Gresham's Law: Algorithm Drives out Thought*

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Gresham's law in economics says, "Bad money drives good money out of circulation." Copper replaces silver; silver, gold Gresham's law in mathematical pedagogy can be stated several ways "Algorithm drives out thought " "The robotic displaces the humanistic " "Cultivation of algorithms replaces concern for thinking and writing."

We view colleges and universities ideally as places that develop the ability to think analytically, to probe independently, to resolve the open-ended problem, to write and speak clearly Though the catalog may not mention them, these goals are in the back of our minds when we picture ourselves as teachers In the catalog we find descriptions of courses couched in terms of their content, such as:

> Linear algebra. Matrices and linear transformations, determinants, complex numbers, quadratic forms

This list, with its focus on topics, illustrates the power of our version of Gresham's law. We can be sure that there will be definitions, theorems and proofs, and algorithms Swept under the catalog is concern with the development of the ability to think and to communicate. So, without a battle, in spite of our best intentions, the combination of curriculum, syllabus, and schedule seems to assure the triumph of Gresham's pedagogic law.

Algorithms, of course, are good and must be taught After all, the world would be an unpleasant place if every time we added two fractions we had to discover the procedure from scratch

But the temptation to emphasize drill over understanding is almost irresistible. It is much easier to teach the execution of an algorithm than the ability to analyze Furthermore, an algorithm can be described in just a few minutes and skill in its execution can be tested and scored easily.

Moreover, the incredible power of calculators and computers may entice us to shape our courses around them rather than around the students As we incorporate these devices into our teaching, we must be sure that their role does not shift from servant to master and that skill in punching keys is not confounded with the ability to think and communicate.

The tendency for algorithm to displace reflection is not new. The student who shows up in our remedial or calculus class may already have experienced twelve years of robotics. Recently in my first-quarter freshman calculus class I assigned an exercise which asked the student to show that a polynomial of odd degree has a real root. The next day a student asked, "Could you work this problem?"

> "What was the trouble?"
> "Well, what's a polynomial of odd degree?"
> "Didn't you take algebra in high school?"

Then a girl in the back raised her hand, "Professor Stein, you don't understand In high school the teacher works one problem on the board and we then do twenty just like it We don't have to know anything." A murmur of endorsement swept the room — from students who had graduated in the top eighth of their class from school throughout California

In one classroom in an above-average high school, logarithms were taught in this way: "Logarithms are tough, but all you need to know is that when you press the log-key you get the logarithm " This is the complete triumph of algorithm over understanding.

Of course, educators have tried to resist the working of Gresham's law. The director of the California Curriculum Commission recently complained, "Youngsters need to know more than just computational skills. We want them to have a sense about what numbers mean " This announcement followed the Commission's rejection of all the textbooks submitted for adoption in grades K to 8 because they did not relate to the objectives that the Commission had published a year earlier, such as:

> The focus of the program is on developing student understanding of concepts and skills rather than "apparent understanding"
> Students should be actively involved in problem solving in new situations.
> Nonroutine problems should occur regularly in the student pages

These objectives, taken from the 1985 "framework", were not new. In 1980 an earlier Commission had urged,

> Problem solving has become the all-encompassing theme of mathematics instruction and is no longer a separate topic.

Twelve years earlier, in 1968, a still earlier Commission had said the same thing in different words:

> Textbooks shall facilitate active involvement of pupils in the discovery of mathematical ideas

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But even before that, in 1963, another Commission had insisted that:

> Pupils should make conjectures and guesses, experiment and formulate hypotheses, and seek meaning. Materials should elicit thoughtful responses and develop understanding.

So the texts submitted in 1986 not only failed to satisfy the demands of the current Commission, but they wouldn't even satisfy the demands set by any of the Commissions going back a quarter century.

However, concern with the displacement of thought by algorithm did not begin in 1963. In describing some of his experiments in the teaching of arithmetic, L.P Benezet, a superintendent of schools, wrote in 1935 [1]:

> For some years I had noted that the effect of the early introduction of arithmetic had been to dull and almost chloroform the child's reasoning faculties. [In my experiments] the teacher is careful not to let teaching of arithmetic degenerate into mechanical manipulation without thought.... The objectives are first of all reasoning and estimating rather than mere ease in manipulation of numbers

Incidentally, pupils in his program for one year caught up with pupils who had spent three years in the traditional arithmetic program

This conflict between the thoughtful and mechanical is as ubiquitous as the conflict between good and evil Once you are sensitized to it, you see it everywhere In one mail delivery recently I found an ad for a college algebra text and a sample of a new journal The ad included this reassurance:

> Numerous algorithms for solving word problems are developed to help students learn and remember concepts.

So algorithm finally disposes of its arch enemy, the word problem

There was an odd juxtaposition between this ad and the title of the journal that came in the same batch of mail: *Teaching thinking and problem solving* with the peculiar implication that we need not think to problem-solve

There seem to be two separate worlds One is the world of Math Commissions with high aspirations, enrichment materials at publishers' booths, conferences on humanistic mathematics, articles that show how to teach thinking, books with the phrase "problem solving" in their titles, and the exciting prefaces of texts. The other is the world of the typical classroom, whether K to 12 or Freshman to Senior at college. Vast storms of reform rage in the first world, but they stir scarcely a faint breeze in the second world. The first corresponds roughly to the world of "thinking"; the second to the world of "plugging in".

The fashionable terms are now "problem solving" and "algorithms" Whatever the terminology, students know the difference. In anonymous course evaluations they write, "This course made me think." They do not write, "This course made me problem solve " The word "think", loose though it may be, is good enough.

But there are many obstacles to teaching "thinking" Some are external to any particular course. As individuals, we can't do much about them: that for twelve years most of our students have learned robotics, with even word problems resolved by mnemonic devices; that society rewards the seemingly practical more than the fundamental; that many students go to college only to get a good job at a time when the economy no longer even promises everyone a job.

The internal obstacles are quite different The prescribed syllabus may move so fast that there isn't time to address such fine points as "thinking" The midterms and finals are squeezed into such narrow time slots that we dare not pose problems that demand fresh thought The text may offer almost exclusively exercises that cultivate algorithms. Indeed, if you thumb through many a high school or college text, you can come upon section after section where every single exercise is routine.

Everything seems to conspire to favour algorithm over thought. The syllabus is worked out and expressed in terms of topics, not in terms of processes Texts, by their very structure, offer answers before the students have absorbed the questions. Homework assignments draw the students' attention to individual exercises rather than to underlying concepts. To cap it off, we're so busy or the classes are so large that we read neither the daily homework (read by undergraduates), nor the midterms (read by graduate students). So, captivated by the clarity of our own lectures, we assume that all is well

For some twelve years most students have been strapped to a table. No wonder they cannot walk on their own two feet We must remember that thinking in a mathematics classroom may be a novel or at least unusual experience.

In spite of these obstacles, external and internal, there are actions we can take as individuals to subvert Gresham's pedagogic law

As we propose a day-by-day syllabus we can *delete topics* to provide more time to give attention to "thinking"

We may even propose a new *course whose main purpose is the cultivation of the student's ability to analyze and write* It can be smuggled into the catalog under the guise of, say, "discrete mathematics".

And we can make a conscious choice as we begin teaching a course Are we going to emphasize facts and algorithmic skills, hoping that incidentally the students will mature? Or are we going to emphasize independence, analysis, and communication, hoping that along the way the students will pick up the facts and algorithmic skills?

In the first case we plan more in terms of our lectures, in terms of what *we* will do. In the second case we plan more in terms of the homework, in terms of what the *students* will do.

In the second case we would examine the exercises and ask "What is the purpose of this exercise?" Is it to check a definition or a theorem or the execution of an algorithm? Such exercises have their place, but they should not be the last word. They represent one coin of Gresham's law; they are designed to have a closed field Blinders are placed on the student to focus attention on particular facts or skills For instance, we may ask the student to factor $x^4 - 1$

An open-field exercise puts no blinders on the student.

We might ask, "For which positive integers n does $x^2 - 1$ divide $x^n - 1$?" An open-field exercise may not connect with the section covered that day; it may not even be related to the course. Such an exercise may require a student to devise experiments, make a conjecture, and prove it. If it has all three parts, it is a "triex", which is short for "explore, extract, explain" or for "try the unknown". But it may have only the first two parts, amounting to "find the pattern". Or it may have only the last two parts. For instance: "If a continuous function defined on the x-axis is one-to-one must it be a decreasing function or else an increasing function?" This could be reworded to become just the third part of a triex: "Prove that a one-to-one continuous function defined on the x-axis is either an increasing function or a decreasing function." Since experiments with such functions are not feasible, this exercise does not lend itself to the full triex form. However, the following exercise does.

"Does every convex closed curve in the plane have a circumscribing square?"

The way we word a problem may determine how closely it approximates a full triex and where it stands on the "closed-open" scale. Here is an illustration in which each variation enlarges the field from closed to open. At each stage the student is offered more responsibility, more chance to develop self-reliance.

> *First formulation:* Prove that if 3 divides the sum of the digits of an integer, then 3 divides the integer.
> (This is the narrowest form, just the last part of a triex.)
> *Second formulation:* If 3 divides the sum of the digits of an integer, must it divide the integer?
> (This opens up a bit of the second part of a triex, but the student can guess, "Of course, why else would the instructor ask?")
> *Third formulation:* Let d be one of the integers 2 through 9. If d divides the sum of the digits of an integer, must it divide the integer?
> (This is a full triex. There are no clues to the answer. The student must experiment and conjecture.)

The following exercise has a closed field: Prove that when a segment AB is cut into segments by dots labelled either A or B, then the number of segments having both labels is odd. It can be recast to have an open field: (a) Draw a segment AB and cut it into segments by dots that you label A or B. Count the number of segments AA, the number AB, and the number BB. (b) Do this several times and on the basis of your experiments make at least one conjecture. (c) Prove your conjecture. See [2, 3, 4] for more examples.

So the simplest way to resist the assault of Gresham's law is to include exercises that are not simply routine. To do this, it helps to go beyond the usual ways we contrast exercises as "easy" versus "hard", "short" versus "long", "new" versus "review", but to think in such dichotomies as "computation only" versus "exposition required", or "closed field" versus "open field".

But choice of exercises comes late in the game. Other steps can be taken earlier.

1. *Curriculum reform.* As we propose a new course or curriculum, we should think in terms of the student, not just in terms of the topics. The temptation is to make a neat outline of chapters and sections, leaving skills in analysis and communication to develop magically on their own.

2. *Planning a course.* As we work out the day-by-day schedule of a course we should put concern for the student's growth at least on a par with concern for particular topics. This means that we may sacrifice some traditional topics to make time for other matters.

3. *Texts.* When writing or adopting texts, we should pay attention to the exercises that provide an opportunity to explore, conjecture, and write. This means checking that there are enough open-field exercises.

4. *Feedback.* The student's work on open-ended exercises requires more careful reading and criticism than do routine computations. An instructor who does not have the assistance of prematurely wise undergraduates or graduate students will have to read papers carefully. This requires time.

These are a few ways to resist Gresham's law of mathematical pedagogy. Perhaps there is another law that reads, "If each of us tries, we can repeal Gresham's pedagogic law."

Bibliography

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