MATHEMATICS LEARNING: IDEAS FROM NEUROSCIENCE AND THE ENACTIVIST APPROACH TO COGNITION

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How do people learn mathematics? How is individual learning related to collective knowledge? What part does biology play in learning processes? What is the influence of the social context on the learning of mathematics? My thinking around these issues has been shaped by ideas coming from biology and neuroscience, particularly those related to the enactive approach to cognition. Ideas emerging from these theories emphasise the complexity of the nature of cognition and learning.

From doing mathematics and trying to teach the subject to other people I became concerned with education and learning. In trying to acquire, share or enhance mathematical knowledge I found myself deeply engaged with questions that have to do with how people construct and develop their knowledge. I began to wonder about both my own and other people's thinking processes as I found difficulties widely spread in the learning of mathematics. I also became aware of different ways of learning and of common misconceptions and interpretations. I started to develop my own 'learning theories' based on what I could notice in schools, lessons, pupils and myself.

Furthermore, my ideas about knowledge have also been shaped by the reading I have done about learning in several areas and disciplines. I have been influenced by work done in the areas of biology – especially those ideas stemming from cybernetics and general systems theory – and neuroscience. I want to explore the way in which my interpretations of these theories have influenced my work as my ideas have emerged and developed in the process of doing research in mathematics education.

During my first years in school I thought that learning mathematics was something that needed little or no effort at all. I went to a Montessori school for several years and I happily worked with numbers and solved, with no difficulty, problems that required mathematical reasoning. I later attended more traditional mathematics lessons, but still had no problem following the courses. I liked mathematics problems because they required no memorising and because solving them gave me a great feeling.

It was not until I entered the world of higher mathematics that I realised that things could be a little more complicated. A variety of activities, like reading, writing, practising, thinking and reflecting were now part of my own learning, which appeared to have turned into a complex process that, surprisingly (to me), needed time to develop. Sometimes things suddenly became clear after a period of time in which there were moments of complete confusion. At other times, things were clear from the beginning or they never became clear at all. Learning, then, was a matter of creating and building; a process that, I felt at the time, had to be worked out mainly by the student.

Later, when I started teaching mathematics, I realised that successes and failures did not depend exclusively on the students' work and internal processes. As a teacher, I had an important role in shaping students' learning. The type of activities I designed, the way I presented them and the way I organised the work in the classroom were all crucial elements; much more than a mere influence on a process that was supposed to be mainly internal.

My ideas about learning became increasingly complex after my experiences with teaching. Although students did seem to go through some kind of internal reasoning process, this could be influenced by the learning environment that surrounded them. Different teachers and different teaching techniques led to different outcomes in the learning of mathematics. Moreover, different schools and different cultures expected and fostered different kinds of mathematics. I began to consider learning as a process where many elements including teachers, teaching approaches, classroom activities and of course, learners, were involved.

These initial ideas were vaguely formulated, it was when I engaged in conversations with other teachers and friends that I became aware of them. But it was reading the literature and talking about what I read that really shaped my thoughts and helped me explain, in a more coherent manner, what I experienced in the classroom as a mathematics learner, teacher and then observer.

Enactivism: ideas from biology and general systems theory

One of the most important influences in my thinking about mathematics learning has been a theory called enactivism. When I came across the enactivist approach, I had already been in contact with some of Piaget's ideas. I have always had an interest in biology, and when I started reading about cognition from a perspective rooted in the biological sciences I was thrilled. However, many of the interpretations I read about Piaget made a strong emphasis on the cognising agent, who seemed to be constructing his or her knowledge in isolation. These ideas did not agree with my previous teaching experiences. Other theories I encountered emphasised the social but neglected individual biological roots, which I considered to be important. Enactivism, with its emphasis on the inseparability of the individual and the

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world, provided me with a middle way that accounted for both individual and collective learning.

The enactivist ideas have been built upon the work of Maturana with Varela. Together they developed what they call the *biology of cognition*. Both were heavily influenced by cybernetics, a conceptual framework developed in the 1940s that brought together an interdisciplinary group of people who were trying to explore new ways of thinking about knowledge and the mind in terms of patterns and organisations instead of description of components (Capra, 1997, p. 53-54).

From this perspective, systems cannot be understood entirely from the properties of their parts, but

> properties of the parts can be understood only from the organization [*sic*] of the whole. (*ibid.*, p. 29)

Within this framework, and following in particular the ideas of Bateson (1979, 1987, 2000), Maturana and Varela developed a characterisation of living beings that situates cognition at the heart of the living process.

Enactivism reflects a strong dissatisfaction with the notion that cognition is fundamentally a representation of the world outside the observer. Knowing, in enactivism, cannot be characterised by the grasping of objects of thought that are in the outside world. However, neither can it be thought of as a process that occurs inside our minds, isolated from the environment. For enactivists, cognition is

> the enactment of a world and a mind on the basis of a history of a variety of actions that a being in a world performs. (Maturana and Varela, 1992, p. 9)

Knowing is a result of interactions with the world, which will be determined by the history of the individual (*ibid.*, p. 18). In this way, biological structure and previous experience will influence the meaning that every person makes of the world. Knowing arises from interactions, which will shape all the participants. The focus is neither on the individual learner nor on the social and cultural context because these are inseparable.

Fundamental concepts in Maturana and Varela's *biology of cognition*, which have informed my ideas of learning are those of structure and organisation:

> *Organization* [*sic*] – denotes those relations that must exist among the components of a system for it to be a member of a specific class.
>
> *Structure* – denotes the components and relations that actually constitute a particular unity and make its organization [*sic*] real. (Maturana and Varela, 1992, p. 47, original emphasis)

In a given system, the structure can change to a great extent, but the system will continue existing only if its organisation is preserved. This applies, for example, to a single student learning in a mathematics classroom, and also to a group (including the teacher and all the students in that mathematics classroom) taken as a whole. In the first case, while the student's structures, that is, the way in which he or she is individually constituted in a given moment, can go through considerable changes, the student must maintain certain characteristics in order to continue existing as a student in that classroom. Changes are constrained by the need for the student to be a member of a particular community.

In enactivism, learning can be seen as the process through which the structure of an organism changes while it continues existing in a given enviroment:

> The problem is [...] to show how an organism, which exists in a medium and which operates adequately to its need, can undergo continuous structural change such that it goes on acting adequately in its medium, even though the medium is changing. Many names could be given to this; it could be called *learning*. (Maturana, 1987, p. 74-75; emphasis added)

Mathematics learning occurs as students change their structures, and therefore their behaviour, in a complex process of interaction with their environment. Modifications in structure are the result of both internal dynamics and external stimuli. Because, in the process of learning, students need to preserve their organisation, that is, they need to continue existing as students in particular classrooms, the changes they go through are related to those actions which are adequate in those contexts. To learn mathematics means, according to these ideas, to be able to act in ways that can be considered mathematical. The criteria that determine the kinds of actions which are adequate, or in this case, which can be thought of as mathematical, are specified by the members of the particular community where the actions take place. The learning of mathematics happens as students act mathematically, as a result of changes in their structures.

This characterisation of learning helped me in explaining the way in which my students differed individually but at the same time changed their behaviour similarly according to my way of teaching and the activities I posed to them. Enactivist ideas suggest that, at any particular moment, students respond to a given stimuli according to their individual structures, and that these structures are constantly changing. When several members of a group interact with each other for a period of time, they construct a history of interactions that allows for their structures to change in a similar way, thus accounting for commonalities that can be observed in groups. Individual biological structure and social interaction cannot give an explanation for learning if considered separately. That is why, when teaching mathematics, the same classroom activities gave rise to different kinds of behaviour in my students, but, at the same time, responses varied only to a certain extent.

Learning, for me, is related to the way in which the structures of systems change. Living beings modify and produce their structures in the process of living, while acting and interacting with their environment. Learning is therefore intimately related to action, in such a way that they are inseparable. This is another key idea in enactivism that has been a strong influence in my thinking.

Cognition as enaction

As I mentioned before, enactivism's response to the challenges of explaining the nature of cognition comes from biology and neuroscience. Maturana and Varela's biological understanding of living systems made them realise that learners do not operate with objects that are determined externally, and therefore cognition is not about representations of external objects. Rather, in enactivism, cognition is action. In addition, this action is emphasised as being 'embodied' in two fundamental senses:

- Cognition dependent upon the kinds of experience that come from having a body with various sensorimotor capacities; and

- Individual sensorimotor capacities that are themselves embedded in a more encompassing biological and cultural context. (Varela, 1999, p. 12)

The first meaning of embodiment locates cognition in our bodies, and prevents us from thinking about it as an abstract notion that is detached from our everyday experience. The second situates our learning in a wider social and cultural context. Both ideas broaden my understanding of learning, enabling me to be more specific about the way in which actions characterise learning processes.

In enactivism, to perceive something does not mean to recover properties of an external object, rather, "perception consists of perceptually guided action" (*ibid.*, p. 12). What this means is that we perceive something in a certain manner because of the way in which we relate to it through our actions. This is beautifully illustrated by an experiment done by Bach y Rita (1962, in Varela *et al.*, 1993):

> Bach y Rita has designed a video camera for blind persons [*sic*] that can stimulate multiple points in the skin by electrically activated vibration. Using this technique, images formed with the camera were made to correspond to patterns of skin stimulation, thereby substituting for the visual loss. Patterns projected on to the skin have no "visual" content unless the individual is behaviourally [*sic*] active by directing the video camera using head, hand or body movements. When a blind person does actively behave in this way, after a few hours of experience a remarkable emergence takes place: the person no longer interprets the skin sensations as body related but as images projected into the space being explored by the bodily directed "gaze" of the video camera. Thus to experience "real objects out there" the person must actively direct the camera (by head or hand). (p. 175)

Furthermore, it is not only perceptions that emerge through our actions, but cognitive structures also arise from recurrent sensorimotor patterns (Varela, 1999, p. 15). What this means is that embodied structures, which allow perceptions to be guided by actions, also give rise to conceptual understanding and rational thought.

For me, these have been crucial points in my thinking about mathematics learning. Cognition arising through actions explains to me how, in a Montessori classroom, learning occurs as children engage actively in different types of jobs. Students can be seen cutting, pasting, tracing letters and numbers on sandpaper, counting beads, piling cubes as well as taking part in many other unexpected activities such as cleaning and sweeping the floor. If anything else, one notices in such classrooms great activity. Learning arithmetic and the basic workings of the decimal number system happen as children repeatedly engage in actions that involve making certain distinctions, which, I believe, help them in developing fundamental concepts that stem from patterns in those actions. The specific distinctions that children make, through playing with the Montessori materials and manipulatives, are, in no doubt, triggered by those objects. However, it is the child's structure that allows him or her to notice particular features, and to engage in certain actions.

> It is this structure – the manner in which the perceiver is embodied – and not some pre-given world, that determines how the perceiver can act and be modulated by environmental events. (Varela, 1999, p. 13)

For example, after hearing the same explanation given by a teacher in any classroom, students develop different interpretations of what they just listened to. We all notice different things and construct different meanings for what we experience. It is possible to observe differences in the way in which different children interact with the different objects in the classroom, maybe learning happens at the interstices of individual preferences and suggested activities.

As in other learning contexts, a student in a Montessori classroom, is not, however, engaging in actions on her or his own, isolated from the rest of the class. Children are in constant communication with their teacher, in this case, their 'guide'. As a student you have to show the guide that you are able to do certain things, to carry out certain actions, before you are allowed to do something new. In addition, certain jobs are sometimes undertaken by more than one student. Finally, there are moments in which the guide asks the class to get together and observe the work that one particular student has done.

For example, when a four or five-year-old student completes what is called 'the one thousand chain', he or she gives the class a brief explanation, including how he or she managed to build a chain that contains one thousand beads, devising a system which allowed him or her to keep track of the number of beads throughout the days or weeks the task took to be completed. This brings me back to the second meaning of embodiment: that our bodies are embedded in a wider context. For a student to continue existing as such in a given environment, he or she needs to act in certain ways. Children need to be able to show the group that they can do certain things, according to their age and stage. In order to exist as members of a particular community, students need to modify their behaviour in agreement with that community. Learning mathematics occurs when a community considers actions as mathematical.

At university, a student who is reading mathematics needs to show, generally through examination papers, that he or she can behave in a way that is acceptable for a community of mathematicians. Moreover, students not only respond to but also change their behaviour according to features of a certain context.

Both the Montessori school child and the university student, through the actions they carry out and which are determined in a given moment by their biological structure, also contribute in shaping the context in which they interact. A child, who announces to the group that he or she has completed a particular job, contributes in creating a context in which other children might feel they can also engage in such a task. University students soon start contributing, with their mentors, to the writing of research papers, which shape the world of mathematics. Learning mathematics is therefore not about an isolated organism changing its structures but about the interplay of individuals and contexts specifying each other.

My current involvement in the development of computer interactive programmes for the learning of mathematics is strongly influenced by these ideas. The team of mathematics educators I work with is devoted to the creation of initiatives that can help teachers to develop contexts in which certain actions, related to the learning of mathematics, can be fostered. Our goal is to broaden students' and teachers' experiences with mathematical ideas. We consider that learning mathematics happens as students and teachers work together and modify their behaviour in particular ways, and therefore we are trying to present teachers with tools that promote mathematical actions through the exploration of different situations.

From the enactivist perspective, the learning of mathematics cannot be understood as the result of a particular teaching practice; however, neither can it be seen as caused by individual pre-dispositions. Individual structure, which determines how a student will act in a particular moment, changes at every instant as a result of the interactions with the environment. In classrooms, students change in similar ways through an intricate process of interaction; their learning being shaped by a culture which they continuously contribute in creating. Enactivism allows me to think of the learning of mathematics as a complex process, which is the result of histories of interactions through which individuals' embodied structures, that is, their actions, thoughts and emotions are manifested and shaped. The experiences I have had with the teaching and learning of mathematics can be seen, under the light of enactivism, as part of these complex processes in which individuals are not separated from their environment, but where learners and context modify each other constantly. This is, for me, a useful way of thinking about learning, and which has allowed me to think of mathematics classrooms as spaces for interaction in which the intricate interplay between individual and context can be often explored and shaped but can never be predicted or controlled.

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If someone claims to know algebra, that is, to be an algebraist, we demand him or her to perform in the domain of what we consider algebra to be, and if according to us she or he performs adequately in that domain, we accept the claim.

(Maturana, 1988, pp. 18-23 online, see references above)