WAYS THAT RELENTLESS CONSISTENCY AND TASK VARIATION CONTRIBUTE TO TEACHER AND STUDENT MATHEMATICS LEARNING

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Laurinda Brown used the notion of relentless consistency to encapsulate a commitment by teachers to engage students in taking responsibility for their own learning. Brown and Coles (2013) wrote:

> In designing and implementing tasks, teachers have, as a base for decision-making, the classroom cultures they have already established with their students. […] So, once a culture in a classroom has been established, through the relentless consistency of practices, and children know what to do to support their learning, this, according to Fullan, frees 'up energy for working on innovative practices'. (p. 617)

In this article, we provide a rationale for the principles and processes of designing and implementing sequences of learning that use the concepts of task variation and learning trajectories to exemplify this relentless consistency with the intention of facilitating and encouraging innovative practices. We are working on a project to develop and research such sequences that not only encapsulate relentless consistency in terms of classroom culture, task design and lesson structure but also in terms of mathematical concepts that are the focus of the learning. Brown and Coles used the phrase *relentless consistency* to refer to actions by teachers in classrooms. We include actions both of teachers and teacher educators working with practitioners.

The following describes our perspective on 'challenge', how challenge is connected to the structure of mathematics lessons, our interpretation of the ways that sequences might support learning, the justification for making specific suggestions for teachers, and the importance of a positive classroom and teacher professional learning culture. An extract from one of the sequences is presented to exemplify our overall approach and to pose some implications for mathematics teacher education. The theme of relentless consistency permeates each of the sections.

The nature of mathematical challenge

Fundamental to our approach is a belief that learning is most effective when students are engaged in higher order thinking as they work on tasks that are appropriately challenging for them. A task or problem is challenging if students do not know initially how to proceed, have not been told how to do so by the teacher, and are expected to make decisions on solutions or solution strategies for themselves. Of course, struggle associated with challenge can be alienating (Smith, Grover & Henningsen, 1996) and so the challenges need to be within the student's Zone of Proximal Development. Many mathematics concepts are difficult to understand, at least initially, and so students benefit when they persist with concepts and tasks that include concentrating, applying themselves, believing they can succeed and connect effort with learning. The tasks and lessons likely to foster such actions are termed 'challenging' in that they allow the possibility of sustained thinking, decision making and some risk taking by students. As Wiliam (2016), quoting Daniel Willingham, wrote:

> Students remember what they have been thinking about, so if you make the learning too easy, students don't have to work hard to make sense of what they are learning and, as a result, forget it quickly. (para. 17)

In other words, students are more likely to make sense of mathematics and remember what they have learned if they work on tasks that are appropriately challenging.

One of the frequently expressed concerns by teachers is that students are motivated by success and teachers see the need for success as contradictory to the notion of challenge. Yet Middleton (1995) argued that, while students are indeed motivated by success, these successes need to be genuine, not merely the completion of simple tasks. More importantly though, for student motivation, is what he termed control, which we interpret as students making their own choices of the type of solution or solution strategies when solving tasks. In fact, we consider that 'engagement' is more a product of control than it is of perceived relevance. Further, this control also connects to the goal of prompting students to generalise their solutions. This is discussed further below.

Challenge comes when students do not know how to solve the task and work on the task prior to teacher instruction. Other characteristics of such tasks are that they:

- build on what students already know;
- take time;

- are engaging for students in that they are interested in, and see value in, persisting with a task;
- focus on important aspects of mathematics (hopefully as identified or implied in relevant curriculum documents);
- are simply posed using a relatable narrative;
- foster connections within mathematics and across domains; and
- can be undertaken when there is more than one correct answer and/or more than one solution pathway.

We propose that teachers incorporate such tasks into their routines frequently. Some examples of challenging tasks, along with an indicative age level at which we anticipate most students would experience challenge, are as follows:

> [Grades 1–2] Three students between them have four 10 cent pieces and two 5 cent pieces (that information is presented pictorially). They each have different amounts of money. How much might they each have?
>
> [Grade 3–4] Some people came for a sports day. When the people were put into groups of 3 there was 1 person left over. When they were lined up in rows of 4 there were 2 people left over. How many people might have come to the sports day? Describe the pattern in your answers.
>
> [Grades 5–6] Five friends shared all of this chocolate (a diagram of a rectangular block, 5 squares long and 4 squares wide), but they each got different parts of the whole. Describe using decimals, what parts of the chocolate each of them might get. Give as many different possibilities as you can.
>
> [Grades 7–8] I know $4a + 5b = 120$. What might be the values of a and b? (Give a range of possible answers)
>
> [Grades 9–10] Five linear functions go through the point (–3,4). What might be their equations?
>
> [Grades 11–12] A function has a turning point at (–2, 3). What might be the function? Give at least three possibilities.

Readers are encouraged to work through each of the example tasks, especially ensuring that you have found all (or at least many) possibilities. While at the level of finding one answer, the questions address the content of the curriculum. In developing a generalisation about the range of possibilities, students are doing mathematics.

Structuring lessons

Using such challenging tasks requires a different lesson format from one that starts from the teacher telling students what to do. In our project, we propose a particular structure that can support teachers in the consistent incorporation of challenges into their repertoires. Specifically, teachers are encouraged to incorporate the following elements proposed by Sullivan, Borcek, Walker and Rennie (2016):

- tasks are posed without instructing students on solution methods;
- students are allowed time to engage with tasks initially by themselves, perhaps later in small groups;
- actions are taken by the teacher to differentiate tasks for students who might require additional support and those who finish quickly; and
- responses to the tasks are observed and selected by the teacher during the lesson to orchestrate classroom dialogue between students, emphasising students' explorations and mathematical thinking.

In other words, fundamental to the notion of challenge is that students have to puzzle over a solution strategy for themselves. We note that there is some evidence that a lesson structure beginning with more explicit teacher guidance can still promote student 'puzzling', provided that the tasks are sufficiently challenging, and the teacher is committed to maintaining this level of challenge as the task unfolds (Russo & Hopkins, 2017). Explicit guidance does not imply the teacher explaining solution pathways but can mean eliciting prior knowledge and insights from the students as a starting point for inquiry. Indeed, both task-first lesson structures and discussion-first lesson structures have been demonstrated to generate substantial learning gains, suggesting that there is more than one way to effectively teach with challenging tasks (Russo & Hopkins, 2018). However, at least according to teacher-observers, there appear to be distinct benefits to a task-first lesson structure compared with a discussion-first structure. In particular, the fostering of mathematical creativity, the discovery of novel solution methods and the rich mathematical discourse generated post-task (Russo & Hopkins, 2017). It is our contention that these benefits, particularly the latter, are critical to realising the full potential of teaching with challenging tasks over the longer term. Perhaps most critically, the explicit intent is that students learn not only from the struggle with the task, but also by listening to suggestions for the solution and solution strategy proposed by other students, with productive discussion orchestrated by the teacher through effective questioning of students.

Smith and Stein (2011) outline five practices that inform the aspect of lesson structure that involves promoting classroom discussion and argumentation. These practices include anticipating the expected responses, monitoring student work (which of course connects to the formative assessment potential of such learning), selecting examples for students to present, sequencing the presentation of those responses and connecting the students' responses to the mathematical purpose of the task on which the students work. To these five practices, we add the notion of encouraging students to listen to others and using efficient ways to allow students to present their completed work to the class.

There is a further key aspect missing from the Smith and Stein practices, which refers to processes for supporting students experiencing difficulty and extending those who are ready. We have noticed that some teachers are concerned at the risks associated with posing challenges for fear that some students might find the challenge unproductive. Many of our suggestions can be described as 'low floor and high ceiling tasks' which can also be effective in including students for whom the challenge is especially difficult. However, we also encourage teachers to pose enabling prompts to students that are a variation on the main task but created by posing a change to the representation, the size of the numbers, or reducing the number of steps. The intention of the enabling prompts is that once the prompt is completed, the students return to the original challenge. An example of such a prompt is presented below. We also recommend that teachers prepare prompts to extend the thinking of students who have solved the initial task. Such extending prompts are best when they prompt abstraction and generalisation of the ideas represented by the initial challenge.

The lessons also follow the common triad of 'Launch' (without telling the students what to do), 'Explore' (in which students engage with the problem by themselves or in small groups) and 'Summarise' (in which the teacher elicits from the students their insights and solutions), with the important variation that this triad can occur more than once in a single lesson.

It is emphasised that our approach does not represent unstructured inquiry but that the teacher has a specific instructional role. We agree with Lerman (1998) who elaborates the Vygotskian opposition to pedagogies that seem to require students to "rediscover the development of mankind for themselves" (p. 69). Rather, Lerman argues that mathematics learning is centrally concerned with "the mediation of cultural tools and of metacognitive tools" (p. 69). For both of these, some active teacher guidance is needed, although after students have experienced the tasks for themselves. The notion that the teacher should play an active role in structuring the inquiry experience in a careful and deliberate way, to ensure that learning is maximised, has broad empirical support. For example, Alfieri, Brooks, Aldrich and Tenenbaum (2011), in their meta-analyses incorporating 164 studies across multiple educational settings, reveal the need to carefully distinguish between structured inquiry-based approaches and unstructured inquiry-based approaches. They find that unstructured inquiry is inferior to more explicit instructional approaches in terms of its impact on assessed student learning, whereas structured inquiry is superior to all other instructional approaches. The authors surmise that, "participation in guided discovery is better for learners than being provided with an explanation or explicitly taught how to succeed on a task" (p. 11).

In our experience, teachers often report spending something like 10 minutes on number fluency games and activities at the start of lessons. We see advantages in the fluency activities preparing students for the upcoming experiences, which can in turn have the effect of reinforcing the initial fluency development. Consequently, the inclusion of such fluency activities coheres with our suggested approach to structuring lessons.

We propose that teachers use the lesson structure described here consistently and relentlessly. Sullivan *et al.* (2016) add a further element in which teachers pose additional similar tasks with the intention of consolidating the learning (Dooley, 2012). We term the set of such tasks a 'sequence'.

The notion of sequences

We are also exploring the possibility that learning will be enhanced if purposeful follow up tasks are posed to consolidate learning. This process for consolidating learning is connected to considering sequences or trajectories of learning over a longer time frame, rather than a single task and lesson.

We propose that teachers pose further tasks that are in some ways similar and in some ways different from the initial task. If, on one hand, the teacher keeps the context the same but varies the concept, this can contribute to understanding and the fostering of connections within mathematical domains. If, on the other hand, the teacher keeps the concept the same but varies the context, this is intended to prompt transfer and the stimulation of connections across domains. Variation Theory informs the design of these learning sequences. Kullberg, Runesson and Mårtensson (2013), for example, argue:

> In order to understand or see a phenomenon or a situation in a particular way one must discern all the critical aspects of the object in question simultaneously. Since an aspect is *noticeable only if it varies against a background in invariance,* the experience of variation is a necessary condition for learning something in a specific way. (p. 611)

Connected to the notion of consolidation and the creation of sequences is what Simon (1995) describes as a hypothetical learning trajectory that:

> provides the teacher with a rationale for choosing a particular instructional design; thus, I (as a teacher) make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions [...] as well as the spontaneous decisions that I make in response to students' thinking. (pp. 135–136)

Such a trajectory is made up of three components: the learning goal that determines the desired direction of teaching and learning; the sequence of experiences to be undertaken by the teacher and students; and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136). The learning goal can be related to the documented curriculum or it can be an outcome of research into particular aspects found to be difficult for some students to learn.

These predictions are not related to students listening to a hierarchy of explanations but to them engaging with a succession of problem-like tasks. In planning (and teaching), the role of the teacher is to identify potential and perceived blockages, prompts, supports, challenges and pathways. In other words, the learning occurs as a product of students working on sequences of tasks purposefully selected by the teacher and contributing to ongoing dialogue with the teacher and their peers on their strategies and products.

We see the following as some of the advantages of specifically planned sequences of learning for students and teachers.

1. Sequences can help students see the 'bigger picture'. One of the disadvantages of teacher directed approaches to mathematics and numeracy which involve teacher demonstration followed by student practice is that mathematics can seem to be broken into sets of micro skills rather than contributing to

a coherent whole. Sequences may help students see connections by making the big ideas and progression of learning more obvious to them.

2. Concepts are learned as much by what they are not, as from what they are (such as, for example, the attribute of height is different from volume). Carefully varied tasks within sequences can emphasise what the central ideas are (and what they are not) thereby allowing students to discern the essence of concepts.
3. Sequences of challenging tasks can prompt 'light bulb' moments. There are no light bulbs if students are told what to do. Students can benefit from working on tasks that are challenging, and progressively see meaning by experiencing connected tasks with success developing progressively, especially where the insights or 'aha' moments are the result of their own thinking.
4. Sequences can reduce the sense of risk experienced by some students. Many teachers report that some students do not embrace challenges possibly fearing failure. One of the goals of the sequences is for students to see that, even if they cannot do the current task, there is a similar task coming and they can learn how to do subsequent tasks by engaging in the current task, even if not successful yet.

There are some attempts at designing such sequences and working with teachers on such designs (Fonger, Stephens, Blanton, Isler, Knuth & Gardiner, 2018) although their published sequences do not seem to challenge students, nor do they focus on important mathematical connections. Part of our goal is to clarify what sequences might look like, how they might be interpreted by teachers and how they support student learning. We also consider such sequences to be supporting relentless consistency in that teachers and students can see that the solution of the second task is made clearer by working on the first, the third from the second and so on.

The rationale for suggesting sequences to mathematics teachers

We propose to develop a range of sequences, each addressing a key mathematical concept. The intention is that these sequences contribute to improvement in classroom teaching and teacher learning generally. By proposing carefully constructed and effectively trialled sequences supported by related professional learning, teachers can experience not only the notion of sequence but also ways that sequences enhance learning opportunities for students. We assume that teachers will adapt the sequences to align with their programs and usual routines and do not see the provision of resources as 'dumbing down', 'spoon feeding' or adopting a 'deficit stance'. In fact, the goal of offering suggestions for teachers is (to draw on the quotation from the start of this article) to free up energy for them to engage with the complexity of converting task and lesson sequences into learning experiences for their students; and adapting the stories and complexities to suit their particular class and student context. Our participating teachers take an active role in the design of the tasks, lessons and sequences, not only improving on the initial designs, but also gaining important insights into the process of sequence creation.

Because of their similar structure, purposefully designed sequences provide several advantages for our research. The sequences will contribute to relentless consistency in structure and culture, will make the assessment of student learning across classes more comparable, and will allow feedback to be gathered from a range of teachers on the ways that any one sequence is implemented in different classrooms.

Classroom culture

As Brown and Coles argue, a central aspect of the relentless consistency is the culture of the classroom. Of course, this culture includes an expectation that the tasks will be challenging; that lessons will be structured in particular ways, especially allowing students time to work on the task either by themselves or in small groups; that students will work on sequences of tasks rather than one-off problems; and that the tasks will be appropriately differentiated. Further, these ways of working are not intended to be every now and again but applied consistently. As the NCTM (2014) notes:

> Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (p. 17)

Another aspect of the relentless consistency is the stance that students take and their willingness to engage with the learning that such a lesson structure offers. Key to this stance is what Dweck (2000) describes as a growth mindset, in which students believe they can get smarter by trying hard and being willing to embrace challenges.

Central to classroom culture are the norms of activity in mathematics classrooms. We consider the mathematical norms to be the principles, generalisations, processes and products that form the basis of the mathematics curriculum, broadly defined, and serve as the tools for other learning. The socio-mathematical norms encompass not only "classroom actions and interactions that are specifically mathematical" (Cobb & McClain, p. 219), but also the goals of interaction that address elements such as culture, social group, language comprehension and usage and classroom organisation, as they relate to the teaching and learning of mathematics. Examples of such norms are that students are encouraged to listen to each other, and the teacher models the process of taking risks.

Another key example of these norms is the expectation that students will make a start on problems without direct interactions with the teacher. To support this, we recommend that teachers establish ways of structuring student responses. For example, we propose that older students be encouraged to write what they currently know at the top of a page, to have rough working space under that and to write a clear synthesised response below that again. We suggest that teachers consider optimal ways that younger students record attempts and solutions. Another approach might be to establish a class code that specifies steps that students must take before they can ask the teacher a question. Another strategy is to inform students of a ‘zone of confusion’ into which all students go for some time and can only emerge with persistence and effort.

In the end, classroom culture is about social relationships. Lerman (1998) emphasises “the centrality of the social relationships constituted and negotiated during classroom learning” (p. 70). In clarifying this social perspective, Lerman makes use of the Zone of Proximal Development metaphor. Even though ZPD is sometimes used to describe teacher choice of an activity to allow students to step onto the next rung on a ladder of many minuscule steps of mathematics learning, Lerman argued that ZPD is connected to creating classroom environments with conditions that are likely to facilitate student engagement in tasks.

An excerpt from a sequence

The following exemplifies the notion of a sequence. While our current project is working with students aged 5 to 8, this is the first of five suggestions from a sequence intended for students aged 10 to 13. The rationale of the sequence is given as:

> While fractions are used to describe parts of a whole, fractions are also numbers that can be represented on a number line using both mixed numbers and improper fractions. The sequence helps to establish the relationship between repeated addition of the same fraction and multiplication. The suggestions introduce students to the notion of fractions as numbers and different ways we can represent those fractions (materials, pictures, numbers, equivalence) as well as the operations (addition, multiplication). The tasks are posed using proportions.

In the first suggestion, titled *Calculating with halves and quarters:*

> the unit is not a whole number. The emphasis is on imagining buckets and drawing pictures and number lines. There are various ways of approaching this (this highlights that teaching a single method is limiting) so students have an opportunity to explain their thinking.

A variety of games and activities are suggested initially to encourage students to count using quarters, halves, using diagrams, number lines and mental processes.

The first task is proposed as follows:

> The recipe for 4 people uses $2\frac{1}{2}$ cups of vegetable stock. How many cups do I need to make soup for 10 people? Work this out two different ways.

It is anticipated that students at this level will not multiply $2\frac{1}{2}$ by $2\frac{1}{2}$ but will use additive methods involving partitioning the fraction either symbolically or diagrammatically or both. Note that the task has potential for students to experience mixed number/improper fraction conversions, equivalence and seeing fractions as representing something tangible. The request to provide two different solution strategies encourages students to consider ways in which their two strategies are similar and different. This approach is consistent with evidence that encouraging students to make comparisons between methods is a powerful way of facilitating mathematical learning (Rittle-Johnson & Star, 2011).

Many of the tasks that we are researching have a ‘low floor and high ceiling’, although this particular example does not have a low floor in that there are no easy solutions to the task.

In anticipating that some students might experience difficulty with this task, the following is suggested as an enabling prompt:

> The recipe for 4 people uses 2 cups of vegetable stock. How many cups do I need to make soup for 10 people?

Note that the prompt is similarly structured to the original task but the need for fraction calculations is removed. The expectation is that, once students have completed this prompt, they will return to and work on the original task.

Anticipating that some students might finish quickly, we suggest a further task similar to:

> The recipe for 4 people uses $2\frac{1}{4}$ cups of vegetable stock. How many cups do I need to make soup for 11 people?

The intention is to extend the students who have completed the original task by prompting generalisation. In this case, because the numbers do not process easily, in solving this, students form a general approach to the solution of such problems. An example of such a generalisation is the process of finding a unit and then operating on that unit.

After various solutions and solution methods to the original task are discussed, including those using diagrams, those that partition and regroup the fractions, those that use a unitary method and those that use repeated addition, the following task is proposed:

> It takes 2 minutes to fill $\frac{3}{4}$ of a bucket. How many buckets can I fill in 10 minutes? Show two different ways to find the answer.

In this case, the context has been varied but the concept is substantially the same (repeated addition/multiplication of a fraction). An example of a solution is that since we need 5 sets of 2-minute intervals to make 10 minutes, the solution involves 5 lots of $\frac{3}{4}$. The connection to multiplication, and even the method of multiplying fractions is more explicit. The intent is that students focus on the concept and see the context (stock, buckets) separate from the mathematics. The hope is that this facilitates transfer of the learning to different concepts or different contexts or both.

It is suggested that teachers then pose the following task:

> It takes 2 minutes to fill $\frac{3}{4}$ of a bucket. How long would it take to fill 9 buckets? Show two different ways to find the answer.

This time the context has stayed the same as in the previous task, but the concept has changed. That is, the task now is to find how many ‘three quarters’ (of a bucket) are needed to make 9 (buckets), which can be done by repeated addition even though it connects to division by a fraction. The hope is that this facilitates understanding. The sequence continues using different fractions (thirds, fifths) and different contexts. The progression of experiences exemplifies relentless consistency.

The teacher education perspective

Brown and Coles also argue that teachers should develop particular lessons that they know exemplify the approach that they will seek to apply relentlessly and consistently. The same applies to teacher educators. Both initial teacher educators and also those leading professional learning experiences for practising teachers should identify particular tasks and sequences that they have found to be helpful for teachers in illustrating approaches that:

- engage teachers in experiencing the nature of challenge;
- model the pedagogies as described above with the prospective and practising teachers being the focus of the mathematics learning;
- explain the important mathematical ideas that underpin particular sequences; and
- allow teachers to experience ways that the sequences represent a learning trajectory.

The overall intention is for teachers to hear not only the theoretical rationale for the approach but also to experience ways that the approach might be applied in their current or future classrooms. The ultimate goal is that the pedagogies become part of teachers' ongoing practice. We note that there are many wonderful resources available both commercially and online that draw on interesting applications of mathematics with potential to engage students in learning. However, many of those resources do not incorporate some of the approaches described above, especially how the suggestions can be effectively differentiated, what might be effective follow up and ways to increase student agency. The intention of the project, therefore, is that teachers can either create new resources for themselves or adapt existing resources to incorporate these features.

Conclusion

In exploring ways to support both teacher and student learning, we are exploring an approach to resource development and teacher professional learning that uses the notion of relentless consistency to encourage innovative practices involving sequences of student learning experiences. Noting that classroom culture plays a central role in this relentless consistency, as described by Brown and Coles, both teachers and students have the responsibility to maintain the orientation that assumes tasks will be challenging and introduced without direct interaction with the teacher; lessons will be structured to promote student engagement and agency; that sequences of tasks will develop and connect important mathematical ideas; and these tasks will be appropriately differentiated for students who experience difficulty and those who are ready for extension. The provision of illustrative resources, not only as a research tool but also for teacher professional learning, is intended to support an environment in which teachers have the 'energy' to engage with the complexity of implementing and adapting the pedagogies in their classrooms.

Acknowledgements

The research that formed the basis of this article was funded by the Australian Research Council, Catholic Diocese of Parramatta and Catholic Education Melbourne (LP 180100611). The views expressed are the opinions of the authors who take full responsibility for the ethical conduct of the research and preparation of the report.

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