Communications

Should a mathematics teacher know something about the history of mathematics?

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My response to Professor Freudenthal's question (FLM,2,1) is conditioned by battles fought over the time available, and the feelings of inadequacy about a teacher-of-teacher's knowledge of the history of mathematics.

My reasons for wanting students training to be teachers to study the history of mathematics are two-fold. First, as one strand in the attempt to get them to reflect on the nature of the subject they will teach It remains true that the personal experience of many students up to the time of entry into training has produced a static "body of knowledge to be learnt" view of mathematics, reinforced by the style of many textbooks So we ask them to undertake mathematical investigations at their own level throughout their four-year course and to reflect on this experience. In this way they try to formulate, solve and extend problems, invent notations and proofs, and find themselves in blind alleys. But some still think that the "real" mathematicians who invented the mathematics in the textbooks did not share similar experiences. It is at this point that studying a limited number of historical source papers — which may have errors in them — can finally convince them that mathematics is a human activity after all, and pose seriously for them the question of what kind of mathematics teaching they should aim for in school.

One example which I have found makes an impact is Cayley's role in the history of matrices. In two excellent papers Hawkins [1975, 1977] has uncovered the problem background and shown that at least five people invented matrices more or less independently Though Cayley was more influential through the problems he tackled than in his use of matrix symbolism it is nevertheless fascinating to see the development of his use of the symbolism by looking at five of his papers. In the first, written in 1846 at the time he was learning about determinants from Cauchy, we see him working out the inverse of a matrix by what we would call the method of transposed co-factors [Hohn, 1964]. But since he does not have a well-formulated notion of a matrix he has to use suffixes in his summations to show whether he is multiplying on the left or the right. It is an impressive tour-de-force of manipulative skill. Nine years later we find a brief note introducing matrix notation, as well as another paper reworking the problem of his 1846 paper using the new notation. In 1858 comes the expository paper which for years led to what Hawkins calls the "Cayley as founder" view, as if he had started at this point. (The students enjoy Cayley's attitude to proof with his one example of the characteristic equation in the 2×2 case.) In the same year he is using the condensed letter notation P, P^{-1} for matrices in further problem solving.

The second reason for my wanting student teachers to study the history of mathematics is one which has been felt more urgently by both students and myself the further we have travelled from the curriculum changes of the early sixties and as they are increasingly under review Without some outline knowledge of history, they cannot understand the issues about curriculum content as it concerns geometry, functions, and modern algebra The history of the function concept is fairly well documented [Youschkevitch, 1976; Monna, 1972] and the realisation that in a traditional secondary school course the concept of function is the one presented by Euler in 1745 shakes the students. The subsequent history of the concept is central to the arguments from the 60's for a change in the treatment of functions, even if its implementation seems largely to have failed! Likewise some sketch of the development of modern algebra in the 19th century is needed in order to make sense of the arguments about that area A knowledge of the history of geometry raises questions about the school geometry curriculum much more deeply than any other way I know

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