

ONE, YOU, SHE: OBJECT RELATIONS AS GROUNDING METAPHORS FOR LEARNING MATHEMATICS

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A concept of some kind of ‘inner representation’ of an ‘object’ is required, since there is overwhelming empirical evidence, in regard to both human and nonhuman species, that attachments or ties may persist throughout long periods during which the object is absent from perception and can provide no ‘reinforcement’. (Ainsworth, 1969, p. 9)

Many psychologists have traced adult behaviour to bodily experiences in early childhood, from Freud’s oral fixation and Klein’s splitting of the suckling breast, to Winnicott’s holding and gaze-mirroring, Bion’s containment, and Ainsworth and Bowlby’s proximity-seeking to attachment figures, among others. In their book ‘Where Mathematics Comes From’ (WMCF), George Lakoff and Rafael E. Núñez (2000) similarly trace adults’ concepts of mathematics back to the actions of our bodies on physical objects as children, and to four actions or ‘grounding metaphors’ in particular: collecting objects into groups, constructing with objects, moving along a path, and using a measuring stick. They suggest that mathematics, rather than existing on some abstract Platonic plane, is instead grounded in the brain’s capacity to use embodied actions on physical objects as metaphors to make sense of the world. WMCF proposes that there are neural mechanisms linking specific sensory-motor experiences cognitively to mathematical concepts and maps out a network of these links. This article attempts to connect WMCF’s cognitive network based on sensory-motor experiences of physical objects, with the affective psychology of experiences of human object relations.

Though Lakoff and Núñez characterise WMCF primarily as cognitive neuroscience, they claim “it also can have an important application in the teaching of mathematics” (p. 7). And for educators of schoolchildren alienated by the abstraction of mathematics, the idea that the brain builds up abstract concepts from bodily actions on objects, which almost every child can enact, is alluring. Especially when these actions align with usage of mathematical manipulatives already established in classrooms—such as collecting counters, constructing with blocks, stepping along number lines, and measuring with rods (Figure 1). Publication of WMCF seemed to support proponents of hands-on manipulatives as engaging and motivating to children, as well as a ‘social turn’ in education in the 1990s, which saw object-manipulation as a facilitator of dialogue.

Curiously, however, Lakoff and Núñez distanced them-

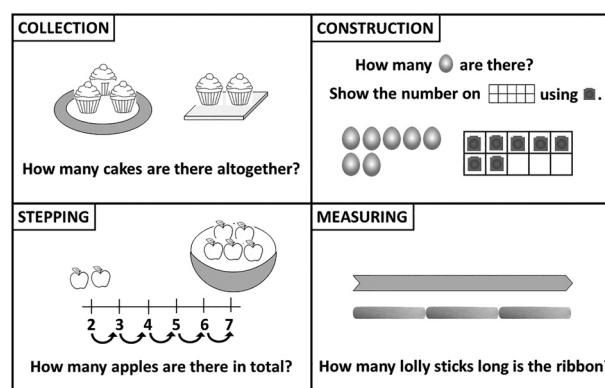


Figure 1. Correlates with Lakoff and Núñez’s four grounding metaphors in representations adapted from *Maths No Problem! Textbook 1A* (Yeap et al., 2014) aimed at 5-year-olds in England.

selves from key schools of thought in mathematics education that embraced embodiment as catalytic to both emotional engagement with, and social construction of, mathematics. In WMCF they emphasise that their approach is not consistent with existing philosophies of mathematics, “Nor is it consistent with recent post-modernist accounts of mathematics as a purely social construction” (p. 9). And although Lakoff had previously explored embodied metaphors for emotions, family, and love extensively with Mark Johnson in ‘Metaphors We Live By’ (MWLB)—for example, “we have the primary conceptual metaphor Affection Is Warmth because our earliest experiences with affection correspond to the physical experience of the warmth of being held closely” (Lakoff & Johnson, 2008, p. 256)—links with emotional metaphors or the psychology of affect are oddly absent from WMCF, as if embodied metaphors for emotions are somehow segregated cognitively from those for mathematics.

Personally, I am puzzled by WMCF’s lack of affective and social metaphors. As a thought experiment, in this article I aim to connect Lakoff and Núñez’s four grounding metaphors (‘4Gs’), which they map to learning arithmetic, to children’s developmental psychology, and its world of affection and family relationships.

Firstly, I suggest that the 4Gs may be based on embodied actions—accurately counting grouped objects, or steps, or

placements of measuring sticks *etc.*—that require fine-motor and linguistic skills typically accessible by children too late in their development to be true *grounding* metaphors for learning mathematics. This opens a role for pre-4G embodied grounding metaphors, to map from the domain of babies—who can apparently innately subitise or sense at-a-glance numbers of objects up to three or four—to the 4G domain of children who can coordinate precise arrangement of objects with verbal counting.

Secondly, to fill this role, I highlight the theory of object relations from child developmental psychology as a plausible source of embodied grounding metaphors for dyadic, child-caregiver ‘two-ness’, and triadic, child-caregiver-other ‘three-ness’ among babies and toddlers. For conciseness, here I use the term caregiver to include any principal attachment figure who suckles and weans a baby, whilst acknowledging that in many cultures breast-feeding (and bottle-feeding) is a shared activity, and among attachment theorists there is debate as to whether babies always prioritise a single attachment figure. I also suggest that these metaphors of dyadic and triadic human relations may help structure early concepts of first-, second- and third-person pronouns, which support Lakoff and Núñez’s “fundamental metonymy of algebra” (p. 74).

Thirdly, I briefly highlight the affordances of these object-relations based metaphors as potential grounding metaphors for developing a pre-counting, ‘subitisable’ mathematics curriculum to underpin the 4Gs.

Here I wish to emphasise that this article is not meant as a critique of WMCF’s (or MWLB’s) theory of metaphorical mapping itself, which I find compelling, but rather a tentative attempt to connect it plausibly with the affective developmental psychology of babies.

Where do grounding metaphors come from?

Lakoff and Núñez are candid about challenges they faced in applying methodologies of cognitive linguistics, used in MWLB, to mathematics: “To those unfamiliar with the methodology of cognitive linguistics, it will not be obvious how we arrived at the metaphorical mappings [...]. In cognitive linguistics, the main technique is building models that generalise over the data” (p. 100). Working across multiple disciplines including neuroscience, cognitive psychology, and developmental psychology, “This is not an easy job [...]. We must propose plausible ultimate embodied groundings for mathematics together with plausible metaphorical mappings” (p. 101).

The process was iterative and collaborative:

In studying arithmetic, for example, we depended on the prior research of Ming Ming Chiu (1996). Chiu’s dissertation set out some first approximations that met a number of constraints. Starting there, we made many successive revisions until the constraints were met. (p. 101)

Lakoff’s former student Chiu’s dissertation, based on metaphors from MWLB, does include some ‘social’ metaphors for mathematics, for example “Arithmetic is a Social Transaction” (Chiu, 2001, p. 118). However, these apparently did not meet Lakoff and Núñez’s constraints for grounding metaphors for mathematics.

WMCF evidently represents an enormously complex, interdisciplinary synthesis of research. However, in their linguistics-based methodology, the plausibility of the grounding metaphors depends critically on the data they have chosen to generalise across. Thus, if new data are included that affect the plausibility of the mappings, the 4Gs might have to be revised to accommodate this.

The two key pieces of data I wish to introduce are the complexity of counting from educational research, and the theory of object relations from developmental psychology.

The complexity of counting

Lakoff and Núñez are clear in WMCF that counting is a cognitively complex task, listing seven distinct cognitive capacities required just to extend subitising to finger counting (p. 51). To map counting further, a grounding metaphor is required.

Introducing the first grounding metaphor—*Arithmetic Is Object Collection*—they say: “No metaphor is more basic to the extension of our concept of number from the innate cardinal numbers to the natural numbers” (p. 54), implying that this metaphor is necessary to conceptualise numbers beyond the innate, subitisable three or four. However, from a pedagogical perspective, WMCF does not detail exactly how object collection maps to counting, until a leap to “every natural number can be conceptualized as a polynomial—that is, a sum of integers represented by simple numerals times powers of some integer B” (p. 82). There appears to be a paradox here: to understand counting above four we need to collect objects, but in order to know how many objects we have collected we need to be able to count above four.

In mathematics education, this assumption that counting can and should be mastered early, before arithmetic, though common, sits awkwardly with research into children’s development. For example, acoustic counting—saying ‘one, two, three’ *etc.* in order—does not always coordinate with pointing at objects (van den Brink, 1984). Patterns of sounds in acoustic counting are often inconsistent with the polynomial system—for example, in English ‘three’, ‘thirteen’, ‘twenty-three’ for 3, 13, 23—requiring exceptions to be memorised. In counting with fingers there is no consensus on which finger is ‘one’, and linguistically some doubt as to whether a thumb is a finger at all. And to read or write numbers beyond 9, we encounter concepts such as zero, powers and place value that underpin the counting system and are themselves complex (Coles & Sinclair, 2017). As the mathematics educator Caleb Gattegno put it in his farewell speech:

We have, for centuries, taught people, by offering counting as the basis of elementary arithmetic. It’s wrong! Shall I say it louder? It’s wrong. Not because I say so, but because counting is a complex activity. It’s a complex activity asking of children more than is required in order to give them a better foundation. (1989, p. 24)

Below I propose that an alternative foundation, and a possible way of mapping counting more closely to the 4Gs, would be to introduce key mathematical concepts needed for counting—such as products, powers and positional value—

before counting, using only the ‘innate’ numbers up to 3. These numbers of objects are subitisable, with little effort, by young babies, freeing cognitive resources, and are also, psychologically, closest to our earliest concepts of objects, according to object relations theory.

Object relations psychology

Lakoff and Núñez’s grounding metaphors involve the body acting on physical objects, or “things in the world” (p. 97). For so-called ‘object relations’ psychologists, the term ‘object’ has a psychological meaning. Originally deriving from Freud’s concept of libidinal drives—sexual desire and aggression—to describe the target or ‘object’ of the drive, the term was adapted by child psychologists Klein, Winnicott, Bion and others to describe the objects of a child’s love, for example, their mother or principal caregivers, and images of relationships with these objects which are internalised, or ‘introjected’ in the child’s unconscious. The term object has various interpretations in different schools of thought in psychoanalysis; however, here I wish to highlight the way Klein uses it to connect a baby’s first loving relationships with caregivers, with the manipulation of physical objects such as toy bricks, via the process known as projective identification.

For Klein, at first babies conceive of objects such as a breast-feeding caregiver as embodied ‘part-objects’, for instance, as a breast. When the hungry baby finds the breast available, the baby is content, and internalises the image of the suckling breast as a ‘good object’, meeting their needs. However, human babies are highly dependent on their caregivers for many months and vulnerable in their absence. When the breast is unavailable, the hungry and helpless baby’s instinctive fear of death causes them anxiety. Struggling to cope emotionally, as a defence the baby’s psyche may split off their anxiety and project it onto the image of the unavailable breast, which now becomes internalised as a ‘bad object’, towards which the baby is aggressive, to the point of even biting or screaming at the caregiver when they return. The baby experiences a split in their personality, a contented, breast-feeding ‘good self’ and a hungry, anxious and aggressive ‘bad self’ (Spillius *et al.*, 2011).

Over time, with the repeated return of the good breast, the baby begins to realise that these two split-off objects, the available and unavailable breast, are in reality one, which is a crucial stage in their development. The baby feels remorse for their aggression against the bad breast and begins to integrate the good and bad objects into one person, the caregiver. At the same time the baby realises that the two split selves they experienced, the good and the bad self, also form one personality, the baby’s own. The baby begins to understand themselves and their caregiver as two separate objects, in a loving relationship.

Also, to psychologically survive periods of no good breast, babies may create a mental image of the good breast as an emotional comfort. This introjected image can be understood as an early ‘metaphor’ or symbol of the union with the caregiver. And according to Winnicott (1990) a baby may also project this mental image of the loving relationship onto relationships with external objects, or ‘transitional objects’, such as sucking a thumb, or holding a

piece of blanket or soft toy, which can similarly be understood as comforting embodied metaphors for the yearned-for union with the absent love object (Coles, 2014).

Soon the infant can project their emotional relationships onto various physical objects in play, which Klein made use of as psychoanalytical tools:

the brick, the little figure, the car, not only represent things which interest the child in themselves, but in his play with them they always have a variety of symbolical meanings as well [...] Play analysis had shown that symbolism enabled the child to transfer not only interests, but also phantasies, anxieties and guilt to objects other than people. Thus a great deal of relief is experienced in play and this is one of the factors which make it so essential for the child. (Klein & Mitchell, 1986, p. 51)

One implication of object relations theory is that the frustrating loss of a love object—the literal ‘abstraction’ or pulling away of the breast for example—can lead to the creation of a mental symbol for it. This can then be projected onto other people and also onto physical objects, charging them with meaning, and enabling the parting and reunion to be re-enacted, reflected on, and to an extent controlled. As Kleinian psychologist Hanna Segal puts it:

The symbols, created internally, can then be re-projected into the external world, endowing it with symbolic meaning.

The capacity to experience loss and the wish to re-create the object within oneself gives the individual the unconscious freedom in the use of the symbols. (1988, p. 167)

The grounding metaphor of Becoming Two: splitting and reuniting with the love object

There is, obviously, much more to object relations than is touched on above, however in order to map metaphorically to mathematical concepts, I wish to highlight the baby’s symbolisation of their original identification with their caregiver’s body, their ‘splitting’ of both the object and themselves into two parts to cope emotionally with their separation, and their eventual recognition of the union of these split parts, as a new ‘two-ness’, a loving, you-I relationship between two embodied individuals.

For object relations psychologists such as Winnicott, the confidence of the child in their own identity, separate from the caregiver, but secure and confident in their love, is crucial to the idea of the transferable ‘unit’ self, with which other ‘Two-One’, you-I loving relationships are formed. This also carries as a metaphor into the unit, or number one, in mathematics (Winnicott, 1990, p. 61). The secure two-ness of the relationship with the caregiver becomes a base from which to explore, and a safe haven to return to when anxious.

As developmentally this predates conceptual schemas of numbers, such as number lines, whether splitting represents a halving or a doubling is moot. Just as cell division and cell multiplication can be understood as the same thing, the *Becoming Two* metaphor of splitting and reuniting can be mapped to doubling and halving and ‘whole-ing’, ‘Two—

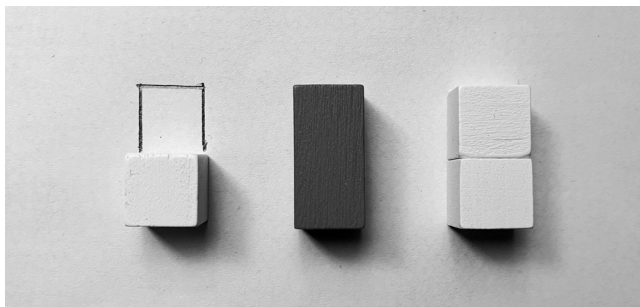


Figure 2. *Becoming Two metaphor for the child-caregiver union.*

One', and 'One—Half', and any power of $\frac{1}{2}$. It is the metaphor of being whole, split into two parts, and reunited in a symbolic relationship or 'two-ness' that is key.

To illustrate this operation physically I have chosen Cuisenaire rods that are neutral in terms of numbering—for instance, the red rod is twice the length of the white rod, and they can be thought of as Two—One, One—Half *etc.* So in Figure 2, pedagogically, a red and two white blocks can represent the multiplicative relationships $2 = 1 \times 2$, $1 = \frac{1}{2} \times 2$, and in terms of powers, $2^0 = 2^{-1} \times 2^1$, and symbolically, $r = 2w$. The rods are simple to map to the physical objects of the 4Gs, and also have established use in modelling both language and, pertinently, family relationships (Paipa, 2010).

Psychologically, in Figure 2 the red rod, centre, is a metaphor for the union of child and caregiver. The right-hand image is a metaphor for the gradually self-sustaining separation into two people in a relationship, facilitated by the Winnicottian 'good enough' caregiver, who gives the child enough love and freedom for them each to develop as individuals, secure in being loved. On the left, the empty outline above the separated block is a metaphor for the loss and anxiety felt by a baby who lacks love and nourishment, and may 'split' and project a 'bad object' to cope.

As a baby is weaned and gains mobility and starts to explore their environment, though physically separated, the crawling or toddling baby is still emotionally attached and tends to seek gaze-contact and proximity to their caregiver, especially when experiencing anxiety, as Ainsworth demonstrated in the much-replicated 'Strange Situation' procedure, in which babies are observed being temporarily left by their caregivers in a room with a stranger (Gillath, Karantzias & Fraley, 2016).

As an extension of the splitting and reuniting metaphor, it seems biologically plausible that, once a baby can crawl away from their caregiver, they maintain awareness of their distance and direction—a line of sight—so they can seek proximity to them. The *Becoming Two* metaphor can thus also map to simple geometric concepts such as the straight line connecting two points, directional angle and the radius of a circle (Figure 3). As Bion puts it:

Mathematical elements, namely straight lines, points, circles, and something corresponding to what later become known by the name of numbers, derive from realizations of two-ness as in breast and infant, two eyes, two feet, and so on. (2013, p. 180)

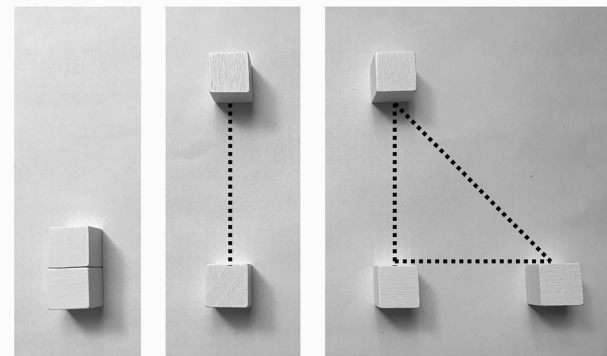


Figure 3. *Maintaining dyadic and triadic relationships at-a-distance through mutual gaze-lines.*

The grounding metaphor of Becoming Three: triangulating an Other

There is also in object relations theory the concept of the third 'other' object introduced into the dyadic child-caregiver relationship, for example another caregiver or sibling, which can be related to *Becoming Three*, or the third-person relationship, with the addition of the she/he/it object to a dyadic relationship.

The concept of number is acquired early in childhood at the time when the infant is becoming aware of its relations with significant others [...] certain numbers, especially 2 and 3, can be carriers of infantile phantasies. The number 2 can represent the relation with the other: the child with the mother, the father or another principal carer. Two is also the parental couple or the rivalry with a sibling. The number 3 can represent the triangulation of mother, father and child, and also the loss of that triangulation through death or divorce or the arrival of a sibling. (Brown, 2008, p. 30)

A baby with a 'good enough' caregiver, who is handled and held with love, reflected in the gaze of the love object, will, according to Winnicott, have the confidence to start increasingly paying attention to other objects, both human and physical. Similarly, the baby will be able, eventually, to tolerate the love object giving attention periodically to a third 'other', and thus observes two others in a dyadic relationship which the baby may recognise as reflecting their own relationships:

To the degree that we ever manage to grasp two-way directionality (*that in a relationship, I impact on you as much as you impact on me*), we do so only from the place of the third, a vantage point outside the two. (Benjamin, 2004, p. 7)

To become a secure triangular relationship, the three people's gazes and attentions shift sufficiently frequently for all three not just to feel loved, but to observe the loving dyadic relationship between the other two. In Figure 4, the white cube represents the observer. If the baby is seen as bottom left of each triad, and the caregiver as bottom right, then in the left-hand triad the baby is relating to the caregiver, with the third other observing. In the centre triad the baby is relat-

ing to the other. In the right-hand triad the baby is observing the caregiver relating to the other.

Both psychologically and mathematically, this metaphor of triangulation is considerably more complex and dynamic than the simpler splitting and integrating metaphor of the dyadic relationship, as it contains three destabilised dyadic relationships within it. Just as a transitional object, such as a dummy or thumb, may help a baby cope with their caregiver's absence, play with transitional objects such as a toy or ball can help babies and caregivers 'practice' integration of a third 'other', transferring attention to and from the third object and each other.

When the third object is human, with its own gaze and emotional life, the processing of three dyadic object relations can stretch the cognitive capacity of a baby, as well as the baby's emotional tolerance. However, once dynamic triadic relationships can be integrated, as a 'three-ness', *Becoming Three* provides an embodied metaphor for extending the 'Two-One', not just to 'Three' but also beyond. By switching the third object we can relate to further objects without losing the emotional security of the original 'Two-One' relation. Over time we feel safe enough in our you-I relationship with this third person that they can substitute the presence of our original attachment figure, so by substitution we can relate triadically to a fourth 'other' and so on. This mature capacity to relate to others independently of the primary caregiver is critical from a psychological and socio-cultural perspective when forging sustainable relationships with new people and communities.

Just as in *Becoming Two*, it seems biologically plausible that a baby would also have awareness not just of the proximity of the principal caregiver, but of any other person or animal approaching that might trigger a retreat to the caregiver. Thus, the *Becoming Three* metaphor for mentally modelling the triadic relationships between three bodies can be extended to geometric triangulation of three points, their distances and angles, and early trigonometry and spatial reasoning. Once language starts to develop these triadic relationships may also offer a proto-algebraic conceptual structure for first, second and third person, 'I', 'you', and 'she/he' pronouns.

The metonymic algebra of I-You-She

As well as a child's formation of a dyadic 'Two-One' relation with their caregiver, and the dynamic projection of this onto a third 'other' to make a triangular relationship main-

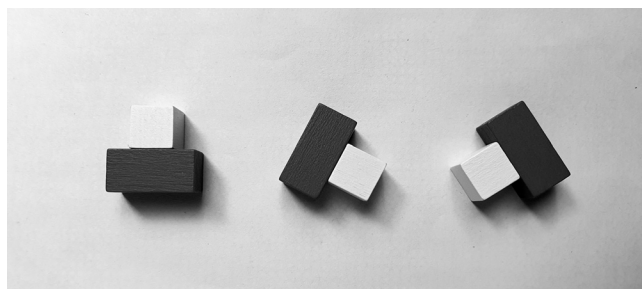


Figure 4. The *Becoming Three* triangulation metaphor for the third-person 'other' she/he/it introduced to a dyadic relationship.

tained through switching of gaze and attention, as language develops there is also the concept in object relations of mentally assuming different roles or perspectives within relationships *symbolically* through playing with linguistic metonymy.

From the earliest experiences of holding a new-born baby face-to-face, there is a reciprocal mirroring of expressions and gestures, analogous to the recursive doubling effect of two mirrors facing each other, or the reflection of one's own image in another's eyes. I reflect you, reflecting me, reflecting you *etc.* Through mirroring the two becomes one, and the one becomes two.

As they grow older and learn to talk, children often play at being adults, re-enacting adult-child conversations but switching the roles of 'I' and 'you', consciously inverting the structure. As Klein describes:

At times he plays games in which he allots roles to the analyst and himself such as playing shop, doctor and patient, school, mother and child. In such games the child frequently takes the role of the adult, thereby not only expressing his wish to reverse the roles but also demonstrating how he feels that his parents should treat him. (Klein & Mitchell, 1986, p. 41)

Later on, children can explore more sophisticated, three-way role-switching in family relationships and beyond in adult social life, as Winnicott observed:

A child moves over to a relationship with the father, and in doing so develops an attitude to the mother which belongs to relating to the father [...] This sort of thing goes on as a to-and-fro experience in the daily life of the child in the home. Of course, it need not be the father-mother relationship; it can be an experience of going from the mother to the nurse and back again, or it may be an aunt or a grandmother or a big sister. Gradually in the family all these possibilities can be met with, and experienced, and a child can come to terms with the fears associated with them. Moreover, the child can come to enjoy the excitements that belong to all these conflicts, provided they can be contained. (1990, p. 138)

This experience of dynamically switching roles within triadic family relationships also involves linguistic switching of the objects referred to by 'I', 'you' and 'she/he' in conversation. In one-to-one conversation, the object referred to as 'you' becomes 'I' in the mouth of the other, and vice versa, as the speakers take turns to talk. In private dialogue with her mother a daughter may refer to her mother as 'you' and to the grandmother, as 'she'. In a separate conversation with the grandmother, these are switched, and the grandmother is 'you' and the mother is 'she'. And in overheard conversation between the grandmother and the mother, the child may be 'she'. As children transfer these patterns of dialogue to others, the objects referred to in the first-, second- and third-person are also re-arranged, to maintain the triadic relationships, not just face-to-face by switching gaze and attention, but also symbolically, by mentally rearranging the objects represented by the pronouns, in a similar way that equations can be rearranged to put x , y or z on the left-hand side.

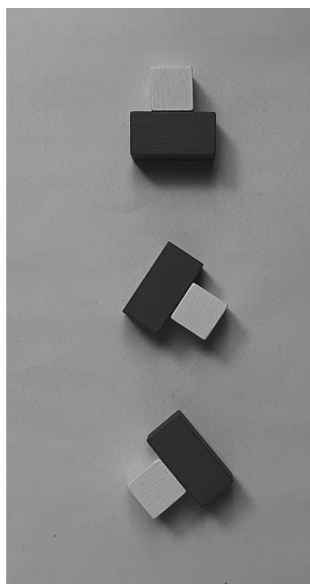
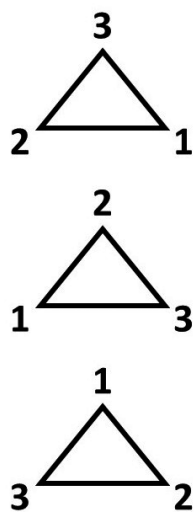


Figure 5. WMCF's representations of a commutative algebraic group with three elements (left, p. 115) compared with the *Becoming Three* triangulation metaphor.

For Lakoff and Núñez, this kind of linguistic metonymy, 'The Fundamental Metonymy of Algebra', underpins mathematics:

This everyday conceptual metonymy, which exists outside mathematics, plays a major role in mathematical thinking: It allows us to go from concrete (case by case) arithmetic to general algebraic thinking. (p. 74)

As WMCF illustrates, the conceptualisation of algebraic structures such as simple groups does not require counting beyond three (Figure 5). So, it is plausible that algebraic concepts may be accessible to young children *before* counting. Mary Boole, editor of her husband George Boole's 'Laws of Thought' (to which WMCF devotes a chapter), recommends that babies start exploring algebra—in the sense of manipulating objects which can stand for more than one thing—even before talking, for example by repeatedly touching a shiny teapot, which can be hot or cold:

Everybody ought to be able to make Algebras; and the sooner we begin the better. It is best to begin before we can talk; because until we can talk, no one can get us into illogical habits; and it is advisable that good logic should get the start of bad. (Boole & Tahta, 1972, p. 57)

Towards a subitisable mathematics curriculum to underpin the 4Gs

In this article I have proposed, drawing on object relations psychology, that our earliest mathematical objects may be our closest caregivers, rather than inanimate objects, and that our relationships with them are affective, reciprocal and dynamic. These are objects that can hold us as much as we can hold them, that can gather us up or put us down, who move and talk. And, unlike pebbles or wooden blocks, they

can appear impermanent, and such is our emotional dependence on them that when they are gone, we create internal images of a permanent union with them to comfort ourselves—our first symbolic relationships—which we may embody by projection onto physical transitional objects.

In England, as in many school curricula, the rush to symbolic counting to twenty and higher numbers starts early, coinciding with starting primary school, and being left alone by our caregiver not just briefly with one stranger—as in Ainsworth's Strange Situation—but all day, with dozens of strangers. If psychological and mathematical object relations are linked, this is bound to be overwhelming both emotionally and cognitively for many children. And, psychologically they may defensively disassociate their anxiety, split it off and project it onto mathematical objects and symbols.

From a curricular perspective, one possible avenue for future research would be to investigate *postponing* counting, to spend more time attending to the relationships between just two or three subitisable objects: halving and wholing, doubling and double doubling, switching positions, stepping from foot to foot, joining points, triangulating and squaring—the common dyadic and triadic rhythms and symmetries of bodies and dances, songs and conversations, and the angles of turning our heads and arms from one object to another and back again. By prioritising secure internal models of dyadic and triadic relationships with both people and physical objects, I suggest the grounding metaphors of *Becoming Two* and *Becoming Three* may emerge as cognitively and emotionally secure bases for concepts of spatial reasoning, products and powers, that would support place-value counting, as well as the 4Gs and the metonymy of algebra.

On the cover of my copy of WMCF is a promotional quote: "Adds body heat to the cold and beautiful abstractions of mathematics". Object relations psychology may offer a way of adding the emotional warmth of loving relationships to this body heat.

Acknowledgment

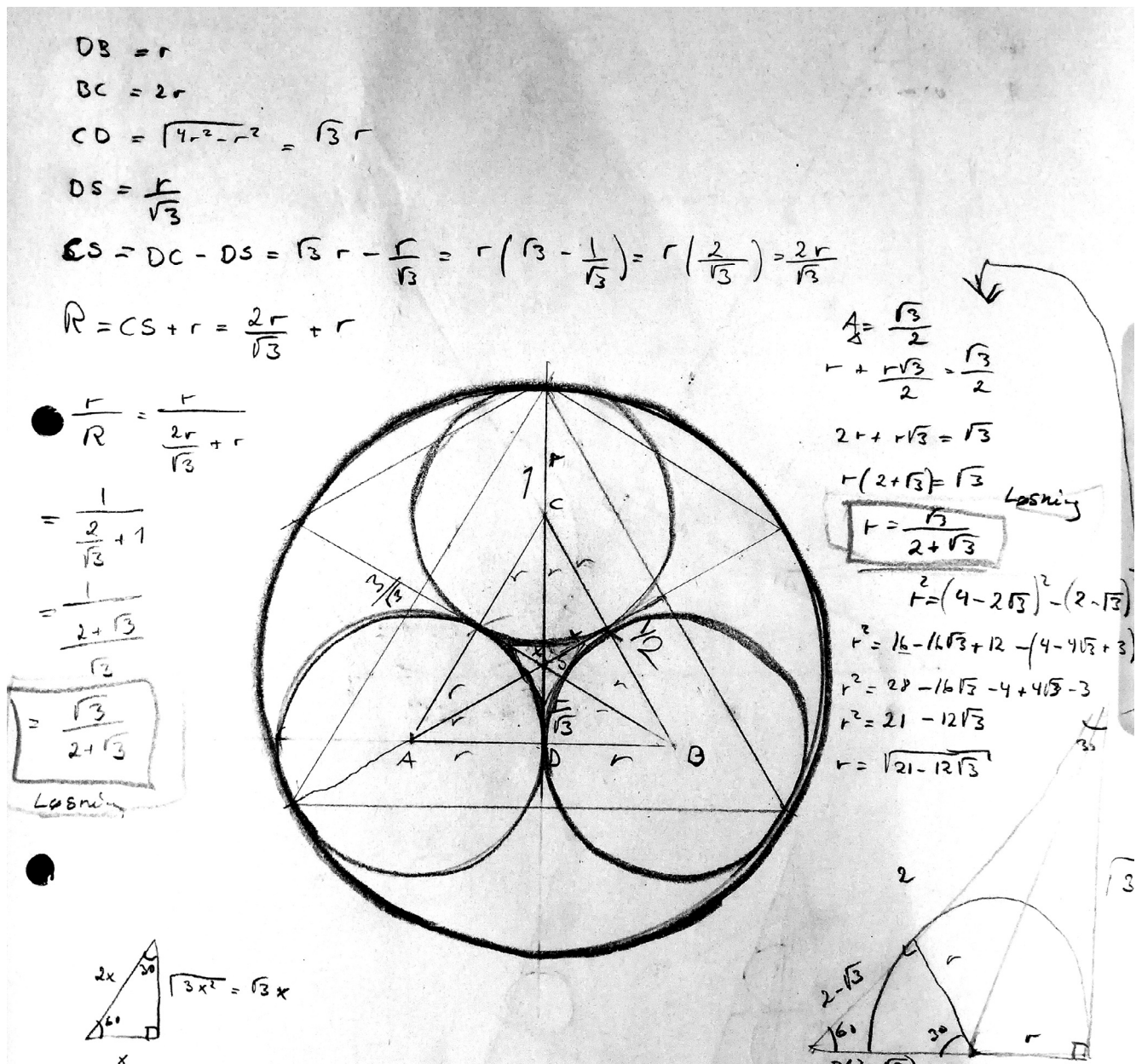
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A construction by Benjamin, age 14.