully how a child at any particular development period behaves in relation to his or her environment.

Using evolutionary biology as a metaphor makes examination of the metaphor itself typically problematic. Our attention is drawn to the constructs in the metaphor while the metaphor itself is taken for granted. This leads us to neglect the fact that evolutionary biology is itself a historical/cultural artifact. It can distract our attention from many issues that feminists have recently demonstrated: for instance, how evolutionary biology has often ignored the role of cooperation in the evolutionary process. The issues of sociobiology have only entered biology slowly as evolutionary theory and its difficulties in explaining behaviors which are detrimental to the survival of the individual but valuable for the group. This concentration on the individual as a microcosm of the species has tended to turn attention away from collective behavior. We can avoid taking the metaphor for granted by placing it in dialectic with other metaphors.

In contrast to the Piagetian metaphor, in Vygotskian theory we find the Marxist construction of knowledge through labor and production as the primary metaphor. It is through human labor, and the use of tools to create products, that knowledge is constructed. Vygotsky [1978] explicitly refers to Marx, stating, "Marx cites that definition when speaking of working tools, to show that man uses the mechanical, physical and chemical properties of objects so
as to make them act as forces that affect other objects in order to fulfill his personal goals” [p 54] Knowledge, according to such a conception, describes the invariances found as a result of transforming products through labor Vygotsky described his commitment to this approach:

The keystone of our method follows directly from the contrast Engels drew between naturalistic and dialectical approaches to the understanding of human history Naturalism in historical analysis, according to Engels, manifests itself in the assumption that only nature affects human beings and only natural conditions determine the historical development The dialectical approach, while admitting the influence of nature on man, asserts that man, in turn, affects nature and creates through his changes in nature, new natural conditions for his existence This position is the keystone of our approach to the student and interpretation of man’s high psychological functions and serves as the basis for the new methods of experimentation and analysis that we advocate [ibid., p 60-61]

A commitment by Vygotsky to historical materialism entails not only an interaction between man and nature, but also a subjugation of nature to man’s control and domination Thus, Engels’ concept of human labor and tool use as the means by which human beings change nature and, in doing so, transform themselves is reflected in Vygotsky Cole and Scribner [1978] acknowledge this relationship and quote Vygotsky:

Vygotsky exploits the concept of a tool in a fashion that finds its direct antecedents in Engels: “The specialization of the hand—this implies the tool and the tool implies specific human activity, the transforming reaction of man on nature, “the animal merely uses external nature, and brings about changes in it simply by its presence; man, by his changes makes it serve his ends, masters it This is the final distinction between man and other animals” [in Vygotsky, 1978, p. 7]

The implications of this underlying metaphor are both positive and negative It is this metaphor that Vygotsky extends to assert that language and other semiotic tools are also subject to historical and cultural forces of change The inclusion of semiotic tools as psychological tools also allows Vygotsky to suggest that labor and production create the basis for consciousness

A labor and production metaphor permits Vygotsky to reexamine development and assert that a classic misconception was to liken development to embryology, and thus to assume the presence of all adult characteristics in embryonic form His rejection of biological maturation as the basis for development was essential in establishing a central role for social interactions Recall that the major goal of Vygotsky is to distinguish humans from other animals—and it is “doing”, replacing biological metaphors with labor and production, that allows one to escape from the determinism of biology to the social constructivism of activity theory [6] His choice of a focus on language, assumed at the time to be a uniquely human accomplishment (except in parrots who can only imitate) confirms and supports his choice of metaphor In the Vygotskian metaphor, it is human society and culture that create the knowledge that constitutes the records that we choose to pass on as received knowledge Our skills in this set us apart from other species As expressed by Wood, Bruner and Ross [1976], in the article that introduces scaffolding, “Our species, moreover, appears to be the only one in which any “international” tutoring goes on” [p. 89]

These shifts entail powerful accomplishments; however, they also contribute to a view of Vygotskian theory which accepts and promotes the differentiation of humans from nature Human beings are assumed to be not quite, but almost, outside of nature, and their capacity for social organization and for teaching is assumed to far exceed that of any other species Recent findings in sociobiology challenge such assumptions And the extensive damage humankind has exerted on the environment has resulted in an increased awareness that man cannot consider the environment to be casually subject to his force and transformation, but must learn to view the environment as a resource to be preserved and valued If one simply contrasts the word “resource” with the Marxist view of “material”, one captures the difference in spirit between these two metaphors of evolutionary biology and labor/production

These two theories, then, express two powerful metaphors for understanding humanity and for modeling and investigating human development We experience ourselves both as biologically developing beings and as productive members of a collective enterprise Hence neither can be eliminated from our consideration It seems obvious that we must consider both when we study our view of humanity and its development

This suggests that both are necessary, and that the challenge is to integrate them I would like to suggest that what has been typically done in debate in mathematics education is to view Piaget’s as an individualistic theory and Vygotsky’s theory as a theory about society Both descriptions seem woefully inadequate and simplistic Neither description does justice to the strength of the insights of the theorist

Alternatively, I would submit that both theories lack attention to a fundamental characteristic of human development that allows one to avoid placing the individual in tension with the social This missing component is the role of reproduction I claim that neither theory pays adequate attention to the importance of nurture and reproduction in human development

First of all, the labor and production metaphor focuses on how knowledge evolves from the transformation of objects through the use of tools in the activity of labor This seems clearly an insufficient description of parent-child interactions, where knowledge is achieved through the guidance of children into adulthood The metaphor of reproduction that I am suggesting extends far beyond the act of birthing, just as Vygotsky extended the idea of tool beyond the physical tool to include psychological tools Feminist scholars such as Jane Rowan Martin [1985] have argued for the importance of a broader definition of reproduction, writing: “Discussions about marriage, home, fam-
ily are missing, as are discussions about society’s reproductive processes—a category I define broadly to include not simply conception and birth but the rearing of children to more or less maturity and associated activities such as tending the sick, taking care of family needs, and running a household” [p. 6].

By this argument, education can be viewed as a nurturing process as much as it is a preparation for work. Caring for the development of the child is more than developing work-related skills; it is the nurturing of a curious, creative, well-adjusted child, capable of responsible and satisfying interactions with others. It is ironic that to date in education (outside of feminist scholars such as Noddings, 1984; Martin, 1985; Laird, 1988), we have appropriated the language of reproduction (conception, development, reproduction), but severed its ties to nurture and care. We have coined a term called “social reproduction” and used it to describe the unconscious replication of cultural norms across generations. In my use of reproduction, I seek to disengage if from its implication of unconscious duplication and to establish its ties to nurturing and growth.

Those who had the opportunity to hear Davydov [1993] speak at the national meeting of the American Educational Research Association in Atlanta, heard a description of Vygotsky in which nurture was referred to frequently. This term is absent from the translations of Vygotsky that are current in English. Davydov attributed his use of nurture to a newly discovered publication of Vygotsky that had been censored. One also heard in this presentation few references to the Marxist roots in Vygotsky, which emphasized far more his connections to Hegel and the dialectic. These changes in presentation, whether due to the release of new work, or to the dis-integration of the Soviet Union and its rejection of Marxism, may signal changes in the interpretation of Vygotsky in directions compatible to those which I am proposing. Vygotsky, I am suggesting, recognized the importance of adult-child interactions. However, his reliance on the Marxist metaphor limited the scope of those interactions to those or production; I am arguing for its expansion to include also the metaphor of reproduction.

Furthermore, I would argue that despite Piaget’s emphasis on evolutionary biology, his treatment of the territory signaled by the metaphor of reproduction is also inadequate. First, he largely neglects many forms of adult and child interaction in development, including imitation. Second, he follows Kant in arguing that one’s understanding of space, time, number, and causality creates the fundamental fabric of cognition, and as such he elevates mathematics and physics to the highest plane, ignoring the importance of human connections. Third, he models mind as the embodiment of abstract mathematical structures. And, finally, as a result, a child’s understanding of reality emerges from its formal, logical characteristics, rather than from the simultaneous construction of conceptions of a variety of human and non-human living beings.

The model that I would put forth holds evolutionary biology as the umbrella metaphor, to signal humanity’s placement in the full ecology of the earth. To deal with knowledge construction and accomplishment, the model is placed within the framework of genetic epistemology. Subsumed within the evolutionary biology metaphor are the two metaphors of labor/production and reproduction in dialectic relation. Each of these sub-metaphors has a feedback loop to the evolutionary metaphor to signal the need for critical examination of this umbrella metaphor from the perspective of the sub-metaphors. For instance, an examination of whether the mechanisms of selection and variation are sufficient to account for human development is encouraged by such a feedback loop. See Figure 4.

![Figure 4](image-url)

**Notes**

[4] A dialectic should not be confused with a dichotomy. A dichotomy implies polarization, conflict in the two positions, and the need for choosing one or the other. A dialectic is a talking-back-and-forth

[5] Besides the bias against the functional qualities witnessed in these passages, we see repeated passages in Vygotsky that refer demeaningly to " primitives," children, and mentally handicapped people. He was progressive for his time; however, interpretations of this theory must include careful attention to modifying the theoretical assumptions that accompany these beliefs.

In addition, it should be carefully pointed out that in Vygotsky’s times, with the introduction of literacy cultures in which physical labor predominated, the penetration would have been from language into the use of physical tools. However, I suggest that in the present time, when communication devices dominate so much of human interaction and computers are increasingly used to represent physical activities (computer games, simulations, computer design, etc.), there is a need to see how the physical uses of tools can penetrate and guide language development. This is particularly the case in mathematics.

[6] This use of biology seems to be deterministic and naturalistic rather than developmental or ecological.

**References**


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