A MATHEMATICS-SCIENCE COMMUNITY OF PRACTICE: RECONCEPTUALISING TRANSFER IN TERMS OF CROSSING BOUNDARIES

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Over the last three years we have been involved in an interdisciplinary research project whose general aim was to study what has been called in the educational literature 'transfer of knowledge', in this case between mathematics and science school practices. This project has been developed in a basic (elementary and secondary) school where Cristina teaches mathematics and Selma teaches science. During the first year of the project, Cristina and Selma acted as teacher-researchers of one of the Year 9 classes (approx 15-year-olds) that they both taught The three of us have spent the last two years making sense of the data We have also set out to offer a contribution to the discussion of transfer from what Engeström (1999a) refers to as a 'strong' situated point of view - that is, using ideas of community of practice as our starting point.

Beyond our intellectual commitment to the situated character of learning, two significant observations have encouraged us to develop this project: first, the controversial notion of transfer within situated learning perspectives; second, the clear diversity of views of transfer in the mathematics education literature relating to these perspectives. In the educational literature, transfer of knowledge refers in general to the use or application in one context of knowledge learned in another. It is suggested by some researchers (*e.g.*, Greeno, 1997; Lave, 1993) that those choosing a strongly cognitive perspective on learning see knowledge as something relatively stable, generalizable to different situations and characterized by personal attributes in the sense that, once acquired, the subject carries it from one place to another. As Boaler (2002a) points out, situated learning perspectives offer an interpretation of knowledge that is radically different: a representation of knowledge as activity, as something that is shared or distributed by persons; something that emerges between persons, the environments in which they are inserted, and in developing activities. From these perspectives it is not that cognitive structures are not considered, but that they cannot be detached or abstracted from learning contexts.

The epistemological assumptions that accompany situated perspectives of learning have challenged, among other things, the interpretation of transfer mentioned above; they have also led some mathematics educator researchers to rethink the issue in terms of these assumptions. However, we still find much variation among the approaches to transfer of mathematics educators taking a situated perspective. This raises theoretical and methodological difficulties for those who aim to research the issue of transfer of knowledge from a situated perspective

At the beginning of our research, we strongly believed that we could plan a collaborative school project between mathematics and science, theorizing it in terms of a community of practice. But we had no clear idea about the direction to take to elaborate a theoretical contribution to the discussion of transfer from a strongly situated point of view During the planning phase, the teacher-researchers' struggles to share both their scientific discourses and their disciplinary experiences led us to think about transfer in Bernstein's (1996) terms, as the transfer of knowledge between two insulated domains or vertical discourses.

Our situated account of transfer represents, more specifically, a reconceptualization of transfer within the school context in terms of boundary crossing between two apparently insulated practices. We suggest that, where transfer between practices is thought to be observed, it can be usefully conceptualised in terms of a community of practice that overlaps these practices In doing so we take as our major challenge a response to Lerman's (1999) question, "how can children learn to be conscious of contexts ... and cross the boundaries of practices successfully?" (p. 94)

Practice, community of practice and identity in school contexts

Three key concepts underpin our work: practice, community of practice and identity. By practice we mean 'doing' something not just in itself, but in a historical and social context that, as Wenger (1998) puts it, gives a structure and meaning to what is being done. For Wenger, practice includes what is said and what is left unsaid, what is represented externally and what is assumed. It also includes the language, symbols, tools, documents, well-defined roles and rules, procedures, regulations, contracts, implicit relations and conventions, perceptions, assumptions, understandings and shared worldviews and beliefs So, when we talk about a school practice, this should be understood in terms of all of these features, within the school context, where participants/subjects include teachers and students (and eventually others) in some way

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Activity, in the sense of activity theory (Engeström, 1999b), is important when we come to characterise a school practice: school practices can be characterised by their activities. Winbourne and Watson (1998) propose an adaptation of Lave and Wenger's core ideas of community of practice (Lave, 1988; Lave & Wenger, 1991; Wenger, 1998), in order that they might become applicable in certain school settings. They suggest that, in some school settings, we can have (or not have), what they call a 'local community of practice'. [2] A local community of practice in a school setting is, amongst other things, continuing activity where the participants - teacher and students - work purposefully together towards the achievement of a common goal. In doing so they share (not in the same way or with the same intensity, to be sure) ways of behaving, language, habits, values and tool-use, and can see themselves as an essential part of the regulation of their activity and progress towards the common goal. This is to say that what is important for the constitution of a local community of practice within a school setting - that is, a community of practice where the goal can correspond to fairly standard curriculum goals and objectives, is that the students and teacher legitimately feel that they are agents of the practice with all that goes with that. Entailments include sharing the doubts, understandings, meanings and experiences associated with the tensions and contradictions that attend such participation - processes that, as we make clear below, are constitutive of identity within the practice.

We see this notion of community of practice as an analytical tool. The presence of a community of practice in a classroom does not necessarily mean that students will be learning what the teacher wants (Winbourne, 2008; Winbourne & Watson, 1998); there are plenty of classrooms where there are communities of practice, but these communities of practice are certainly not those the teacher might want to see produced; these are practices where the constitutive activity has nothing to do with the learning of mathematics or science, and, indeed, may be in opposition to this. ('Annoying the teacher' can be just such an activity, producing a community of practice of this kind.)

Here we are using notions of identity influenced by Gee (2000) and worked out in recent writing (Winbourne, in press): [3]

> Being recognized as a certain "kind of person," in a given context, is what [Gee] mean[s] ... by "identity." In this sense of the term, all people have multiple identities connected not to their "internal states" but to their performances in society. (p. 99)

We continue to make use of a view of identity as the aggregation of the smaller 'becomings' (or identities) identified with a learner's participation in a multiplicity of communities of practice, local and not so local, some of which are locatable within school classrooms and most not (Lave & Wenger, 1991; Winbourne & Watson, 1998). Boaler (2002b) suggests links from identity to both affect and practice in mathematics classrooms, and we find this particularly helpful to our study. For Boaler, identity formation is a process strongly situated within the practice and corresponds to relationships the students develop with the disciplines. These relationships or the students' mathematical identities include: "the knowledge they possess, as well as the ways in which students hold knowledge, the ways in which they use knowledge and the accompanying mathematical beliefs and work practices that interact with their knowing" (p. 16). In our view, the affective dimension of these relationships comprises not only beliefs, but also other affective components such as emotions, motivation, disposition and attitude (Frade & Gómez-Chacón, in press).

In relation to the concept of transfer, we do not provide a comprehensive review of the transfer literature here, but focus on researchers who have similar theoretical interests. [4] Our ideas are illustrated through two accounts of learning episodes. As this article is more a theoretical essay than a research report, the data provided are intended to be illustrative of ideas rather than demonstrative of empirical findings. Further to this point, we are aware that it is difficult to extract theoretical claims through data reporting, given the complexity of the collective dynamic discussed, and so efforts to validate claims through presentation of data would detract from the article's more issues-oriented intentions. Having said that, the episodes presented should be seen as examples to help us to ground, develop and illustrate our argument that transfer can be viewed as a collective phenomenon within a (local) community of practice - a phenomenon that may 'include' but goes beyond the head and heart of the individual 'successful' learner. We conclude with some discussion of how we might plan to develop interdisciplinary work using ideas of community of practice.

Some situated views of transfer of knowledge

There is clear disagreement over transfer in the mathematics education literature relating to situated learning. For example, Greeno (1997) proposes that it is more appropriate to treat the issue in terms of 'generality of knowing' than 'transfer of knowledge'. For him, *generality* is better than *transfer* to describe how much the apprehension of aspects of a specific kind of practice depends on resources available either within that practice or in quite different kinds of practice. Further, Greeno's shift from *knowledge* to *knowing* is intended to highlight the participation of individuals in interaction with others and with material and representational systems. Greeno, Smith and Moore (1993) approach transfer in terms of the patterns of participatory processes across situations, whereby a process of transfer corresponds to a transformation of the situation in which an activity was learned. By 'activity' they mean the interaction between the learner and the systems (*e.g.*, representational and material) available in the situations. The activity, in its turn, has to be transformed depending on the ways the situation is transformed. They suggest that the difficulty of a transformation for students depends on how they are attuned to constraints and affordances in the initial learning activity. They discuss possibilities for *constraints* and *affordances* through a re-examination of some 'classic' research on transfer.

Lerman (1998) suggests that *transfer-ability* in mathematics is a specific activity that can be learned. Such ability is related to the potential to read texts with mathematical eyes no matter the form of presentation. This is only possible if the subjects are appropriately positioned within the discursive domain of mathematics. Lerman (1999) draws on anthropological (*e.g.*, Lave 1988) and linguistic/discursive (*e g*, Walkerdine 1988) perspectives to elaborate these points, noting that Lave treats the issue in terms of meanings within practices. Lerman (1999) further notes that Lave suggested a more flexible notion of boundaries between communities of practice. This notion sees "the range of practices in which any individual engages to be overlapping, mutually constituting and related" and "offers the possibility for conceptualizing transfer across ... boundaries, where practices have family resemblances to each other" (p. 96). He observes that, although Walkerdine approaches the issue in terms of a "disjuncture between practices and of discontinuities of meanings across boundaries" (p. 97), she is concerned to bridge these gaps by identifying areas where there might be overlaps and to show how the teacher can provide a structure in the school discourse to allow transfer.

Carreira, Evans, Lerman and Morgan (2002) analyse transfer of knowledge from two theoretical perspectives: semiotic mediation and discursive practice. Their research was carried out with students taking a calculus course as part of their business degree. The students were observed during an activity of mathematisation, which made use of open questions to guide investigation of a particular model. According to the authors, from a perspective of semiotic mediation, transfer is a process that does not result automatically from learning. Transfer is seen as something that can be, and needs to be, taught if the aim of teaching is to help students to make connections between semiotically and conceptually different practices. From the discursive practice perspective, transfer is seen as a process depending on a *chain of signification* (Evans, 2000) – that is, a process that, in this case, links concepts originating in the discourse of one participant with the discourse of another participant. Carreira *et al.* conclude that there is an association between transfer and metaphorical meaning, from the perspective of semiotic mediation.

Boaler (2002b) uses inverted commas to say that students are able to 'transfer' mathematics under certain circumstances. In her research, carried out in two secondary schools in the UK in the 1990s, Boaler examined pedagogical practices focusing on modes of teaching, students' beliefs and mathematical understanding. One school practice was characterized as based on the use of the textbook, where exercises were based on past examinations. The other was based on open projects involving problem solving and discussion in groups. Boaler observed the students' performances when they encountered a range of mathematical test situations. She noted that the students involved in the textbook based practice performed well in situations similar to those of their textbook, but found difficulties in using mathematics in open situations applied or based on discussions. The students involved in the problem solving practice, however, were found to have developed more flexible forms of knowledge, which they could apply in different situations. Boaler connected the learning of the students in these practices and their performances in the test situations with transfer of knowledge. She concluded that the students were able to 'transfer' mathematics partly because of their knowledge, partly because of the practices they have engaged in, and partly because they had developed a productive and active relationship with the discipline.

We understand Boaler and Lerman to be arguing that transfer of knowledge is best seen as a practice that can be opened up to students – but that this depends, essentially, on a mode of teaching that encourages students to make parallel investments: in their mathematical identities, in the development of their knowledge, and in their participation in the practice. Winbourne (2008) also shares the idea that transfer and predisposition to learn are strongly linked. He proposes that the issue of transfer may be better approached than in terms of the types of knowledge the students carry with them from one context to another; rather, "as participants in mathematical practices, they carry with them identities that predispose them [or not] towards looking for and making use of mathematical knowledge in a range of contexts" (p. 100).

The wide range of approaches to transfer was the stimulus for our development of the interdisciplinary research we report here. None of these approaches explicitly takes communities of practice as the starting point or the unit of analysis – and so, according to Engeström (1999a), none takes a strongly situated point of view. True, communities of practice (or some of their characteristics) appear as the setting for some of these approaches, but they ultimately depict transfer as an individual achievement only. We prefer to see transfer as a product of the interactions between the individual and the collective within a practice – that is, a phenomenon that emerges from the practice, from the fact that an essential feature of the practice *is* making resources available for involving and encouraging the individuals to make connections. Theorising transfer from this perspective and taking community of practice as our unit of analysis, we hope both to show how the establishment of a community of practice can produce the conditions for transfer and to support our claim that transfer is primarily a collective, cultural phenomenon.

Boundary crossing

Evans (2000) describes three main forms of transfer: from pedagogic contexts to work or everyday activities, from out-of-school activities to the learning of school subjects, from a specific school subject to another. Due to our theoretical commitments with situated perspectives, we opt to interpret these three forms of transfer in terms of crossing boundaries between practices. We are particularly interested in interdisciplinary school research and, for this reason, we focus on the third form: crossing boundaries between school subjects, and crossing boundaries between what might appear as insulated, non intersecting school practices.

Bernstein's (2004) theoretical perspective accounts for the social production of such subject boundaries and their associated pedagogies. It helps us to understand the reproduction and production of the culture of these school subject practices in terms of the rules and the discursive order that regulate their inner logic. Also, Bernstein's (1996) concepts of *classification, framing, vertical discourses* and *recontextualisation* are helpful. *Classification* refers to the degree of boundary maintenance between school subjects. *Framing* refers to the degree of agency teacher and pupil possess over the selection, organization, pacing and timing of the knowledge shared within these practices. Framing can be examined at a number of levels of the pedagogical relationships of a certain practice. Whatever the level, what is important is to see framing as associated with both teachers' and students' dispositions to cross (or not cross) subject boundaries. The degree of agency can be strong or weak, depending on these dispositions. For Bernstein the specialized symbolic codes of the academic disciplines of mathematics and science are examples of *vertical discourses*. These codes can be used to identify the boundaries between these practices as well as to theorise about ways to minimise the degree of insulation between them. Finally, *recontextualisation* is the process in which the instructional discourses of subject disciplines are inevitably shaped by the regulative discourse operating in the institutional context. It is a concept related to the discursive practice of the teachers, and it helps us to emphasise the way that teachers' speaking and thinking about their subject are seen as inevitably shaped by the dominant ways of speaking and thinking within their institution and the institutional context. Evans (2000) argues that, for learners to cross boundaries, they need to grasp or apprehend – and this does not need to be a conscious, explicit or articulated process – something of the nature of recontextualisation.

The notion of transfer, reconceptualised in terms of Bernstein's perspective of boundary crossing, brings with it pedagogic baggage that may well play a significant part in how teachers and students seek to bring it about. From the perspective of situated cognition (*e.g.*, Lave, 1988, 1993; Winbourne & Watson, 1998), transfer of *knowledge*, as was suggested earlier, sounds an unhelpful idea. However, given that some people do appear, within the school context or between school practices, to be able to act *as if* they are transferring knowledge or crossing boundaries between subject contents, how might we account for this from a situated cognition perspective?

Bernstein can help us to explain why it is that a major challenge for teachers and students in schools is to do what looks like transfer or boundary crossing. Boundaries may be socially produced, but they are no less real for this in the experience of teachers and students. So, why is it that some people are so disposed to do what looks like boundary crossing that, for them, boundaries appear completely permeable? Why is it that, for others, boundaries have a solidity that makes the very thought of crossing impossible? We think that it is helpful to account for this kind of boundary crossing in terms of a community of practice. We will use two stories of learning episodes as a basis for illustrating and developing this idea. First we describe briefly the research environment and what we take to be the contexts of these episodes.

The research environment

The research was carried out in an urban basic school located in a university of a large Brazilian city. At the time of the research the school had aproximately 700 students, notably mixed in gender and socio-economic terms. As a part of a university, the school functioned as a sort of laboratory for teaching and learning, encouraging and supporting educational inquiry and developing innovations based on sound research.

Teachers worked with the same social mix of students and fielded the same concerns from parents and authorities as any other public school. Having said that, the students at the school were used to working within a research environment and all that this entails. The qualities that seem to distinguish this school, according to the reports of the many researchers, teachers and student teachers who have experienced it, are the high level of participation of students in all classes and their readiness to express their ideas and points of view.

The research was conducted in a Year 9 class that Cristina and Selma both taught. Cristina had been teaching this class of 28 students since Year 8. During mathematics, students usually sat in pairs or in small groups of four or five. They sat individually only when they had individual tests. The class used a textbook as a reference for the topics to be studied, not to guide what should be taught. The work on the book was sometimes preceded by provoking questions from Cristina followed by whole group discussions. At other times, student pairs or groups were asked to read a certain topic of the book with the aim of improving their abilities to read mathematical texts and to interact with their classmates. After such activity, Cristina would promote collective discussion through which students and teacher could share their understanding of the text and select textbook exercises that were thought to be helpful. Occasionally other didactical materials were also used to complement the textbook work, including interactive texts prepared by Cristina that contained some routine and non-routine problems, along with activities in the computer laboratory. Whatever the didactical material used, the atmosphere of the mathematics lessons was marked by lively interaction between and amongst the teacher and the students (a characteristic that, as mentioned above, was a feature of the school). In this sense we can say that this mathematics classroom was, at least, open to the production of (local) communities of practice.

The history of Selma's relationship to the class is similar to Cristina's. She had taught them in Year 5. The science lessons were usually taught in the science laboratory of the school, though some were taught through fieldwork. Both in the laboratory and in the field, students worked in groups of about four. As with mathematics lessons, the students sat individually only during individual tests. They made similar use of a textbook – as a reference for teaching rather than a framework for defining activity. Also like mathematics, this was supplemented with other materials. Sometimes Selma would precede the use of these materials with provoking questions or explanations. At other times she would work with the students through collective discussion. Selma's lessons were also marked by lively interaction. Like Cristina, she encouraged the students to express their points of view, valuing the contributions these made to collective discussion. In this sense we can also say that Selma's science classroom is at least open to the production of (local) communities of practice.

Whilst the use of a textbook was a central tool in both Cristina's and Selma's practices, this is not to suggest an objective/target-led approach. Their teaching was primarily focused on their students' mathematical and scientific (and other) needs. Again, while we claim that the two classes were fertile ground for the production of communities of practice, we should stress that neither Cristina nor Selma taught in any "paradise": as in all groups or communities, their classrooms were the site of tensions, conflicts and complex power relations

Mathematics and science interdisciplinary work

The objective of the research was to investigate how and under what circumstances collaborative work in mathematics and science might encourage students to cross the boundaries between these disciplines The subject matter chosen by the teacher-researchers was *proportionality* in mathematics and *density* in science. This choice stemmed from the assumption that, although the same concept from the point of view mathematics, density probably would not be recognized as such by the students. The teachers' intention was then to develop a mathematics-science learning environment in which the students could see the correspondence between these two concepts - that is, that the density of a homogenous material is the ratio of proportionality between mass and volume, that mass and volume are directly proportional quantities In this way the teacher-researchers planned also to contribute to the development of the students' reasoning about intensive quantities: two variables related on the logic of co-variation (see, *e.g.*, Howe, Nunes & Bryant, 2005)

Cristina and Selma spent considerable time - an average of 2 hours each week between the months of June and November 2005 - planning and organizing the materials and activities for their class Activities included readings and discussions of materials each planned to offer the students, efforts explicitly to bring together the language codes of the disciplines, and discussions of how and when bridges could be built. These interactions occurred at school, at Selma's home, at lunch times, and through many phone calls and Skype conversations, most of them outside working hours. Some of these were recorded in writing, others were recorded on video. When Cristina or Selma had a new idea about their collaborative work, regardless of the hour or day, they communicated with each other We think it is significant that the two colleagues were very close, both in daily-life and their shared educational values

The school curriculum for Year 9 suggested that the children should study proportionality before density. For this reason only, Cristina set off on the research activity in her classroom first. The proportionality activities began in August 2005; the density activities began two months later.

Cristina made an interactive text about direct proportionality that, among other things, invited the students to discuss some 'special ratios'. After introducing and exemplifying the concept of ratio as a quotient between two numbers, the text stressed that average speed, energy expenditure during a period of time, demographic density and π (3.1416 ...) were examples of these special ratios, and asked the students: 'Do you know what these ratios mean? Do you know other special ratios?' To work on the text/exercises the students were divided into small groups (5 at maximum). When the groups finished this work Cristina encouraged them to talk about it. The proportionality activity took 4 class-hours. Data were collected in the form of students' written exercises, video recording of group discussion, and video recording of interviews with some students and two undergraduate students who were doing their teaching practice in Cristina's class.

Selma gave 8 class-hours to the topic of density Here the students worked through activities from their science textbook and carried out laboratory activities in small groups in which they calculated the density of materials and did some experiments to check the relationship between density and the buoyancy of these materials in water. During each of these experiments, Selma said 'floating and sinking, see?', and the students seemed to enjoy this as many of them repeated 'floating and sinking'. The science activities were recorded on video and the students' individual written exercises were collected. During all classes on density Selma drew the students' attention to the fact that the mathematical concept behind density was proportionality that had recently been studied in Cristina's class In doing this she revisited the concept of proportionality - something she already planned with her colleague that she would do - adjusting her language (scientific) code as much as she could to be close to Cristina's (mathematical) language code.

Before starting the data collection, both teachers talked with the class about the research They talked in broad terms about their objectives and procedures, but they did not discuss the specific focus of their research with the students. Their intention was to build on the children's high levels of participation and encourage them to become active agents within the collaborative practice the teacher-researchers wanted to produce. From the start, Cristina and Selma discussed this plan explicitly in terms of the constitution of a (local) community of practice; the collaborative practice was conceptualised in terms of strong characteristics of a community of practice, rather than simply two teachers and their classes working together This is an important point for us to emphasize because the same is not true of all collaborative or interdisciplinary work in schools

The learning episodes: Aline and Julia

We start our account of Aline's learning episode after the work on proportionality had been collectively marked. The class had been discussing the special ratios mentioned above and Cristina had made careful notes on the blackboard. Cristina asked the students if they knew any other special ratios. Aline said, 'density', and so did a number of other students. Cristina asked what they meant by density and these students said 'mass divided by volume' Cristina asked Aline to talk further about the connection between proportionality and density She was not able to do so; her face showed she was in doubt. The teacher stimulated student discussion about the densities of water, iron, oil, and other physical materials, emphasising (and illustrating) the fact that for each of these materials the density is the constant of direct proportionally between mass and volume. As Selma had done, Cristina also encouraged this 'linking' discussion, trying to adjust her mathematical language code to fit with Selma's scientific code. [5]

At the end of the class Cristina asked Aline and two more students (one boy and one girl) to talk with her. Among other things, she asked them when they had first identified density as a special ratio; had it been on the day they had been working on the text, or on the day when they had marked their work together? All of the students said that they had made the connection when they had marked their work as shown in the transcripts below:

Aline: I've already studied this in chemistry. Then I saw the ratio. Then when I compared this with the ratios that were on the blackboard yesterday, acceleration [for example], then I remembered density

Henrique: I think it was more because the discussion was more open. I think it helped. Everybody giving an opinion, saying something, so remembering a little bit here, a little bit there

Carolina: It's because I saw her [Aline] speaking, I only remembered it when she spoke

Aline's utterance, 'Then I saw the ratio', seems to mark the moment when she made the connection between what had previously been two apparently unrelated concepts. The rest of her utterance indicates a crossing of the boundary between the two subjects. First she recognises that she had studied density in chemistry, and then she suggests that density is a particular case of proportionality. What Aline (and other students) seemed not to know was that for a specific physical material mass and volume are in a relation of proportionality

Within the traditional view of transfer - the use or application in one context of knowledge learned in another - one could say that Aline's response, 'density', does not suggest a powerful example of transfer. This may not be evidence of transfer in the traditional sense, but what is significant for us is that the learning practice proposed by the teacher might have allowed and encouraged the students not only to cross the boundaries between school mathematics and school science, but also to 'circulate' between them. In this sense, and in response to Lerman's previous question (1999), we say that Aline and other students learned to be conscious of aspects of the contexts of their activity and so cross the boundaries of mathematics and science practices. Furthermore, this consciousness, shared by the teacher-researchers, was a defining feature of that practice.

The utterances of Henrique and Carolina strongly suggest that a *semiotic chain* or *a chain of signification,* which carried meanings between mathematics and science discourses, and was triggered explicitly by Aline, might have been established in the classroom (see Evans, 2000). [6] We see the establishment of this semiotic chain, like the establishment of the semiotic chains set up by other students when Aline first said 'density' in the classroom, as characteristic of the mathematics-science community of practice that was being formed. We offer two reasons for this. Firstly, these chains of signification were produced around a common understanding of density as a particular case of proportionality. Secondly, the students' interventions about density in the discussion in class can be seen as manifestations of their agency in the practice: it was regulated and developed according to these interventions. So, the practice allowed improvisations, sharing of doubts, understandings and meanings. Finally, all of these led us to claim that the connection made between proportionality and density both in the mathematics classroom and in interviews with Cristina was not only an achievement of Aline. Indeed, the utterances of Henrique and Carolina explicitly show that a process linking these concepts had emerged from discourse within the class. So, the boundary crossing between mathematics and science practices we are associating with this connection was a feature of a collective phenomenon produced by the practice that involved Aline, Henrique, Carolina, and the other students who echoed Aline in saying 'density' in Cristina's classrooms; other more peripheral participants, who may well have been silent, nevertheless heard the discussion, listened to their classmates and teacher, and so shared in that collective achievement

Later in this conversation, Cristina asked if they had worked on density before. They all had, in two different contexts - when they were 11 in a science class (taught, coincidently, by Selma), and recently in an additional private 'cramming' course they were doing for entry to high school. They all commented that Selma did not like formulas, that she preferred to work on understanding; they voluntarily contrasted her approach with the 'straight' (procedural) methods of the cramming course: '*this is for this, that is for that and you have to use this* [formula for density] *in this way*'. More interestingly, these students seemed aware that they had learned different things, albeit with the same label, in the two settings. We suggest that this may also be evidence of students' awareness that their teachers' differing approaches reflected what we might call the inevitable result of recontextualisation.

Julia's story starts during a technical outing to a hydroelectric plant, by coincidence planned by Selma at the time the class were studying density. The objective of this visit was to observe the phases of the process of energy transformation in real life. The first stop the group made was at a reservoir. Their task was actually to watch how water passed through sluices designed to regulate flow into a canal. Sadly, when the students got closer to the reservoir they came across the dead body of a dog floating in the water. The body floated amidst papers, plastic bottles, pieces of wood and other rubbish. The sight of the dead body unsettled the students. Some of them began to wonder how the dog - presumably a good swimmer - had drowned. Others were disturbed by the teacher's observation that the Brazilian attitude to the environment allowed the river to become a rubbish dump. And then Julia exclaimed: *'floating and sinking!'* As happened with Aline, this utterance seems to mark the moment when Julia made a connection between two apparently non-intersecting school science practices: the lessons about density at school and the outing to the reservoir. This surprised Selma whose agenda was no longer density, but the processes of transformation of energy. But, she used Julia's observation to revive the talk about 'floating and sinking' and new problems arose: why were all those materials floating? The students had no difficulty recalling their studies about density. Some said, '*because they were less dense than water*'. Others said: '*But, what about the* *dead body of the dog, why does it float?*' Selma and the students talked about the dead body of the dog and applied what they had learned about density in the science laboratory. Here again we have a suggestion that a semiotic chain, triggered explicitly now by Julia, which carried meanings between two different science topics might have been established in the context of the visit to the hydroelectric plant. Selma also extended the talk to include other scientific relationships, including that between buoyancy of bodies (including human bodies) and the structure of lungs. This talk took over a good part of the visit, and all students engaged in the conversation. The story concludes with Selma, back at school again with the class, referring once more to the events of the hydroelectric plant about 'floating and sinking'.

In a way similar to that described in Aline's story, the teacher used Julia's comment 'floating and sinking' to blur the boundaries of the two science practices mentioned above; without this intervention these practices might have stayed unconnected within the vertical scientific discourse. Through this intervention, the teacher set out to create an atmosphere that would allow and encourage students to move between these practices. This allows us to say, too, that in this way the students were encouraged to become conscious of the contexts of their activity and to cross boundaries.

So, what could lead Aline and Julia to make these connections? Why did other students engage so quickly in the conversation that followed these connections?

A mathematics-science community of practice

In Bernstein's terms, we might say that Cristina and Selma, having translated for each other their specific discipline codes and worked together to prepare and organize collaborative work, had set up 'something' that enabled the crossing of the boundaries. Our theoretical approach is that this 'something' can be seen as a mathematics and science community of practice (MSCoP), which had some durability and stability. The comments and the connections that were made by Aline, Julia and others are signs of the students' participation in such a MSCoP.

We see the predispositions of the students, their readiness to become active participants in these kinds of practices, as key requirements for the constitution of the MSCoP. The students' apparent awareness of differences in teaching, of the ways in which these differences are produced and why, might be taken as evidence of their apprehension of something of the principles of recontextualisation we mentioned earlier. It may also be another facet of the students' predispositions and so important in the constitution of the MSCoP. Such active student participation might be due to their consciousness that they were allowed to be agents of this MSCoP: both teachers were able to catch on to their students' comments to socialize new understandings. Cristina took the opportunity presented by Aline's response, 'density', to encourage the students' interaction towards producing a common understanding of the density of homogenous physical materials like water, iron and oil as the constant of direct proportionality between mass and volume. Similarly, Selma profited from Julia's comment, 'floating and sinking', to encourage the students' interaction towards producing a common understanding of buoyancy of human bodies and the structure of lungs. In both cases the students (perhaps more obviously in Selma's class), guided by the teachers, dictated the unfolding of activity within the MSCoP.

Another question of continuing concern to us is what schooling might have to do with the development of such dispositions. Bernstein's ideas of framing and classification are very helpful when working on issues of transfer at the school level. In the school where Cristina and Selma teach, the curriculum is one that maintains specialised codes with subject disciplines that are strongly classified, but this was no impediment to weakening the boundaries between particular disciplinary codes. The dispositions of teachers and students - for the teachers, using Wenger's (1998) words, to act as boundary crossers, or *brokers*, of their disciplines; for the students, as they engaged in the collaborative project - showed that the agency of all participants over the practices established depended much more on the people involved at the local level than action at other institutional levels of schooling (though we should not disregard action at these higher levels).

Wenger's (1998) notion of brokers - those who facilitate coordination and open possibilities for new connections and meanings between practices - brings with it the concept of *boundary objects*. These can be thought of as objects that inhabit several practices and satisfy the informational requirements of each of them (Tuomi-Gröhn & Engeström, 2003). The boundary object under discussion here is the concept of proportionality. Indeed, if we follow Ludvigsen, Havnes and Lanh (2003), we may surmise that initially the student participants may not have recognized proportionality in their mathematics classroom and density in their science classroom as the same concept - and may well have attributed exclusive meanings to each of them. Our interpretation is that the teacher-researchers, having broken the frontiers of their disciplines to encourage connections between proportionality and density, allowed the students to connect these apparently disjoint concepts, making sense of them as the same concept across the two practices. When discussing Wenger's (1998) idea of *boundary encounters*, such as conversations and meetings, Tuomi-Gröhn and Engeström (2003) say:

> Although Wenger's concept is useful for designing different forms of boundary encounters, it is not, however, enough to explain the dynamics of learning and change in these encounters. It is necessary to examine the boundary encounter as a discursive space and a hybrid learning context. (p. 5)

We suggest that Aline's and Julia's stories satisfy the requirements suggested by Tuomi-Gröhn and Engeström. These stories provide illustrations of such boundary encounters as a discursive space in terms of mathematics and science discourses of proportionality and density.

Finally, turning to the empirical studies to which we have referred, in each some community of practice (or, at least, some characteristics of a community of practice) was also set up to enable students to cross boundaries between some apparently unconnected school practices or contexts. In Bernstein's terms, the degree of agency can be characterized as strong in all these studies as, in a way similar to our study, there was a clear disposition of both teachers (intentional and careful planning of the mathematical activities) and students (to interact in the activities) to engage in the activity out of which these communities of practice emerged This led us to believe that our proposal to account for crossing boundaries in classrooms in terms of community of practice and Bernstein's pedagogy can indeed be a fruitful path

Final discussion

We hope we have established the construct of an interdisciplinary CoP to help us to understand students' crossing of boundaries between some specific, apparently insulated school practices. However, much has yet to be developed if we want to give to this construct the kind of theoretical status that will render it useful in future research. Here we begin that process of development with discussion of some ways forward

We suggest that we can further develop understanding of an interdisciplinary community of practice such as this MSCoP by focussing separately on the role of the teachers and the role of the students in its constitution. In relation to the teachers, our research strongly suggests two central roles to be explored in order to set up any kind of such CoP: participating teachers' attunement in relation to both general education and their discipline values; and participating teacher's disposition to cross the boundaries of their own discipline, including their willingness to adjust their language codes Concerning the latter, we have been suggesting exploring the issue in terms of Bernstein's (1996) works about integrated curriculum codes: "Integration ... refers minimally to the *subordination* of previously insulated subjects ... to some *relational* idea, which blurs the boundaries between the subjects" (p. 93) So, while Cristina and Selma may not on their own have set up an integrated code through their collaborative work, we think they did, at the very least, draw attention to and try to work around the collective code within which they were working.

As for the role of the students in the constitution of MSCoPs, our research suggests that students' participation in it spins around the affective domain, encompassing such elements as their dispositions 'to enter' into a MSCoP, to interact socially, their relationships with disciplines and teachers, and their openness to making connections. Hence the literature on affect (*e g.*, Zan, Brown, Evans & Hannula, 2006; Frade & Machado, 2008) should inform future discussion. In this sense, students' boundary crossing is much more a collective phenomenon - a matter both of their collective engagement in a community of practice and the affective relationships they develop with the practices to be crossed - than mere individual achievement

Notes

[1] Supported by FAPEMIG and CNPq

[2] See Frade and Tatsis (2009) for a discussion of this adaptation in terms of Lave and Wenger's notion of community of practice.

[3] Grootenboer, Smith and Lowrie (2006), Ingram (2008) and Sfard and Prusak (2005) present reasons for considering the concept of identity in mathematics education research Grootenboer *et al.* point out that the concept of identity has been used by researchers from a range of different paradigms, and thus has been conceptualized from multiple perspectives They discuss three particular views of identity that, in their opinion, are particularly influential in mathematics education: psychological/developmental, socio-cultural, and poststructural They link these views to a number of theorists and observe that some research uses combinations of views We take a socio-cultural view of identity, though one consistent with the poststructural approach described by Grootenboer *et al.* - namely that "identity formation is ... [a] dynamic and somewhat unstable [process]" (p. 613)

[4] Good reviews of transfer can be found in Evans (2000), Greeno (1997) and Tuomi-Gröhn and Engeström (2003)

[5] Recordings of Cristina's classes show a change in the way she talked mathematically about proportionality. It became embedded in scientific talk. Cristina had already studied Selma's material and talked over her understanding with her Cristina set out to talk about density in her class using the terms and expressions Selma had taught her In the case of Selma a similar change can be seen in the recordings of her classes

[6] It was because Aline triggered the episode that we gave it her name The same is true for Julia's episode.

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Elusive solutions?

Consider what method(s) you might use to solve the following simultaneous equations:

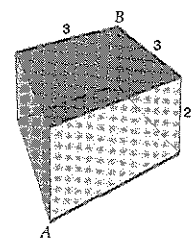
$$x^2 - y^2 = 0 \quad \text{and} \quad (x - \alpha)^2 + y^2 = 1$$

Is one method more 'efficient' than another, and how many (real) solutions are there?

(first appeared as a "show that there are n solutions" type of problem in a textbook for first-year university students about 30 years ago; selected by Leo Rogers)

Shortcutting

The diagram to the right shows a solid rectangular prism with square cross-section measuring 3×3 and height 2. A and B are opposite corners. Find the length of a shortest path on the surface of the prism from A to B. Explain your calculation



(unknown origin; selected by Malgorzata Dubiel)