The Utility of Mathematics Education: Some Responses to Scepticism

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A comforting feature of the latest version of the National Curriculum in England (D*f*EE and QCA, 1998) is that, unlike its predecessors, it explicitly addresses the purposes of teaching and learning mathematics, if only in a cursory way. More specifically, it contains a statement of the "importance of mathematics" (p. 60). [1] The statement is inevitably compressed and is really a collection of 'sound bites' that reflect much of what has been said over the centuries. Whether or not it is sufficient to satisfy the pupil who seeks the point of learning mathematics, or the parent, teacher or policymaker who is concerned with the place of mathematics within education is highly unlikely. To be told, for example, that mathematics "equips" one with "a uniquely powerful set of tools to understand and change the world" or that one will from time to time enjoy moments of "pleasure and wonder", may have a rather hollow ring. Further elaboration is, understandably, not possible in the document. On the other hand, to leave matters here is to suppose a tidy answer can be given to the question "why *must* everyone learn mathematics?", which is misleading.

Although philosophical work has been carried out on the aims of education in general (White, 1982; Wringe, 1988; White, 1990), very little sustained work of this kind appears to have been carried out in mathematics education. It would be exaggerating to say that there is little consensus on the matter, but there is sufficient disagreement, lack of clarity and modesty to warrant further enquiry (Andrews, 1998; Huckstep, 1999).

Justifications for engaging in any activity fall into two broad categories: either the activity is a means of achieving another desired goal, extrinsic to the activity, or the activity has intrinsic value. More informally, the activity is either *useful* for something else or it is carried out *for its own sake*. A satisfactory rationale for learning mathematics involves establishing a symbiosis between these categories. Yet Howson (1982) claims that what he calls the 'practical' and the 'contemplative' aspects of mathematics provide a source of tension which runs deep in mathematics education. Speaking of mathematics since the fifteenth century, he writes:

> The subsequent history of mathematics education in England is largely a chronicle, on the one hand, of how this problem was ignored - with the result that a bipartite system of mathematics education was effectively created - and, on the other hand, of how individual educators have constantly sought to effect a reconciliation. (p. 5)

The current article seeks no such reconciliation. It examines and attempts to resolve some difficulties in supposing that mathematics *is useful* (or 'practical'), since all is not well with this notion.

Aims and purposes

In their article 'On the aesthetics of mathematical thought', Dreyfus and Eisenberg (1986) write that:

> One of the major goals of mathematics teaching is to lead students to appreciate the power and beauty of mathematical thought. (p. 2)

Following Poincaré (1946/1956) and Papert (1980), the authors are at least partly concerned with the internal role that aesthetics has in the processes of problem solving, thereby suggesting that aesthetics, in this respect, plays a regulative role in reaching mathematical results. This also suggests the aesthetic ideals of mathematics may be silent on the question of what the results they spawn are *for*.

Indeed, aesthetic awareness thus conceived seems to be perfectly compatible with a thoroughgoing utilitarian view of the *purposes* of mathematics. Yet one of the persisting notions of aesthetics is that it involves a certain kind of enjoyment *for its own sake*. Nevertheless, it is possible to uphold a fairly balanced position on the relationship between the beauty and the utilitarian value of mathematics, as writers such as Sawyer (1955) have done. A discussion of the merits of such a balance is not the main concern of this article. Important as the question of aesthetics in mathematics is, I am ushering it in as a stalking horse in order to explore a more fundamental issue.

What I want to do is to highlight some more general difficulties that emerge in justifying the learning of mathematics on the strength of its supposed utility. Firstly, I want to maintain a distinction between aims and purposes [2], a distinction drawn out by such questions as 'What are we trying to do *in* mathematics education?' and 'What are we trying to do it *for*?' The first is a question internal to the enterprise whilst the latter is external, or extra-mathematical. To draw pupils' attention to the aesthetics of mathematics may be an aim - something we are trying to do within mathematical education - but this remains distinct from the question of what such a mathematical education is for.

One immediate response to all this might be to reply that even if we can distinguish aims and purposes, why should it be supposed that the latter question is worth asking? After all, the answer to it seems clear. As Perry famously put it at the beginning of the twentieth century:

For the Learning of Mathematics **20**, 2 (July, 2000)
FLM Publishing Association, Kingston, Ontario, Canada

> the study of Mathematics began because it was useful, continues because it is useful and is valuable to the world because of the usefulness of its results. (cited in Griffiths and Howson, 1974, p. 17)

Indeed, the two questions 'What is X for?' and 'What's the use of X?', though not identical in meaning, are nonetheless somewhat interchangeable in uncritical discourse. We are so prone to equating the reasons for doing something with its uses, that the questions 'What is mathematics for?' and 'What's the use of mathematics?' have a close similarity in meaning in education, as elsewhere. It is not surprising that the purposes of mathematics frequently comprise simply a catalogue of uses to which it may be put.

Davis and Hersh (1980), with a touch of irony, write:

> A pedagogue - particularly of the classical variety - might tell us that mathematics is useful in that it teaches us how to think and reason with precision. An architect or sculptor - again of a classical sort - might tell us that mathematics is useful because it leads to the perception and creation of visual beauty. A philosopher might tell us that mathematics is useful insofar as it enables him to escape from the realities of day-to-day living. A teacher might say that mathematics is useful because it provides him with bread and butter. A book publisher knows that mathematics is useful for it enables him to sell many textbooks. An astronomer or a physicist will say that mathematics is useful to him because mathematics is the language of science. A civil engineer will assert that mathematics enables him to build a bridge expeditiously. A mathematician will say that within mathematics itself, a body of mathematics is useful when it can be applied to another body of mathematics. (pp. 79-80)

Nevertheless, writers do from time to time raise questions not only about aims but also about purposes. Moreover, Costello (1993) seems to be at home with asking for aims - or what he calls 'intentions' - he is less comfortable with determining the purposes of the subject:

> There are plenty of people who believe that mathematics education has lost its way. Newspaper reports in recent months confirm this feeling; but the saddest part of this concern is the suggestion that we need to go back - to recover some perceived strengths of a previous age. Of course, this is irrational prejudice. But it does oblige those of us concerned with teaching mathematics to examine our intentions. We need to ask *"What is progress?"*, or even more pretentiously *"What is it that mathematics education has to contribute to civilisation?"* (p. 41)

But even if the question of purposes does have something of a 'pretentious' air about it, prominent writers have nonetheless set out some extensive answers to it. For example, in his classic *Mathematics in Western Culture*, Kline (1972) shows how mathematics has been indispensable in the development of such disparate areas of civilisation as fine art, welfare and scientific research. No one could gainsay this. As if this were not enough, various writers still, from time to time, ask non-rhetorically what mathematics education is *for*. What is more, there has been fundamental disagreement amongst some of those who provide answers.

Notwithstanding the all-embracing view of usefulness adopted by Perry above, writers who touch on questions relating to the rationale for mathematics do not always equate the purposes of mathematics with the uses to which it may be put. They sometimes identify not only the *kinds of uses* but rather the *kinds of reasons* for learning the subject. Occasionally, some of these writers even take the radical position of rejecting usefulness as one kind of reason. Thus, it has seemed to one mathematics educationalist recently that:

> one of the biggest stumbling blocks to mathematical education in this country is the continuing tradition of justifying mathematics on the basis of its usefulness. It is a myth and we have peddled it for decades. (Andrews, 1998, p. 3)

The scepticism that Andrews expresses here arises from reflecting upon the extent to which he himself makes use of mathematics in everyday life. As a teacher of mathematics, he clearly knows and understands how to use and apply a good deal of mathematics. Yet so often when it comes to using this mathematics, he admits that he often unnecessarily relies on other people or machines to supply him with answers. He believes that this is the case with most pupils. From this, he draws the conclusion that much of the mathematics that we teach cannot be justified on the basis of its usefulness, simply because *as a matter of fact* it is not used very often by those who learn it.

It thus seems that one concern of mathematical educationalists is that although mathematics is *potentially* useful, their pupils might not choose to use it. Of course, whether or not pupils do use mathematics as much as we might hope is an empirical question, but Andrews' supposition is a plausible one to dwell upon. Before seeking the counsel of despair, we should ask how we might respond to it. At least three possible responses are worth considering.

Firstly, we may agree that the relationship between mathematical knowledge and its use by pupils in their daily lives is contingent upon their own decisions. But we may suggest that one reason for the pupils' failure to make continual use of mathematics is that they are less *aware* of these uses than we might otherwise suppose. Indeed, we might even argue that mathematical understanding is to some extent a function of the awareness of mathematical purposes.

Secondly, we may agree that the usefulness of mathematics is contingent upon pupils' own choices to make use of it, and upon the importance of teachers' making them aware of such uses. It might be held that pupils' choices to use mathematics in appropriate situations should not simply be left to chance: so that not only should they be *taught that* mathematics is useful but they should also be *taught to* use it. Thus, alongside capacities to use mathematics *settled dispositions* to use it should also be taught.

Yet a third response is to suggest that the question of the choice to use mathematics is not an issue. Mathematics, it is argued, is in a sense *inescapably* already 'in use' in our lives. So that the purpose of teaching mathematics is not so much to show that it *can* be used, nor that it *should* be used, but is to *reveal* that it is used in some special, non-obvious way

I shall discuss each of these replies in more detail, although most of my attention will be directed to the third response

Use and cognition

As I have already remarked, whether pupils do or do not choose to use much of the mathematics that they have learnt is an empirical question. It certainly would be a matter of concern if pupils seldom chose to use *any* mathematics in their daily lives. Yet it may be safely supposed that their use of *some* rudimentary mathematics is inevitable. One hardly need bemoan that pupils are taught to count yet that they do not often make use of it in their lives.

It has recently been stressed that the purposes of even such rudimentary aspects of mathematics as counting do need to be made explicit to pupils. Indeed, it has been argued that pupils' beliefs about the purposes of counting are *internally* linked to their understanding or *cognition*. According to Munn (1997), young children are competent with number yet have an inability to count objects when randomly arranged, failing to understand conservation number by tending to judge the relative quantities by sight

This apparent contradiction Munn argues arises from researchers' focusing on what she calls 'external' factors, and can be removed by attending more to subjective considerations of pupils' beliefs about counting She argues that what have been seen as errors in understanding are internally linked with pupils' ideas of the purposes of counting. Munn, in her research, asked the following questions to the children: 'Can you count?', 'Can you count these blocks?', 'Could you give me (one, two, three, etc.) blocks?', 'Do you count at home?', 'What do you count at home?'. She did this in order to gain some insight into children's *beliefs* about counting, and found that few pupils understood 'the adult purpose of counting' before formal schooling, even those who were competent in counting out several blocks.

The pupils did reveal their *own* reasons for counting, some simply enjoying it, for its own sake - what Munn calls 'counting to please the self' - others counting merely as part of a social activity The upshot of this is that development in children's counting is partly a function of their awareness of its purpose Munn concludes that teachers should "make the purpose of counting explicit for children" (p 17) and also that they should "stimulate children to develop their own numerical goals" (p. 18) So even in one area of mathematics that seems to be least in need of justification, the question of purpose is not simply a reasonable one to ask, it is one that has a bearing on a pupil's understanding

In making an empirical connection between cognition and purpose, Munn shows that the pupils are not simply motivated by being aware of the purposes of what they are learning but that knowledge of purposes is an integral part of the teaching. Munn's view is not an isolated one, nor does it apply only to mathematics in the early years.

Sierpinska (1994) discusses a similar situation within the broader aspect of what she calls 'theory' in higher mathematics Like Munn, she argues that understanding is often judged by external factors when, for example, it is a matter of determining whether what students know is consistent with a particular theory which they are learning. But she adds:

> When it comes to understanding not a particular concept of a theory or a particular method but the theory as a whole, when, for example, one asks the question 'what is the point of this theory?', then the evaluation must be more subjective [...] The judgment depends on one's philosophical attitudes towards scientific knowledge [...] *on the goals of learning mathematics* (p. 113; *my emphasis*)

At different ends of the mathematical spectrum, then, writers have assimilated certain questions of the purposes ('goals' or 'the point') of mathematics into the question of understanding it.

The philosopher John Passmore (1980), too, in outlining different ways in which a pupil may not understand something, suggests that a more general conceptual connection between understanding and purpose can be made. He distinguishes (p. 198) not only between a pupil's misunderstanding, failure to understand and partial understanding, but he also briefly raises the case where a pupil *sees no need to understand*. Later, he remarks:

> 'I don't understand the point of ...' is indeed a very common form of not understanding. And often enough [...] *the answer is far from being obvious*. (p. 205; *my emphasis*)

All these writers, discussing the matter at different lengths and levels, suggest that the point of learning mathematics may not be as straightforward as we might suppose It would be putting the matter too strongly to claim that pupils' lack of inclination to use mathematics when appropriate is due to *ignorance* But we might need to reconsider how much it is necessary to reflect upon the point of what we teach

Use and compulsion

Suppose that we do teach mathematics the applications of which, though not necessarily obvious, are drawn out in the classroom, yet on the whole such mathematics does not get used by many of those who have learnt it Does this mean that as educators we have failed in some way? This would certainly be the wrong conclusion to draw if our intentions were simply to enlarge pupils' *options* Indeed, some liberal educational theorists have in the past argued that the particular lifestyle which pupils adopt must, within certain limits, be left open. Living a lifestyle that eschews all but the most minimal use of mathematics, though surely lamentable to a mathematics teacher who believes in the usefulness of mathematics, is on this view up to the pupil to decide The most educationalists can *compel* is sufficient engagement in those activities to place pupils in a position where they can make choices. Any further compulsion would be morally unjustifiable (White, 1973)

The amount of prescriptive detail that may be packed into a short space is often quite startling As far as numeracy is concerned, just one word in 'The framework for teaching mathematics from Reception to Year 6' (DfEE, 1998) seems to rule out as insufficient that we simply set out to enlarge pupils' options by developing knowledge and understanding. As far as primary school is concerned, at least, numeracy *by definition* requires not only:

> a confidence and competence with numbers and measures [but also] an understanding of the number system, a repertoire of computational skills and *an inclination* and ability to solve number problems in a variety of contexts (p. 18; *my emphasis*)

Clearly, ability is one thing and inclination is another. I may have one but not the other. But suppose that I am able, and very confident in being able to carry out all of a large range of number operations, and that I understand the number system – do I fall short of being numerate if I am not *inclined* to use these myself in situations where they arise? To suggest that I am thereby not numerate seems absurd. Yet to insist that I must be so inclined is to prescribe a certain kind of lifestyle, one in which whenever a problem is encountered for which mathematics could provide the answer I am disposed to attempt to solve it mathematically. [3]

Moral education must take the distinction between inclination and ability seriously. Few would now agree with Socrates that good conduct is a function of knowing what is right, and doing wrong simply a matter of ignorance. But other thinkers since then have pointed out that one can know what is right and even know how to how to do it, but nevertheless be weak-willed, or incontinent, when it comes to doing it (or not doing it when restraint is required). Thus, one could with conviction argue that a morally educated person is one who not only knows what is right, and can do it, but also *does* do it. But to view the avoidance of mathematics in situations that warrant its use hardly seems to be a morally indefensible case of backsliding. A quasi-moral approach could be adopted. We could set out to develop certain intellectual courage in our pupils, so that they do not shirk away from difficult problems when they arise nor seek the assistance of others when they could solve the problem themselves. But beyond this, it is difficult to see what else ought to be done.

It is at this point that writers sometimes turn to different kinds of reasons for justifying the subject. They persuade their pupils that mathematics should be pursued *for its own sake*, for enjoyment and fascination, in short as a means of entertainment. However, I shall resist the temptation to discuss theories of this complexion. What I want to consider now is a rather more novel attempt at justification that retains a serious air about it and yet can be contrasted with any view of mathematics as entertainment or a pastime or indeed as a source of aesthetics.

Use and self-consciousness

There is a way of side-stepping the issue of whether or not pupils should be expected to continue to make use of mathematics, in more than a rudimentary way, during their daily lives. This is to argue that we teach mathematics not simply to provide pupils with opportunities to use it *if they so wish*, nor that we need to ensure that they will use it, but rather to reveal to them that they already do use it in some special way. This response rests upon the idea that mathematics is inescapably already in use in their lives, and studying mathematics is a form of *self-knowledge*.

Brent Davis (1995) finds the usual, commonplace assertions about the utility of mathematics unsatisfactory and seeks a need for learning the subject which, as he puts it:

> should not be understood in the utilitarian terms of equipping children with the skills necessary for adult life, nor in the political terms of providing the understandings needed for democratic citizenship (p. 6)

If Davis' argument is sound, then it ought to be of some considerable significance to those who are sceptical about the utility of mathematics. However, convincing *alternative* (rather than supplementary) justifications for learning mathematics are difficult to assemble and we should not be surprised if difficulties emerge in any such attempts.

Davis introduces his distinctive, non-utilitarian need for mathematics by reflecting upon what we might call the 'unsayable' in mathematics. From a transcript of a classroom episode, he focuses on a remark of a pupil, Jiema, who appears to permit degrees of equivalence in fractions. That is to say, she asserts that 3/9 is not as equivalent to 2/6 as certain other fractions are, in particular 1/3 and 4/12. Davis interprets this response in a novel way. In accordance with the view that pupils' misconceptions are worth dwelling upon rather that being ignored, he welcomes this one as being at odds with, and thus as a 'challenge' to, the strictly categorical, or indeed *dichotomous*, way in which mathematical and logical propositions have been traditionally viewed since at least the time of Aristotle.

Viewed in this way, one fraction either is or is not equivalent to another fraction; there are no intermediate states. I take it that Davis himself does not believe that there should be degrees of equivalence, only that the unorthodox nature of Jiema's assertion somehow throws the whole question of strict categorical ways of thinking back at us. It certainly reveals something about the nature of discourse and thus appears to Davis to have considerable educational value.

In fact, reflection upon categorical discourse was already treated as a source of significant educational conclusions in antiquity. From the fragments of what remains of Parmenides' writing (ca 500 BC), we find him asserting that what is *not* is not sayable or thinkable (Barnes, 1987, p. 134). Whatever exists must always have existed, otherwise there must have been a time when what *is* was *not* and this cannot even be *conceived*. Thus, change is impossible!

As a recent writer has put it:

> What Parmenides suggested was that ordinary talk about the things around us is vitiated by a deep contradiction. We say of things, in the normal way, that they are, say, large but not, say, green. But how can something both be and not be, any more than it can walk and not walk? No doubt this question seems naïve, even childish, and to be susceptible of being explained away as a confusion of a fairly elemental kind. And indeed no doubt Parmenides' question is, in a way, confused, but the confusion is far from elementary, and the ruthless seriousness with which Parmenides took it was to have a profound effect. For Parmenides advanced, in a perfect example of metaphysical

> thinking, from the humble origins of his argument to the extraordinary conclusion that the world is not in fact made up of a large variety of things but of one thing only, which exhausts the whole of being. [...] this doctrine retains its fascination for all the logical cold water that has been poured on it. (Lawson-Tancred, 1998, p. xvi)

This far-flung comparison of Davis' view with that of Parmenides is made, firstly, to hint at the metaphysical reasoning in educational theory that is necessary to articulate such a bold account as Davis has set out. From the humble origins of Jiema's utterance, we have a conclusion of enormous scope. But there is more to come. This famous pronouncement of Parmenides lay behind an attempt to provide a justification for the learning of mathematics in antiquity that did not appeal to practical utility. Indeed, it is one of the first of its kind on record.

In *The Republic*, Plato lures his interlocutor into the Parmenidean dilemma of trying to say the unsayable, and it seems to me that a persistent and articulate Jiema speaking to her fellow classmate might equally have said something like the following:

> 'And what about the many things which are double something else? If they are double one thing can't they be equally well regarded as half something else?' 'Yes.' 'And things which we say are large or small, light or heavy, may equally well be given the opposite epithet.' 'Yes, they may be given both.' 'Then can we say that any of these things is, any more than it is not what anybody says it is?' (Plato, 1955 edn, pp. 275-6)

Far from accepting the Parmenidean dichotomy, Plato entertains something more like Heraclitus' position, *viz* everything is in a state of flux. Those things about which we seem to be able to say both that they are and that they are not, are, for Plato, only *becoming*. Moreover, Plato claims that all objects of sense perception are in this category. For this reason, he equates knowledge with the apprehension of objects in a supersensible realm of Forms, since it is only these objects that are literally changeless.

I am not suggesting that Davis is a Platonist, not least since this is something he clearly denies early on in his article. Nevertheless, there is an important point of agreement between certain aspects of their theories. Both dwell upon assertions of the intermediate state between *what is* and *what is not*. It was Plato's view that the perception of such objects as his fingers, for example:

> don't simultaneously issue in a contrary perception (p. 329)

However, he noticed that other perceptions, particularly, though not exclusively, those involving quantitative comparisons are different in this respect. We have already seen some examples of this. Another example of his concerned our perception of singleness or unity since, as he put it:

> we see the same thing both as a unit and as an unlimited plurality. (p. 331)

In this way, every object both *is* and *is not* a single entity. What this showed was that in perception everything is *becoming* for the percipient. Thus, certain things *become* a double or a half, light or heavy, consist of one part or many, according to the way in which the perceiver views them. But this only led Plato to believe that perception is not a source of *knowledge*.

Plato held the now-unfashionable view that the only candidates for knowledge are those objects that simply *are*. Objects that *are not* are a source of error or ignorance. Somewhere in between there is that which is *becoming*, which more modestly gives rise to belief (or opinion). Since perception does not provide us with what is, but only what is becoming, Plato infers that the world which we perceive is not reality. The value of mathematics, or at least in terms of the kinds of perceptions that mathematical statements when applied to objects of sense experience provide, is that it:

> draws the mind upwards and forces it to argue about numbers in themselves, and will not be put off by attempts to confine the arguments to collections of visible or tangible objects. (p. 332)

Since I took the liberty, earlier, of putting Plato's words into Jiema's mouth, perhaps the favour should be returned. Plato, rather than Davis, might just as well have concluded his non-utilitarian rationale of mathematics by claiming that the subject "offer[s] a rich ground for exploring what tends to be taken-for-granted" (1995, p. 6).

Few if any would accept Plato's metaphysical picture of reality. So there are notable differences between the accounts of Plato and Davis. What Davis believes we take for granted, and which mathematics offers a rich ground for exploring, is not our assumption that perception constitutes reality. He is not urging us somehow to look beyond perception but rather within it. He is partly concerned with the mathematical basis of that perception, not its validity as a source of knowledge. Even if he does suggest that mathematics is ubiquitous in perception, he does not want to challenge the status of that perception. He is not suggesting, like Plato, that we should eschew the value of sense perception in favour of an ultimate 'reality'.

So what is it that engagement in mathematics challenges for Davis? What does its purpose become when it is considered thus? Why does he believe we *need* mathematics? The answer to this question is less clear, but whilst for Plato it is perception itself which has shortcomings, for Davis it seems to be the nature of mathematical assumptions which are at issue. They are in principle always subject to revision, as the discovery of alternative consistent geometries has conclusively shown. What is not clear, however, is the extent to which a pupil can, and indeed needs to, take part in the revision of mathematical assumptions. Stripped of the metaphysical background of the ancient thinker, Jiema's remarks do not seem to have quite the same kind of force in Davis' theory as they would in Plato's.

I have already said that Davis seems to be suggesting that an important purpose of mathematics lies in its being a vehicle for a special kind of self-knowledge. He prefers to look at such instances as Jiema's remarks not so much as a revelation but as something which *challenges*. All the same, what is challenged, he suggests, is "not only what we know and believe, but who we are" (p. 5). Whether or not this may be characterised as self-knowledge, one thing is clear: the *inwardness* invoked in this remark certainly distinguishes his conclusions from Plato's

Of course, the presentation of mathematics as a vehicle for self-knowledge does not constitute a strictly utility-free justification. If mathematics is such a vehicle, then it does have a certain kind of utility for the educator and thus ultimately for the pupil. But we can surely let this objection pass. We can also let pass the way in which Davis allows mathematics to constitute a *need*. Needs are always prerequisites which satisfy ends which are themselves either the prerequisites for yet further ends or are ultimate ends which invoke value judgements. To couch mathematics in terms of needs does imply that ends such as self-knowledge are of value to the pupil.

But there remains a nagging doubt about insisting that the over-riding point of mathematics is satisfactorily articulated in terms of its being a form of self-knowledge. Although for Plato such reflection depended upon a mature approach to the subject, it is in the elementary areas of mathematics that this typically takes place. So that to benefit from the 'paradoxes' or the 'unsayable' in mathematics, few convincing reasons are given by Plato for sustained learning in more advanced areas of mathematics for this purpose, even if such areas are not ruled out as appropriate 'sites' for contemplation and self-knowledge.

The same is true of Davis' account. As he acknowledges:

> even with the most basic of topics, occasions for mathematical anthropology do arise. (p. 6)

But as I have remarked earlier, it is not the more rudimentary parts of mathematics that are typically in need of justification. In short, these arguments seem to be used to justify mathematics at a place where utility arguments are at their strongest. What still seems to be missing from both Plato's argument in antiquity and Davis' recent theorising is an account of why there is a continued need to undergo a sustained and progressive study of mathematics for those precise goals that both theorists seek

Conclusion

Despite the attempts of purists or pragmatists to understate or reject the justification of learning mathematics on the strength of its supposed utility, the view that mathematics is of crucial importance in every pupil's education because it is useful is as resilient as ever to criticism. Whilst it is clearly not sufficient to argue that simply because mathematics *can* be used and *has* been used there are strong enough reasons to suppose that it *will* be used by every pupil who learns it [4], usefulness has a certain *prima facie* attractiveness not found in other justifications.

We are able to point out transparent cases where many individuals use at least some in their everyday lives and where some individuals use much of it. Moreover, in this respect, it enjoys an unrivalled position in the curriculum. Self-knowledge is a relatively clear and broad notion within the fine arts, and the social sciences, particularly psychology, but it remains rather more obscure in mathematics, especially when it is compared with the familiar notion of utility. This is no reason to give up on radically alternative justifications for the learning of mathematics, but I hope to have begun to show what we are up against in such a quest.

Notes

[1] Evidence that there has been some ratiocination over this statement lies in comparing the final version with the draft proposals. In the earlier document, it was the "distinctive contribution" of mathematics that was being set out rather than just its "importance".

[2] I owe this general distinction to T. W. Moore (1982), but it has also been employed in the context of mathematics education. In the document *Mathematics 5 to 11. A Handbook of Suggestions* (DES, 1979), for example, an explicit distinction is made between: 'purposes of teaching mathematics', 'mathematical aims' and 'objectives'. The lattermost is clearly a sub-class of aims and is not directly relevant to this article

[3] Indeed, if the way Andrews (1998) describes himself (above) can be characterised as a lack of inclination, and it is difficult to see how it could be described otherwise, this definition of numeracy renders him innumerate!

[4] As Davis (1995) points out:

> we have repeatedly demonstrated ourselves to be poor predictors of the sort of competencies that will be needed even a few years hence. (p. 6)

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