THE DIAGRAM AS STORY: UNFOLDING THE EVENT-STRUCTURE OF THE MATHEMATICAL DIAGRAM

ELIZABETH DE FREITAS

The mathematical diagram is an odd creature. For many, it is an obtuse image—silent, still, and recalcitrant, as though its encoded meaning entailed a "speech which holds its tongue" (Rancière, 2007, p.11). It is neither an image that bears testimony to history, nor a pure object of art. Consider the diagram in Figure 1. It appears to be located in a plane of virtuality and yet it is present to (and for) the human eye. It sits static on the page, denying all temporal contingencies. Its lines lack all sign of motion, as though impervious to narrative, action and event. How does one story the diagram? How does the diagram trace the actions of its maker? What happens when we take up and tell the diagram as though it were an unfolding narrative? How would we take up the diagram in Figure 1 as a series of actions?

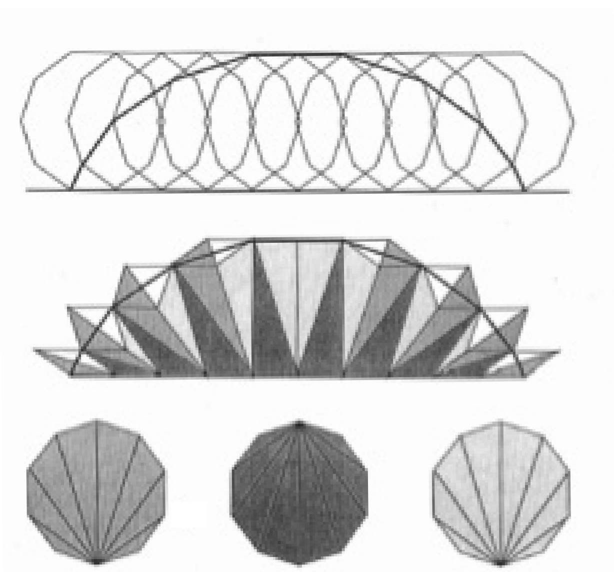

Figure 1. A mathematical diagram (from Nelsen, 2000, p. 31). Image © Mathematical Association of America 2012. All rights reserved.

In this article, I focus on the complex intersection of story and diagram as a way of troubling our assumptions about the fixed and static nature of diagrams (to thereby set the diagram in motion) and as a way of exploring new ways of engaging students in mathematical diagrams. Does storying the diagram animate the diagram and embolden the narrator? Might such an approach help students construe the diagram as an event and themselves as actors or agents? Does the mathematical diagram come alive for the student by being situated in the messy world of events? And more broadly, what are the consequences of temporalizing the diagram as an event? What might be lost by imposing a narrative structure onto these images?

Although Solomon and O'Neil (1998) have argued that "mathematics cannot be narrative for it is structured around logical and not temporal relations" (p. 217), Healy and Sinclair (2007) have shown how students leverage stories as a means of making sense of dynamic visual representations, recounting how students "flesh out" the characters involved (parts of the dynamic image) and attribute purpose to them while narrating their relationships. Others have shown how mathematicians frequently talk as though they are animating or personifying the mathematical entities with which they are engaged (Hofstadter, 1997; Sfard, 1994). This work seems to suggest that narratives do indeed function as a means of accessing the logical structure of mathematics, although there remain many unanswered questions about how narratives perform this function. This paper takes up these questions and examines *static* mathematical diagrams that appear to have no narrative or event structure at all. In the study recorded by Healy and Sinclair (2007), the *dynamic motion* of the visual images is cited as a likely cause for the students' sequential and dramatic recounting. In this article, instead of explicitly dynamic diagrams, I explore how the *implicit* motion and *tacit* event-structure of static diagrams might be accessed through narrative. I focus specifically on two fundamental facets of narrative: (1) the recounting of causally related sequences of events, and (2) the positioning of the narrator through point-of-view and voice. I will argue that both of these facets are highly relevant to diagramming.

To illustrate these ideas, I discuss video data in which a mathematician engaged with a set of three diagrams through gesture and speech when answering the following questions: where is the beginning and the end of this diagram? What sort of motion, action or unfolding do you see in this diagram? How would you tell the story of this diagram? Where are the agents in this story? The three mathematical diagrams, including the one shown in Figure 1, were selected from *Proofs Without Words* (Nelsen, 2000) because they were free of notational and indexical labels, seemed to be relatively abstract compared to graphical renderings of physical processes, and covertly conveyed a mathematical relationship that might have been interpreted as their intended meaning. Indeed, the three diagrams are considered by Nelsen and others to be visual proofs of particular mathematical relationships and, as such, they can be verbalized in terms of a sequence of logical inferences. This decoding of the immediacy or all-at-once nature of the diagram in terms of an inferential sequence can also be construed as a temporal decoding if the diagram is seen as an event—in other words, if the diagram is decoded in terms of a sequence of constructions, elaborations and embodied actions. Whether these diagrams constitute proofs in themselves, or are simply visual aids to the algebraic or other mathematical expression they inspire, is not the issue I wish to discuss [1]. My aim is to rethink the nature of diagrams and their role in mathematics, and to examine the ways in which the language of time and event might be re-introduced in the study of mathematical proof and explanation.

Inscriptions

The drawn line—short, long, dashed, solid, curved, dented, bumpy, jagged—is the constitutive element of the mathematical diagram. Like other drawings, or visual displays perceived as drawings, the diagram is structured through the varied use of the line. The line can separate one region from another, determine perspective, location and orientation, point to depth and other dimensions, designate a congruency or symbolize infinite extension. Implicit in our perception of the line diagram, unlike that of a photograph or other imprint technology, is an awareness, whether wrongly presumed or not, that a hand has drawn these lines and made these marks, and that it did so by tracing a point across the surface. The act of drawing occurs over time; in the drawn line we see a mark that records a two-dimensional movement in space:

> Drawing is done with a point that moves [...] a tool acting as some kind of surrogate for the hand with its fingers, has made a mark that records a two-dimensional movement in space [...] such movement is the fundamental nature of drawing [...] (Rawson, in Maynard, 2005, p. 190)

This implicit role of the hand in diagrams points to the ways in which diagrams are both semiotic and spatio-temporal. What is a mathematical diagram? This is both a philosophical and a material-semiotic question, and thus demands a response that reckons with both kinds of concerns. From a semiotic perspective, various scholars consider diagrams as a kind of inscription, where inscriptions constitute meaningful marks insofar as we "see" them and make sense of them. Unlike speech, inscriptions and gestures rely on visual perception for their meaning potential. As part of our visual modality, inscriptions are often assumed to be a representation (more or less accurate) of some other "reality" independent from them.

Many mathematics education scholars have drawn on Peirce's semiotics to study the manner in which diagrams function as complex signs that leverage iconic, indexical and symbolic registers (Hoffmann, 2005; Radford, 2004).

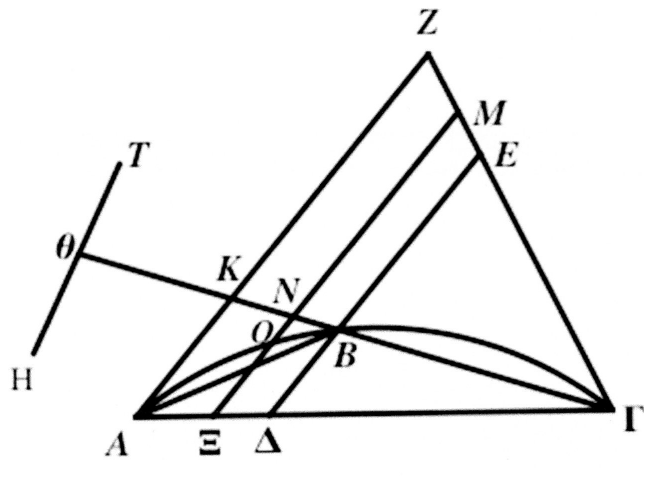

Figure 2. A diagram as physical model (a version appears in Netz & Noel, 2007, p. 102).

O'Halloran (2005) considers diagrams as one of many inscriptions used in mathematics discourse. She divides these kinds of inscriptions into (1) abstract graphs (curves in a coordinate grid), (2) statistical graphs (histograms, *etc.*) and (3) diagrams (geometrical figures, topological contours, graph theory, Venn diagrams). Radford (2004) states that "diagrams are at the heart of Peirce's remarkable endeavour" to restore perception as the fundamental engine in learning and cognition (p. 1). And yet Peircian semiotics does not go far enough in this endeavour, in that it tends to remain tied to Kantian notions of the logical *a priori,* and fails to fully embrace "man as diagram" (Radford, 2004, p. 2). In the next section, I further develop a more embodied materialist approach to diagramming.

Against representation

According to Netz and Noel (2007), ancient Greek mathematical diagrams were non-pictorial, in that they were not strictly representational and often failed to adhere to the determinations of some other reality or ideality. Consider the diagram in Figure 2, from Archimedes' palimpsest, where K does *not* bisect ZA, despite its requirement to do so in the proof.

These diagrams functioned less as pictures or images of resemblance, and more as engines in proofs; less like instances of a concept, and more like generalized objects (Netz, 1998). Such diagrams were used to present the "broader, topological features of a geometrical object" and were simultaneously treated as physical models (Netz & Noel, 2007, p. 105). Later, Western traditions grew suspicious of the diagram as a source of epistemic certainty, due, in part, to neo-Platonist theories of representation that conceived of the diagram as a flawed copy of an ideal mathematical entity. Theories of representation forever demote diagrams to inaccurate, inert and immobile copies of abstract entities. In order to trouble the staid stillness attached to such copies, and thereby invite a narrator into the physicality of the diagram to tell one of its stories, I follow Rotman (2008) who suggests that we embrace the material semiotic body as it engages with mathematical diagrams:

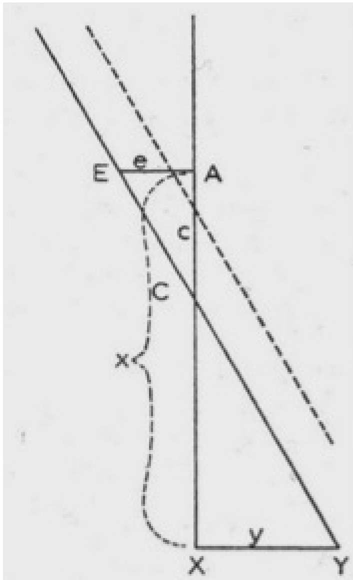

Figure 3. Leibniz's diagram justifying the infinitesimal calculus (a version appears in Leibniz, 1969, p. 545).

> Here the issue is not one of representation—the apparatus of embodied metaphors, similes or metonyms supposedly 'behind' the mathematics—but on what is revealed by the physical activities themselves, the moving around, visualizing, talking, and gesturing involved in learning and communicating the subject. (Rotman, 2008, p. 34)

Merleau-Ponty (2002/1962) made similar claims, arguing that "the subject of geometry is a motor subject" (p. 443). Rotman discusses Merleau-Ponty's argument that our conceptual understanding of *triangle* must be considered in terms of our perceptual encounter with the fluid "lines of force" that emerge as we explore the triangle as part of our hold on the world. In proving that the angles inside a triangle add to 180 degrees, for instance, we engage the triangle as though it were part of a vibrant medium of potential movement and repositioning, as though it could be folded, stretched, foregrounded, filled. Instead of encountering the triangle as "fixed and dead", Merleau-Ponty perceives the virtual potential of the triangle-space (Rotman, 2008). He argues that abstractions and sensible matter are bound together in ways that make them mutually dependent. Thus, the diagram is not merely an illustrative supplement to the meaning of triangle, but a constitutive factor in our corporeal (semiotic and performative) understanding of the concept. Diagrams are neither mere metaphors nor representations, but rather instruments for carving up matter.

Châtelet (2000) pursues this line of thinking, arguing that inventive diagrams are material experiments, cutting up space, folding surfaces, and multiplying dimensions. The mathematical diagram is not simply a representation, nor an illustration or code—there is no algorithm or rule for determining it. Reducing a diagram to a representation "ignores the corporeality, the physical materiality" (Rotman, 208, p. 37) of diagramming as a creative activity. Diagrams are allusive and allegorical, elastic and never exhausted. For Châtelet, diagrams act as interference or intervention, in that they are potentially creative events, conjuring and shaping the sensible in sensible matter. He suggests that innovative diagramming techniques have pushed through the confines of axiomatic systems and state-sanctioned practices to allow for new forms of doing mathematics and science. Like Rotman, he sees the diagram as that which is never entirely actualized, since it stands somehow outside of representation. According to Châtelet, diagrams and abstractions of other kinds cannot be extracted from sensible matter through an act of reduction or subtraction, because diagrams are a kind of *capture technology,* a machine for grasping, trapping, contracting, folding, and twisting matter. For instance, Leibniz's diagram justifying the infinitesimal calculus cuts up matter in terms of fundamental differential relations embodied in a continuously changing pair of triangles (see Figure 3).

If we see a diagram in these more material terms then we can begin to make sense of it in terms of action and event. Instead of the staid and inferior copy of an abstract entity, it becomes a thing in motion, or a site of activity. The encounter between hand and diagram becomes a highly embodied encounter. Bender and Marrinan (2010) argue that diagrams—a term they use broadly to refer to all three of O'Halloran's kinds of inscriptions—function both as representations *and* objects situated in the world of the observer (p. 7). They claim that the diagram, which they define as a typically reductive rendering, usually drawn, with few if no colors, notated, coded and labeled or referenced through explanatory captions, has played a crucial role historically in concretizing processes and in visualizing the temporality of action and motion. [2]

The black lines of the diagram carve out volume in the empty abstract white medium. The white surface upon which the diagrams seem to sit *is* the virtual space where the user of the diagram performs his or her thought experiments. This whiteness is the "motor-energy" of the diagram, which offers itself up to diverse users and invites action in and through its apparent emptiness. The whiteness is the location of such action, "in order to open up spaces for creative misuse" (Bender & Marrinan, 2010, p. 24). But diagrams, unlike photographs and paintings, fail to position a viewer in one designated or legitimate location from which to observe, and thus they invite a more active and yet virtual engagement: "users of diagrams, unlike viewers, are functional components inseparable from the system in which they are imbricated. They are empowered to initiate a process of correlation even as they realize their subjectivity presence is liminal—almost non-existent" (Bender & Marrinan, 2010, p. 72). In other words, their engagement with the diagram does not entail their being positioned as an observer in a particular location, in the way they would with a photograph. When faced with the apparent indifference of the diagram to their observation, that is to say, the way in which the diagram refuses to position and fix a viewer, users of diagrams leverage modes of engagement that differ from those associated with other images. Some may turn away, while others accept their ambivalent status, engaging the diagram, entering the space and tracing multiple paths across its surface.

Narration and observation

Narration functions differently in different kinds of narrative discourse. Styles of narration address the questions: Who sees? Who experiences? Who tells? Who knows? Some narrating is narrative bound or embedded while other narrating is authorial or voiced from outside the narrative situation. Character-bound narrators cannot perceive everything (and are not omniscient) and thus they can only tell the story as that which seems to be present to their perceptions. The narrative situation is replete with information visible to other perspectives: for instance, to other character-bound narrators and to the authorial narrator. This information cannot be completely accessible to any given embedded narrator because of the particularity of their point of view.

In this section, I focus on one kind of narration: free indirect narration. Free indirect narration describes the kind of narration by which a third person or second person narrator (extradiegetic, or outside) speaks through the embodied positioning of a character within the story. The "point of view" captured by such a narrator is somewhat paradoxical, since the narrator is both inside and outside the story-space. In such cases, the narrator can suddenly move from a bird's eye view to the foregrounding of experiential immediacy. Free indirect narration is precisely the kind of narration that animates a diverse set of voices so that a story can be told from multiple and unexpected perspectives. The "free" refers to how the narrator drops the "he said" or "it seemed to her" or "you were …" and speaks directly, a technique known as *focalization* (Genette, 1983) [3]. For example, consider the difference between:

(a) Direct: He laid down his bundle and thought of his misfortune. "And just what pleasure have I found, since I came into this world?" he asked.

(b) Indirect: He laid down his bundle and thought of his misfortune. He asked himself what pleasure he had found since he came into the world.

(c) Free indirect: He laid down his bundle and thought of his misfortune. And just what pleasure had he found, since he came into this world? [4]

Focalization allows an indirect narrator to speak in third or second person and then suddenly shift and speak "as if" they were embedded in the story world, often without point-of-view tags such as "he said" or "it looked". In return, the character pushes back, as their limited perspective determines the limits of the observable story-space. These moments of focalization allow the character to impose a point of view on the narrator and thereby restrict the narrator's capacity to observe. Focalization refers to how the narrator focuses on and through the mind and body of a character, zooming in gradually until the extradiegetic narration turns into a story told from a grounded perspective within the story-space. At such moments in a story, when the authorial narrator embodies a character within the story-space, the narration focuses on observation, affect and perception. If one considers focalization the "perceptual possibilities and selections actually communicated by the narrator and the characters narrated" (Clarke, 2008, p. 32), one can trace the moves of focalization as crucial devices in making or modifying the meaning of the story. One can look for how focalization encodes, through omission and opening, facets of the story-world, sometimes working against the narration by the creation of "narrative blind spots" (Clarke, 2008, p. 32). It is this paradoxical material embodiment of an extradiegetic narrator which lends itself to the storying of mathematical diagrams, since this style of narration embeds the narrator in multiple locations, and animates what might have otherwise been silent.

According to Bal (1997), when an authorial narrator shifts to a figural or character-bound narrator through focalization, she or he tends to penetrate the agents and things in the story world and reveal their temporal becoming. This technique of "virtual voicing" can be found throughout twentieth century literature. By way of this technique, the story-world becomes more complex, teeming with motive and motion. Through free indirect discourse, observations can be dislocated and set in motion in such a way that there is no one actually possessing them. Free indirect discourse is often construed linguistically when the past tense is combined with present-tense deictic tags such as *this, here* or *now* to achieve awkward but compelling constructions such as "this was now here" (Bender & Marrinan, 2010, p.70). This juxtaposition of past and present grammatical markers creates a discourse that disrupts the usual epistemic status of the character and narrator. Time and space are out of joint in free indirect discourse. Although typically highly descriptive of a character's position and perceptions, free indirect discourse does not belong to the character, and thereby dislocates these perceptions from the voice of the character. Such statements construe a sensory experience that does not belong to the character in some traditional sense. Instead, the virtual voice of a temporarily stretched or distributed narrator introduces these "sensibilia" into the story-world. Bender argues that this kind of "impersonal narration of thought" in free indirect discourse parallels the function of the whiteness within diagrams (Bender & Marrinan, 2010, p. 71). In each case, the device dislocates and disorients the system's observer location, and creates a space for alternative observers to enter.

In the case of telling the story of the diagram, does one position oneself as an authorial narrator or an embedded narrator? What might be gained by embedding oneself and attaching one's point of view to particular vantage points within the diagram space? Would that allow for additional insight? Or would it simply add to one's narrative blind spots? Should one instead strive for a free indirect narration, a kind of "virtual voicing", so as to proliferate the points of view that one virtually occupies, and to thereby reconstruct the diagram as though doing so through multiple internal and relational perspectives? With free indirect narration, might the space of the diagram fold and thicken, as though saturated with a kind of distributed agency and motion? Moreover, a free indirect narration might elicit the hand through gesture, and thereby allow the student to re-enter the diagram space as a space of drawing and construction. In such cases the student would become a "motor subject" engaging the diagram as though it were part of a vibrant medium. Could storying the diagram—animating the diagram and emboldening the narrator—tap into the "motor energy" of the whiteness where the diagram sits? I

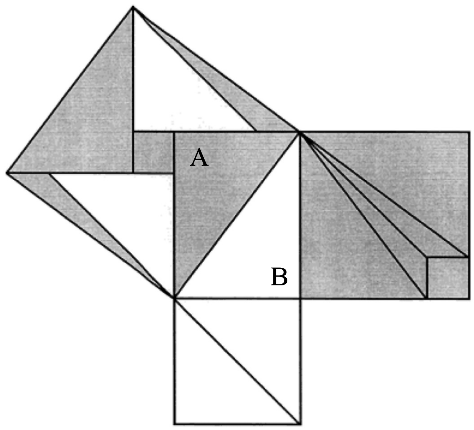

Figure 4. *Liu Hui's visual proof of Pythagoras' Theorem, 3rd century AD (from Nelsen, 2000, p. 4, labels added). Image © Mathematical Association of America, 2012. All rights reserved.*

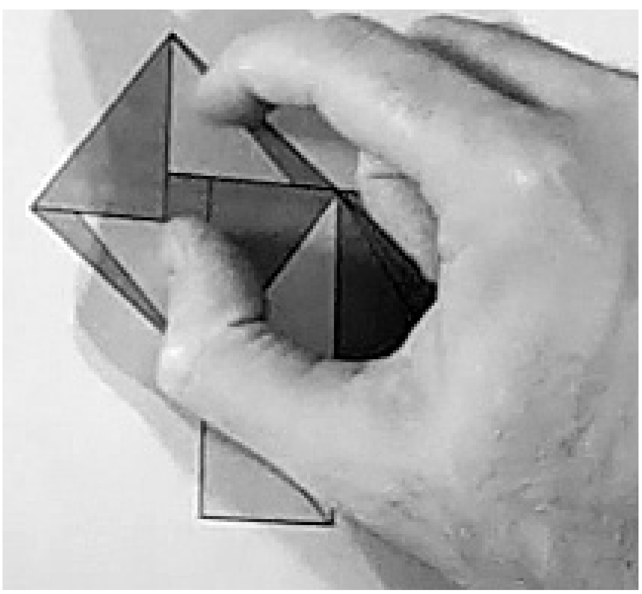

Figure 5. *Marcus's pincer gesture.*

show below how the use of free indirect narration and virtual voicing allows a mathematician to treat the diagram as a non-pictorial "generalized object".

Narrative positioning within the diagram

Marcus, a mathematician, responded without hesitation to the question, "How would you tell the story of this diagram?" He also instantly animated the various characters of the diagram, saying "these guys", and repeatedly attributed agency to these characters with expressions like, "the sides turn into squares". Marcus switched back and forth from the first person "I" to the indefinite "you", shifting the point of view of the narrative in and out of the story-space. The snapshot reveals the most common gesture that Marcus used while telling the story: a pincer gesture (see Figures 5 and 7). As Châtelet (2000) suggests, "one might be tempted to say that the pincers or compasses give a point of view to the hand, by associating an angle in which the interval is 'seen' with the grasp" (p.151). Although I do not have the space to elaborate, one could follow the shifting location and orientation of this gesture across the diagram to track the accompanying point of view taken up in his narration.

When asked for Figure 4, "Where is the beginning and end of this diagram?" Marcus first picked the shaded triangle (labeled A) and then shifted to the white triangle (labeled B), explaining that "it feels a lot better because now I have squares off each end." After first picking the dark triangle as the starting point. Marcus shifts to the white triangle. The whiteness of this triangle was first seen as empty and then seen as the "motor energy" of the diagram, as he begins to conceptualize the Pythagorean relation between its parts. When asked, "how would you tell the diagram as a story?" Marcus replies:

> Yeah I would say that you start with a triangle and what you're going to do is look at the square on each side and look at the areas of those squares and then somehow I would want to dissect it so that this square's area plus this square's area gives us this one. Yeah that actually helps a lot because now if I subdivide this one this way then these two guys move up here and if I subdivide this one these guys go here and this goes here. (Points to corresponding areas of diagram.) So these two areas would combine to give us this one. So I think I would go that way.

Marcus uses *virtual voicing* to tell the story of the diagram. He locates himself in the white triangle (embedded character) and then commands the narrator to occupy this perspective or point of view and "look" or observe the surrounding squares. He then switches back to first person, as the active agent in the story, and populates the story-space with active agents ("guys"), finally switching to the plural "we" as though he was speaking on behalf of a collective. The paths he traces across the diagram constitute an embodied encounter within a mathematical space. He leverages himself (or the collective of mathematicians) through the use of "we" and various other linguistic and gestural markers, while embedding himself in the diagram.

With Figure 6 (overleaf), Marcus was asked the same questions, identifying the lower image as the beginning of the story because he felt that the dotted line was an invitation to perform an action (cutting). When asked to explain, he states:

> Um, maybe because I think the lower one sort of has a concept of cutting. When you start putting in these dotted lines you sort of naturally think of what if I cut out these outside parts so now I'm in this square frame and things that are outside of it sort of need to go away. And this one up here not only do I not want it to go away but I want to put it inside.

We can see the shifting of free indirect narration in his use of pronouns and in the way he positions himself within the diagram. When Marcus says "so now I'm in this square frame," he locates himself in the diagram and looks out from

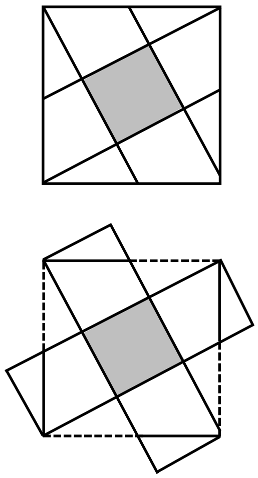

Figure 6. A shaded square one-fifth the area of the given square (from Nelsen, 2000, p. 22). Image © Mathematical Association of America, 2012. All rights reserved.

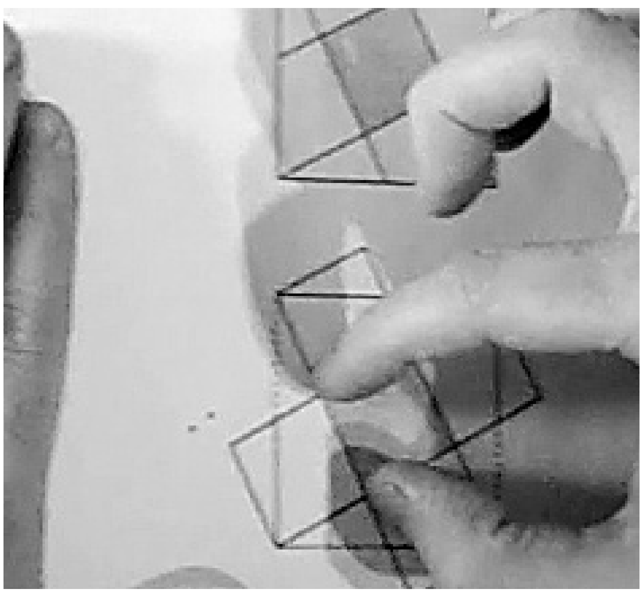

Figure 7. Another pincer gesture.

this point of view at the "things that are outside of it". The indefinite you, however, functions as the external narrator in "when you start putting in these dotted lines". Thus, he switches between the authorial narrator and the embedded narrator, and his point of view and perception switch as well. The pincer gesture here is meant to carve up the page and move the triangles.

In Figure 8, Marcus draws extensively on the color scheme to tell the story. Although he tells a story about how the parts are related, he does not spontaneously see any motion or trace of motion in the static diagram. For instance, he describes the top image as "a lot" of decagons and not in terms of one decagon translated or rotated across the page. Other participants in the research who were not mathematicians described this part of the image in terms of a moving decagon. Like all of Marcus's stories, he tells it in terms of

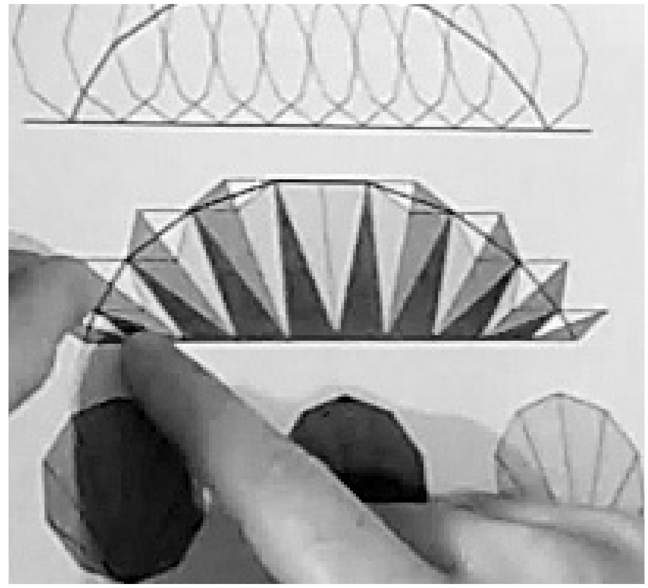

Figure 8. Talking about "these guys".

a series of actions, sometimes performed by the indefinite you, sometimes by the parts of the diagrams themselves, and sometimes by a wandering "I" or "we" that occupies various positions within the story as well as outside it:

> Wow, ok so I think we're starting up here and we have octagons, no ten-gons, whatever those are called, decagons. So you chuck a lot of regular decagons together and you draw this guy by intersecting these key points (follows finger along the arch that goes through the top figure) and now we're gonna take that and shade somehow and we take each of these places where it intersects a decagon and we create these guys. And let's see, the bottom parts are always the dark guys and then semi-dark and then light and somehow we shade all of this stuff together. And then the idea is that if you pull out the individual colors you can make three of these shapes and so then this is probably a story about if you wanna talk about the area under this curve, you just need to know the area of these three regular ten-gons.

Marcus began more hesitatingly with this diagram ("wow"), and his language is more extradiegetic than in the other examples. This seems to suggest that comfort with mathematical diagrams and diagramming might be correlated with the use of free indirect discourse. Also relevant to this point is his claim that telling the story depends on *knowing* something rather than seeing something. It may be that Marcus is occupying the position of an authorial narrator in this example, for whom knowing rather than experiencing is the mode of engagement. To tell the story, according to Marcus, "you" need to know certain things—in this case the area of the regular "ten-gon". We can contrast this refrain with the more experiential perspective: "In order to tell the story, you need to have been there".

Conclusion

Is it possible or desirable to imagine the diagram as composed of characters with quasi-subject status, each ascribed actions and motions and even inclinations? Could focalization allow a student to engage with the diagram as an embedded character narrating from within the diagram? Is free indirect discourse—the switching between various embedded narrators and the extradiegetic voice of an authorial narrator—advantageous in navigating the story-space of the mathematical diagram? This paper suggests that treating the diagram as story allows us to think about the event-structure of the mathematical diagram, and to explore the ways a diagram is a material site of engagement. Not only does this animate the diagram, and allow us to think about diagrams outside a theory of representation, but it also allows us to study the recounting of causally related sequences of actions in terms of the positioning of a narrator through point-of-view and voice.

In the case of Marcus, we see virtual voicing at work as he engages with the event-structure of the diagrams. When he tells the story of the diagrams, he temporalizes and animates them, and locates himself in their experiential world. Indeed, narrating the diagram becomes a way of dissolving its fixity and inviting intervention and reconstruction. Temporalizing the diagram, and seeing it as a set of sequential acts, allowed Marcus to disaggregate its parts and then set them in motion. Marcus switches narrator positions in order to project himself and inhabit various perceptual coordinates within the story-space, as though he were an embedded character whose perspective and manual grasp could move about within the space, allowing him to traverse various paths across the diagram. This analysis points to the need for more research on how we might enhance student engagement with diagrams through attention to the role of narrative and voice.

Notes

[1] See, for example, Brown (2008) for explorations of the age-old debate.
[2] Of course there are exceptions to this characterization, such as the 1847 Oliver Byrne edition of Euclid which used color coding to show proofs.
[3] These techniques are used extensively in fiction. An excellent example of focalization through free-indirect discourse can be found in Virginia Woolf's *Mrs. Dalloway.*
[4] These examples are from en.wikipedia.org/wiki/Free_indirect_speech

References

Bal, M. (1997) *Narratology: Introduction to the Theory of Narrative.* Toronto, ON: University of Toronto Press.

Bender, J. & Marrinan, M. (2010) *The Culture of Diagram.* Stanford, CA: Stanford University Press.

Brown, J.R. (2008) *Philosophy of Mathematics: A Contemporary Introduction to the World of Proofs and Pictures.* New York, NY: Routledge.

Châtelet, G. (2000) *Figuring Space: Philosophy, Mathematics and Physics* (R. Shore & M. Zagha, Trans.). London, UK: Kluwer Academic Publishers.

Clarke, B. (2008) *Posthuman Metamorphosis: Narrative and Systems.* New York, NY: Fordham University Press.

Genette, G. (1983) *Narrative Discourse: An Essay in Method* (J.E. Lewin, Trans.). Ithaca, NY: Cornell University Press.

Healy, L. & Sinclair, N. (2007) If this is our mathematics, what are our stories? *International Journal of Computers for Mathematics Learning* **12**(1), 3-21.

Hoffmann, M. (2005) Signs as means for discoveries: Peirce and his concepts of "diagrammatic reasoning", "theorematic deduction", "hypostatic abstraction", and "theoric transformation". In Hoffmann, M.H.G., Lenhard, J. & Seeger, F. (Eds.) *Activity and Sign: Grounding Mathematics Education,* pp. 45-56. New York, NY: Springer.

Hofstadter, D.R. (1997) Discovery and dissection of a geometric gem. In King, J. & Schattschneider, D. (Eds.) Geometry Turned On! *Dynamic Software in Learning, Teaching and Research,* pp. 3-14. Washington, DC: The Mathematical Association of America.

Leibniz, G. (1969) *Philosophical Papers and Letters, Volume 2* (L. E. Loemker, Ed. & Trans.). Dordrecht, The Netherlands: D. Reidel Publishing Company.

Maynard, P. (2005) *Drawing Distinctions: The Varieties of Graphic Expression.* Ithaca, NY: Cornell University Press.

Merleau-Ponty, M. (2002) *Phenomenology of Perception.* New York, NY: Routledge. (First published in English, 1962.)

Nelsen, R.B. (2000) *Proofs Without Words II: More Exercises in Visual Thinking.* Washington, DC. The Mathematical Association of America.

Netz, R. (1998) Greek mathematical diagrams: their use and their meaning. *For the Learning of Mathematics* **18**(3), 33-39.

Netz, R. & Noel, W. (2007) *The Archimedes Codex: How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist.* Philadelphia, PA: Da Capo Press.

O'Halloran, K. L. (2005) *Mathematical Discourse: Language, Symbolism and Visual Images.* London, UK: Continuum Press.

Radford, L. (2004) *Rescuing Perception: Diagrams in Peirce's Theory of Cognitive Activity.* Paper presented at ICME 10. Copenhagen, Denmark.

Rancière, J. (2007) *The Future of the Image* (G. Elliot, Trans.). London, UK: Verso.

Rotman, B. (2008) *Becoming Beside Ourselves: The Alphabet, Ghosts, and Distributed Human Being.* London, UK: Duke University Press.

Sfard, A. (1994) Reification as the birth of metaphor. *For the Learning of Mathematics* **14**(1), 44-55.

Solomon, Y. & O'Neill, J. (1998) Mathematics and narrative. *Language and Education* **12**(3), 210-221.

The words "do not disturb my circles" are said to be Archimedes' last before he was slain by a Roman soldier in the tumult of the pillaging of Syracuse. The timeless tranquil eternity of the not-to-be-disturbed circles in the midst of this account of hurly-burly and death is emblematic of the contrast between mathematics and stories: history, legends, anecdotes, and narratives of all sorts thrive on drama, on motion and confusion, while mathematics requires a clarity of thought that, in many instances, comes only after prolonged quiet reflection. (p. vii)

Doxiadis, A. & Mazur, B. (2012) Introduction. In Doxiadis, A. & Mazur, B. (Eds.) *Circles Disturbed: The Interplay of Mathematics and Narrative,* pp. vii-xix. Princeton, NJ: Princeton University Press.