ON THE MATHEMATICS TEACHER'S USE OF GESTURES AS PIVOT SIGNS IN SEMIOTIC CHAINS

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A teacher and his fifth grade students are exploring together a model in which the multiplication $n \times m$ is represented by an n by m rectangular piece of paper. The children have combined two rectangles into a larger one. The teacher asks "That big slip of paper corresponds to an operation: which one?" gesturing with his forefinger and drawing a circle on an imaginary plane on his other hand, which is kept open and horizontal. A gesture, together with words, is exploited by the teacher to shape what we will call a pivot sign in a semiotic chain.

Adopting a Vygotskian perspective, we assume the process of teaching/learning to be considered as a whole, and we believe that the teacher has the role of creating the conditions of possibility (Radford, 2010) for this process to take place. Hence, we focus on the role of gestures made by the teacher, who is the only actor in the classroom having the possibility of fostering the development of what we call a multimodal semiotic chain. We consider gestures as signs that contribute to the students' and teacher's semiotic activity within the mathematics classroom. Signs are considered through their functional role in the subject's cognitive activity and we refer to the Theory of Semiotic Mediation by Bartolini Bussi and Mariotti (2008), integrated with Semiotic Bundle analysis (Arzarello, Paola, Robutti & Sabena, 2009). Specifically, we focus on the role of the teacher's gestures—considered in interaction with other semiotic resources—in supporting students to develop mathematical meaning, starting from the activity with an artefact.

Gestures as signs in semiotic chains

The starting point of the Theory of Semiotic Mediation (TSM) is that any artifact may be related to two different systems of signs:

- The contingent signs produced by students while using the artifact to face a specific mathematical task (*artifact signs*).
- Those signs that may be referred by an expert to mathematical contents connected with the artifact (*mathematical signs*).

While the former signs are idiosyncratic to the students using the artifact, the latter ones are culturally shared. The double relation of the artifact with students' activity while facing a task and with the general mathematical culture is called *polysemy* of the artifact (Figure 1). Such polysemy is not evident for the students, but the teacher may act "as mediator using the artifact to mediate mathematical content to the students. In our view: the teacher uses the artifact as a tool of semiotic mediation" (Bartolini Bussi & Mariotti, 2008, p. 754).

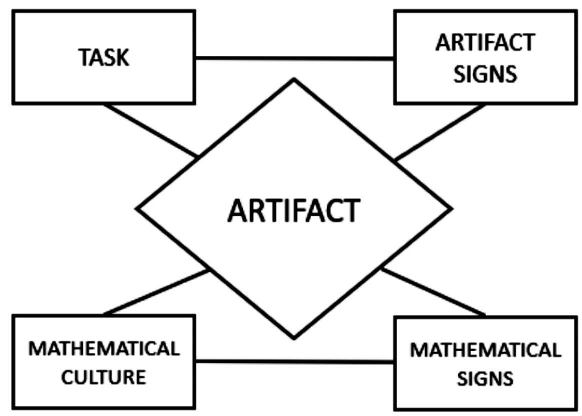


Figure 1. Polysemy of the artifact according to Bartolini Bussi and Mariotti (2008).

According to this theory, the development of coherent relationships between the two different systems of signs is the fundamental educational goal of the process of semiotic mediation orchestrated by the teacher. However, this process is not straightforward: empirical evidence shows that, in order to connect artifact signs to mathematical ones, teachers make use of *pivot signs.* Pivot signs may refer both to the activity with the artifact (specific to a class-group) and to a mathematical domain. In other words, pivot signs are the fundamental 'links' used by the teacher to construct a *semiotic chain* [1] that evolves under the teacher's guidance, with the aim of promoting the switching from situated signs to mathematical ones. This usually occurs during classroom discussions. For instance, Bartolini Bussi and Mariotti show that after using an abacus to represent two-digit numbers, students refer to the beads that are put in the second stick as 'tens-beads'. This word refers both to a part of the artifact (beads) and to a mathematical sign (tens). The teacher can use this sign to relate the position of beads on the sticks of the abacus with the position of digits in a number. In this sense, the word tens-beads can work as a pivot sign.

In TSM, gestures are considered as signs and have been described as having the function of artifact signs and pivot signs. Artifact signs are indeed defined as including "many different kinds of signs, and of course, non-verbal signs such as gestures or drawings, or combination of them" (Bartolini Bussi & Mariotti, 2008, p. 756). Maschietto and Bartolini Bussi (2009) show how gestures can act as artifact signs when used to describe the functioning of a perspectograph. On the other hand, Bartolini Bussi and Baccaglini-Frank (2015) show that an iconic gesture recalling the activity with the artifact may work as a pivot sign. This is done by the teacher, who co-times her gesture with different words: initially she refers to the situated context with the artifact (in this case, a moving robot) and then she uses the same gesture referring to the mathematical domain (angles). In our previous empirical studies, we observed that gestural repetition appears during the construction of semiotic chains in mathematical discussions, shaping what we have called *multimodal semiotic chains* (Maffia & Sabena, 2015). In our analysis of semiotic chains, we take a multimodal perspective relying on the notion of *semiotic bundle* (Arzarello, 2006), *i.e.* "a system of signs […] produced by one or more interacting subjects and that evolves in time" (Arzarello, Paola, Robutti & Sabena, 2009, p. 100). According to this definition, gestures and their mutual relationships constitute a *semiotic set*; they interact with other semiotic sets (*e.g.* spoken words, mathematical symbols) that can be a part of the semiotic bundle.

Farrugia (2017) adopts a multimodal perspective on the Theory of Semiotic Mediation to analyze a teacher's gestures during some sessions on subtraction with 5 years old children. She affirms that "it would be interesting to explore if, and how, gestures serve as pivot signs to help the children […] move from the immediate context to the mathematical domain" (p. 7). This paper is a first step toward an answer to Farrugia's suggestion.

Context

We decided to re-analyze video recordings taken from a previous long-term teaching experiment in which the first author acted as a participant-observer. In some cases, the researcher took the role of the teacher and led the classroom discussion. The choice of this data is due to two main reasons: first, the camera is always recording the teacher-researcher, so the data are suitable for our aims. But this means we have no information about students' gestures during the interaction. Secondly, given that one of the authors is an active participant, we can discuss the intentionality in the use of gestures when he acts as a teacher. At the same time (differently from what would happen with *ad hoc* produced data), the researcher-teacher's gestures may be considered as spontaneous.

In the following, we will present two excerpts from a discussion that took place in a Grade 5 class in February 2014 in Italy. The students have just worked individually on a task with a Laisant's table (Maffia, 2019). This is a sort of 10×10 visual times-table in which multiplications are represented by rectangles: the height of the first cell is one unit and the length of its base is also one unit. Each row is one unit higher than the previous one and each column is one unit wider than the previous one (Figure 2).

The task consists of the following instructions:

1) color two cells (freely chosen by children) in the same row of the table;
2) cut two slips of paper with the same dimensions of the colored cells;
3) glue them together (without overlapping) to form a new rectangle.
4) color a third cell in the table: the one corresponding to the obtained rectangular piece of paper (Figure 3);
5) write the corresponding multiplications on each colored cell (this request is familiar to the children because they have already previously worked with this table).

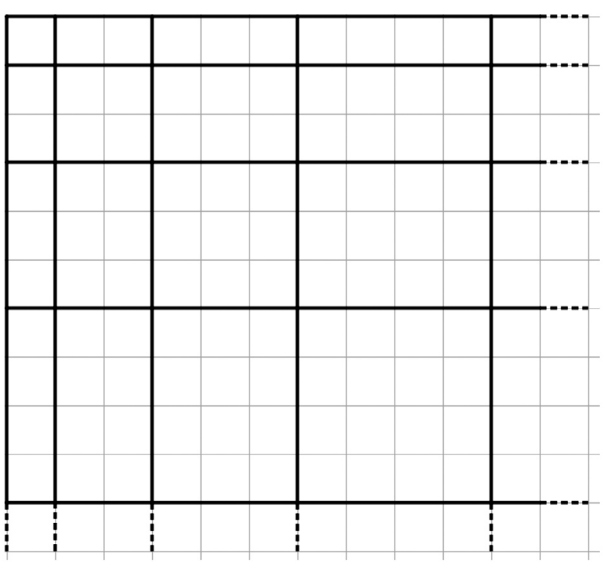


Figure 2. First four rows and columns of Laisant's table.

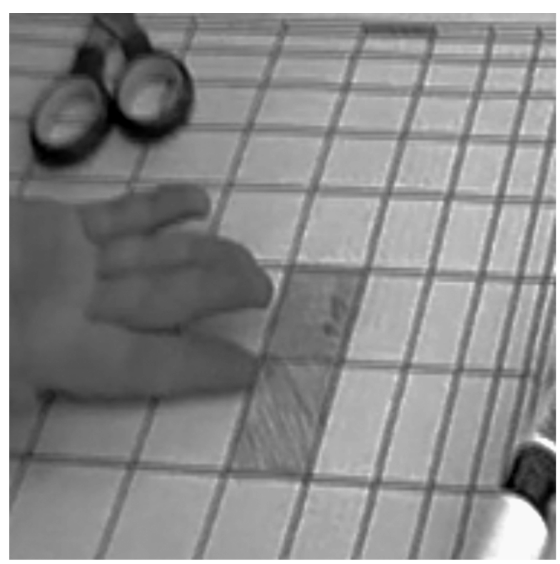


Figure 3. A student's coloring of the cells corresponding to 6×1, 6×7, and 6×8.

The aim of the task (and of the following class discussion) is to guide the students in relating the gluing of rectangles to the idea of ‘sum of multiplications’. The final objective is to make them recognize that the result of such an operation is still a multiplication: a common factor is given by the height of the rectangles, and the other factor is the sum of the factors given by the bases of the rectangles in the initial multiplications. In short, we wanted them to notice the structure (in the sense of Maffia & Mariotti, 2018 [2]) of what they already knew as distributive property.

In the following, we will present two episodes that show paradigmatic examples of the different uses of gestures as pivot signs.

Iconic and metaphoric gestures as pivot signs

The discussion starts with the intervention of a student—we will call him John—reporting his particular example from the individual activity: he joined the rectangles corresponding to 1×1 and 1×2 obtaining 1×3. Another student, Alex, doubts the fact that 1×3 is the correct result of the gluing of 1×1 and 1×2. Hence, the teacher focuses the attention of the rest of the class-group on Alex’s doubt. In the transcript, we underlined the word(s) uttered simultaneously to the *stroke phase* of gestures. According to McNeill (1992), the stroke phase is the central phase of the gesture, *i.e.* a peak between the preparation phase and the phase of returning to the initial position.

1 *Teacher* So, Alex tell me if I understood your doubt. You say: *[…]* I am not so sure about what comes out if I sum *[his hands move as if they were compressing something, Figure 4a]* two multiplications. Isn’t it?

2 *Alex* Yes, and an addition too.

3 *Teacher* Eh, then I do an addition, when I say: “I sum” *[his hands seem to touch the surface of the compressed object, Figure 4b]* I mean “to do an addition”.

4 *Alex* Ah yes.

5 *Teacher* So, let’s try to consider John’s example to understand. Maybe he can help us to understand what he meant. He says: if I have 1×1 *[he writes 1×1 on the blackboard]* and 1×2 *[he writes 1×2 at a little distance]*

6 *Fred* 1×1 makes 1 and 1×2 makes 2 and if you join them it makes 3.

7 *Teacher* Ok. This is it. *[He writes =1 as vertical under 1×1, see Figure 4d]* We know that 1×1 makes 1 *[he moves his hand vertically, from the top to the bottom, pointing to the blackboard]* and *[he writes = in vertical under 1×2]* 1×2 makes *[pause] [he turns, facing the students]*

8 *Pupils* Two!

9 *Teacher* Ok *[he writes 2, obtaining the inscription in Figure 4d]* then, what your peers were saying is that I can do the addition. *[moving his two hands vertically, with palms facing each other, Figure 4c]* Is it right?

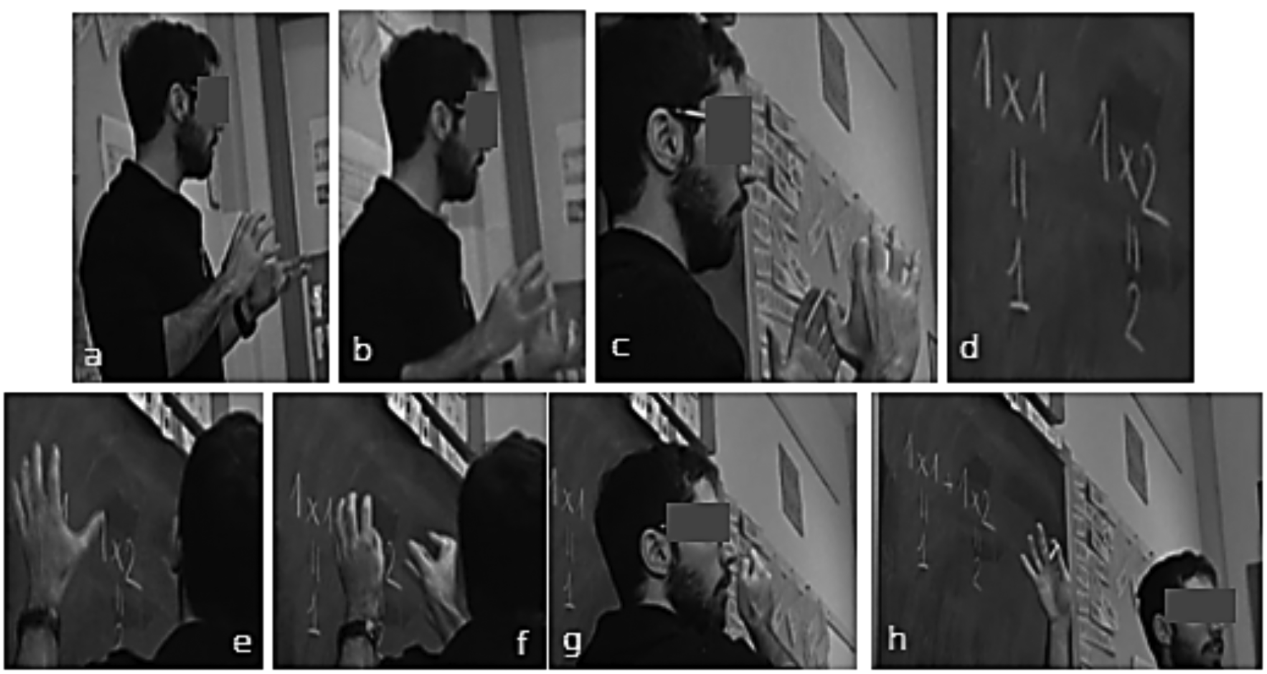


Figure 4. Teacher’s gestures in the first excerpt.

[murmurs] How can I signal that I must do the addition *[repeating the same gesture close to the inscriptions on the blackboard, Figure 4e]* between these two multiplications *[closing his hands, as grasping the inscriptions to make them closer, Figure 4f; then he turns, Figure 4g]*?

10 *Mary* So *[pause]* to write them?

11 *Teacher* Yes, how can I write it *[he points at the blackboard]?* Tell me Mary, I am listening to you.

12 *Mary* You must do *[pause]* how is it called? You must put a plus there.

13 *Teacher* Yes, I wanted to say that I would like to do this *[rotating his forefinger around the first multiplication]* plus *[he writes + just after the multiplication]* this *[rotating his forefinger around the second multiplication]* Is it ok if I write this way? *[he points to the inscription with the open hand, Figure 4h]*

14 *Pupils* Yes *[pause]* No *[pause]* Yes.

15 *Erick* Actually, it would be ok, but we need brackets.

16 *Teacher* Ah!

17 *Nick* Brackets!

18 *Teacher* Wait a minute. Erick was suggesting something. *[…]* Could you tell me where I have to put brackets?

In this episode, the teacher performs repeated gestures while referring to mathematical addition: his two hands get closer, as if he were grouping something or putting together two things (lines 1, 3, 9, Figure 4a, b, f). Gestural repetition has been focused on by psychological studies on gestures and has been called *catchment* (McNeill, 2010). Catchment is herein defined as the recurrence of some gesture's features during a conversation and has been interpreted as giving cohesiveness to the whole discourse.

The teacher's gesture appears synchronic with the words 'sum' and 'addition' (the second one is suggested by Alex, in line 2, and then repeated by the teacher, too). It may be interpreted as *metaphoric* because it is aimed at recalling the intuitive model of sum as the union of two sets. Again, according to McNeill (1992), gestures may be classified as:

- *iconic* gestures, which depict the semantic content of the discourse,
- *metaphoric* gestures, those that are realized while talking about an abstract content,
- *deictic* gestures like pointing to objects or positions in the space. [3]

While keeping the catchment, the teacher's hands change their shape and open vertically (line 9, Figure 4c). Here, they are apparently indicating, iconically, the two sides of the rectangular figures. Through this iconic reference, gestures recall the activity with the artifact, and can therefore be interpreted as artifact signs. These artifact signs are performed while the mathematical term 'addition' is uttered. In the same utterance, the teacher faces the blackboard and he repeats the reference to the addition (he says "I have to do the addition"); but this time, such reference is accompanied with the 'grouping gesture' pointing to the written multiplications 1×1 and 1×2 (Figure 4e, f). We remark that the word 'addition' (that is a mathematical sign) is firstly used referring to the artifact context, and then with reference to the mathematical symbolism that is written on the blackboard. When the word is uttered, similar gestures are performed (catchment). In this example, gesture catchment helps the teacher in keeping the reference to the activity with the artifact, while talking about mathematical operations. The gluing activity is recalled iconically by the gesture (Figure 4c), and the mathematical operation is also reported in the written symbolic expressions. The synchronic matching of speech/inscriptions (mathematical signs) with the iconic gestures (artifact signs) creates a *pivot sign* within the emerging multimodal semiotic chain.

We can also notice the presence of deictic gestures and words: The teacher points at the written multiplications, referring to them with the word 'this' (line 13) together with circular gestures indicating the whole multiplicative couples. These gestures of circumscribing the operations indicate that students should consider them as whole objects. It seems that students can grasp the communicative aspect of the gesture-speech couple. Indeed, they propose inserting brackets (lines 15, 17, Figure 4e). Obviously, from a mathematical perspective, such brackets are not necessary at all. They may play a cognitive role, serving to highlight the link with the activity with the artifact: the multiplications that are inside the brackets are the same multiplications that were written on the students' rectangular pieces of paper. In this sense, brackets crystallize the ephemeral circular gesture (line 13) in a stable sign; these signs are connected within the semiotic chain, acting as pivot signs connecting the mathematical signs (the written symbols) to artifact ones (the slips of paper).

Pointing gestures as pivot signs

The discussion goes on and some children propose to calculate the multiplications that are in the brackets and to write down '1+2=3'. However, the teacher decides to change the discussion path and to focus the students' attention on the relationship between addition and multiplication. Another part of the discussion starts after the teacher's question presented at the beginning of this paper:

19 *Teacher* I would like to understand what happens with the multiplications *[…]* When we took the slip of paper 1×1 *[placing his hand on the first bracket as grasping it, Figure 5a]* and we put it next to the slip of paper 1×2 *[moving his hand on the*

second multiplication, Figure 5b] what slip [pause] well, these were cells *[he repeats a pointing to the two multiplications in brackets],* then we pasted them *[getting his index fingers close to each other, Figure 5c]* and it comes out *[the right forefinger draws a circle on the imaginary plane created by the left hand, which is kept open and horizontal, Figure 5d]* a big single slip of paper *[repeating the same gesture]* That big slip of paper *[repeating again the same gesture]* corresponds to an operation: which one?

20 *Pupils* One times three.

21 *Daniel* To three times one!

22 *Teacher* Daniel 'says three times one' *[the teacher writes 3×1 on the blackboard, Figure 5e]*

23 *Daniel* Equals three.

24 *Teacher* Do you agree?

25 *Pupils* Yes.

26 *John* No, it is 'one times three'.

27 *Teacher* John suggests 'one times three' instead.

28 *Pupils* Yes, yes, 'one times three'.

29 *Teacher* Daniel, do you agree?

30 *Daniel* It is the same.

31 *Teacher* So, can I write 'one times three', Daniel?

32 *Daniel* Yes, here we shouldn't put 'three times one' because we said that it is horizontal.

While talking, the teacher performs several gestures. First, he uses two pointing gestures to point at the operations on the blackboard (line 19, Figure 5a, b). These pointing gestures are followed by iconic gestures recalling the activity with paper and glue (Figure 5c, d). We can notice that the deictic gestures (pointing to the mathematical expressions) are performed while the words 'slip of paper' are uttered. These words play the role of artifact signs, referring to the activity with paper and glue. The teacher states that the focus of the discourse is on the mathematical relationships ("I would like to understand what happens with the multiplications"), but only his hand is pointing to an arithmetical operation: the whole sentence is about those actions that were enacted with paper and glue ("we took the slip of paper [...] we put it next to [...] then we pasted them"). These verbal artifact signs are linked, through the pointing gesture, to the mathematical signs that are written on the blackboard. In other words, we can say that the deictic gestures of pointing act as *pivot signs* because they are linking artifact signs (uttered words) with the written operations (which are mathematical signs).

At the end of this utterance, the teacher makes explicit the interpretation of the activity with the artifact in terms of mathematical meanings; he asks: "That big slip of paper corresponds to an operation. Which one?". Again, the teacher's words are enriched by the repetition of a gesture (Figure 5d) that is recalling the action of gluing the pieces of paper.

Students' interventions suggest that the link between the mathematical signs on the blackboard and the activity with the artifact is seized by them. Indeed, John and Daniel's discussion (lines 20–32) ends when Daniel agrees on choosing 1×3 because "it is horizontal". The sentence in line 32 has hybrid features and can work as a pivot sign: the word 'horizontal' refers to the position of the rectangle as it is shown in the table, the number 1 corresponds to the height and 3 is the length of the base. Daniel determines the result of a mathematical operation referring to the artifact (in which the rectangle could be defined as "horizontal" because its base is longer than the height). Even if the written signs that are introduced by the teacher are mathematical ones, this analysis with a multimodal perspective shows us that there is a constant reference to the artifact through words and gestures, so the semiotic chain shifting from artifact signs toward mathematical ones is still under construction.

Discussion and conclusion

As exemplified above, we can identify two different ways the teacher uses *gestures as pivot signs in multimodal semiotic chains* to link artifact signs to mathematical ones:

- *Pointing gestures indicating mathematical inscriptions* while the speech refers to the context of the

Figure 5. Teacher's gestures in the second excerpt.

artifact: *gestures work as pivot signs* because they link mathematical signs, expressed in a semiotic modality, to artifact signs expressed in a different modality (Figure 6, left). *E.g.,* in line 19, the teacher's gestures (Figure 5a, b) link the arithmetical signs that are written on the blackboard to the spoken artifact signs "slips of papers".

- *Iconic/metaphoric gestures recalling the artifact and the related activity,* while the co-timed speech contains mathematical terms, and/or they are performed closely to mathematical symbolic inscriptions (Figure 6, right). *E.g.* in line 9: the teacher's hands refer to the vertical sides of the paper rectangles (Figure 4c), while his words are "do the addition".

In the former case, one semiotic resource (words) refers to the context, another semiotic resource (written signs) refers to mathematics, and pointing gestures are used as a third sign to link them. In the latter case, the link between artifact signs and mathematical signs is obtained through the *simultaneous presence* of different semiotic sets in the semiotic bundle: it is the couple (gesture; word), or (gesture; symbol), to act as a pivot sign. We underline this aspect by referring to this case as *pivot bundle.* An example of pivot bundle is highlighted by the dotted line in Figure 6, right.

As in Farrugia's (2017) study, we observed metaphoric gestures with the role of pivot signs, but that is not the only case. We showed that also deictic and iconic gestures may have a role in structuring semiotic chains.

As described above, a pivot bundle can be observed when different semiotic modalities refer to two different contexts: a modality is used to refer to the artifact and another to a mathematical domain (which is the goal of the teaching /learning sequence). In psychological studies on gesture, researchers speak about 'gesture-speech mismatch' when gestures are conveying different information than the co-timed speech. Gesture-speech mismatches are claimed to be a useful tool in the mathematics teacher's hands because they can convey two different pieces of information at the same time (Singer & Goldin-Meadow, 2005). In this paper, we step away from cognitivist perspectives on gestures, to take a semiotic view integrating TSM with a multimodal analysis that is carried out using Semiotic Bundles as a theoretical lens. From this semiotic perspective, our results confirm the importance of mismatching or non-redundant gestures (Kita, 2000) in mathematical activities. However, including in our analysis also the other semiotic sets (as the written mathematical inscriptions), we can speak more generally of mismatch or non-redundancy *within* the semiotic bundle, but—more importantly in our view—we can reinterpret these cases in light of the development of meanings from the situated context of the activities with artifacts to a mathematical domain. In other words, we can interpret the teacher's gestures as a strategy to foster such development.

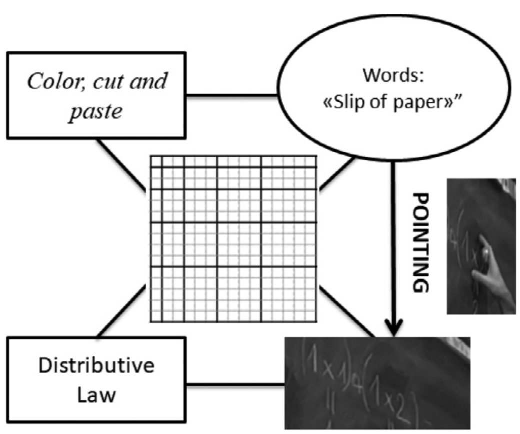
Our empirical data support the fact that gestures may be exploited by the teacher as pivot signs, in the two ways outlined above. We are convinced that this phenomenon involves students as well. Further research is needed to establish if teacher's gestures create the expected link also for students, and the influence on their learning.

Notes

[1] According to Presmeg (2006), in a semiotic chain "a sequence of abstractions is created while preserving the important relationships from the everyday practices of the students. The chain has as its final link some mathematical concept […] Using this process, a teacher can use the chain as an instructional model that develops a mathematical concept starting with an everyday situation and linking the two" (p.166). Bartolini Bussi and Mariotti (2008) adopt this definition including artifact, pivot, and mathematical signs in such a chain.

[2] This idea of 'structure' refers to the possibility of conceiving the distributive law as a mathematical object rather than just a process; it is coherent with the idea of structural conception made by Sfard (1991). We are not referring to formal mathematical structures.

[3] We want to stress that these categories are not mutually exclusive. For instance, pointing is used to refer to different possible cases of an argument (Arzarello & Sabena, 2014). In this example, the gesture is both deictic and metaphoric.



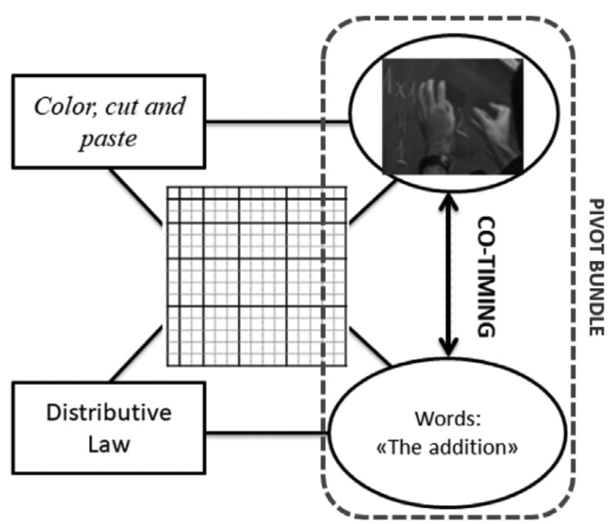


Figure 6. *Two different ways of using gestures as pivot signs.*

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