MATHEMATICAL PRACTICE AS SCULPTURE OF UTOPIA: MODELS, IGNORANCE, AND THE EMANCIPATED SPECTATOR

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School mathematics would always seem to fail. We are usually disappointed. Those who have experienced school mathematics are not always able to demonstrate that they have developed particular conceptual understandings, nor that they have mastered certain skills; indeed, it often appears that a large number of students are not able to demonstrate these things. Few, if any, students would be easily described as having integrated mathematical ways of thinking or the disposition to ask mathematical questions into everyday activities or to seek specific social, cultural or political contexts that are particularly suitable for such ways of thinking. Relatively few students follow school mathematics experiences by deciding to pursue further mathematically-based studies, to pursue careers that require further study of mathematics, or to pursue mathematical forms of recreation and entertainment. It is not unusual to find people who have achieved advanced levels of mathematical education who describe themselves as "non-math" people, or as not good at mathematics.

There are two common interpretations of this situation: the first and more common one blames inadequate educational practices. This frame presumes teachers have not taught well, or that learners have not applied themselves, or that social and cultural inequities have led to environments not conducive to learning. Faith in the fundamental structures of school as an institution, and a similar faith in the ability of educational researchers and teachers to refine commonly used practices over time, maintain the dream of ultimate success. The second interpretation blames the schooling enterprise itself, suggesting that the very assumptions of schooling as an institution undermine its goals, and that there is a need to abandon, modify or replace fundamental structures and conceptions. I claim the "problem" with mathematics education is mathematics education itself, and offer an alternative interpretation based on the ideas of the French philosopher, Jacques Rancière, and the American sculptor, Josiah McElheny.

The theatre of mathematics education

Rancière's (1991) critique of the pedagogic stance confronts the act of teaching and the assumption that a learner needs to learn. We would all agree that most young people have not yet gone through the same school mathematics experiences as their teachers; students do not already know mathematics in the sense that their teachers do. Yet a paradox remains: treating learners as if they need to be taught creates passive recipients of explanations; students are constructed as people who do not know and, because of this, the implicit message that we are all "ignoramuses" at the core is carried through school experience into everyday life. This critique goes beyond teaching and learning to the hegemonic role of explanation and the desire to manipulate that pervades most social relationships (Bingham, Biesta & Rancière, 2009; Biesta, 2010). Mathematics education is complicit in the creation of what Rancière (2009) calls an explanation society. Our educational practices presume people should be changed into actors who use mathematics to make meaning out of experience. We dream of transforming a society in which people do not readily use mathematics into one in which they do. Ironically, we thereby perpetuate a culture of mathematical ignorance.

As Rancière suggests, "To explain something to one who is ignorant is, first and foremost, to explain that which would not be understood if it were not explained. It is to demonstrate an incapacity" (Bingham, Biesta & Rancière, 2009, p. 3). While Rancière is writing of political theatre and its efforts to promote action for social transformation, we can use the same ideas to consider our efforts toward a parallel revolution in the practices of mathematics education. Playwrights and curriculum designers share a common dream: "Perhaps [they] will know what is to be done, as long as the performance draws them out of their passive attitude and transforms them into active participants in a shared world" (Rancière, 2009, p. 11). Theatre innovators as diverse as Brecht and Artaud enact a common pedagogical stance that structures the audience as passive spectators, when they might have been supported in their already ongoing revolutionary action outside of the theatrical experience. However, even if we tried to facilitate such action, as in a Freireian critical pedagogy (McLaren, 2000; Oakes, Rogers & Lipton, 2006), we would still be manipulating people into behaving as if they are observers of their own world, again frustratingly undermining our ultimate goal. In each theorization, spectators need guidance and explanation in order to "progress".

Boal's (2008) "theatre of the oppressed" appears to dissolve this paradox with spect-actors who participate in events, taking over roles in scenes as a sort of rehearsal of what they would afterwards be able to do in "real life"; yet this, too, maintains the need for the theatre event to have been crafted and placed before the spect-actor, without which no individual transformation or refining of beliefs would transpire. The last half century since Brecht and Artaud has witnessed alternative theories of theatre as education, social change vehicles, therapy, and community development techniques, each breaking down clear boundaries between spectator and actor, playwright and performer, audience and director (Boal, 1995; Salas, 1993; Fox & Dauber, 1999). Yet the eternal question remains: how can we theorize the facilitator, conductor, joker, director, as juxtaposed to the learner, audience, spectator, change agent, without recreating a passive audience of ignoramuses?

For both theatre and school, the gulf between passivity and activity must be overcome. In the pedagogic stance, audience or students must be "motivated": in the context of mathematics education, students must be motivated to look at the world mathematically, to transform themselves from mathematically passive spectators into mathematical actors. The "ignorant schoolmaster" [1], in Rancière's formulation, intentionally works in ignorance of the differential between teacher and students, the one who knows and those who need the explanations. When we work as ignorant schoolmasters, we deny any sense of inequality in order to enable learning and transformation. Refusing to disappear from the pedagogical process—to do so would be to abdicate any role or responsibility in society—the ignorant schoolmaster reaffirms equality of capacity and intelligence. At the heart of this subtle switch is the serious notion that students come to school with capacities to be honoured and nurtured, along with our own, as we co-create and recreate our world.

There is plenty of precedent. Skovsmose's (1994, 2005) early work in critical mathematics education attributed a "critical competence" (Skovsmose 1994, p. 61) to both teachers and students. The broad study of "funds of knowledge" (González, Moll & Amanti, 2005; Wink, 2010) grounded mathematics education in culturally relevant and culturally specific pedagogy (Leonard, 2008; Appelbaum, 2009), helping teachers apply learning about their students' lives to apprenticeship in the shared development of mathematical skills and concepts that grow out of the organic texture of these lives. Early efforts of the National Council of Teachers of Mathematics in the United States to generate *Curriculum and Evaluation Standards* (NCTM, 1989) moved from a curriculum that explicitly taught problem solving skills, to mathematics learned in problem solving contexts. Similar reorientations to critical thinking in mathematics avoid teaching the skills and dispositions of critical thinking in favour of classrooms where learners bring critical thinking *to* the classroom (Appelbaum, 2008).

Beyond such a shift, we need to abandon the notion that school mathematics is in any way preparation for later uses. In particular, we must erase any trace of a belief in expertise, and embrace our ignorance of specialized mathematical practices. Rather than learning how to act as a mathematician, our students *are* types of mathematicians, such as artist-mathematicians, scientist-mathematicians, architect-mathematicians, and writer-mathematicians. To act with an awareness of the gulfs that exist among artists, mathematicians, scientist-mathematicians, and so on, creates inequities where someone is in a position to explain to another, maintaining a passive, receptive audience and perpetuating the pedagogic stance. Similarly, to act with an awareness of the differences between a practicing architect, writer, and so on, and a young mathematician in our classrooms, is to regenerate the same failures of the pedagogic stance that we have come to accept as the *status quo*. Ignorant schoolmasters delight, instead, in the ways students *are* artist-mathematicians, scientist-mathematicians, writer-mathematicians, and so on. Rather than identifying what students do not yet know, or wallowing in the pleasures of well-designed curricula that cajole students into learning difficult-to-understand mathematics, we can document how students already demonstrate mathematical skills (Donald, *et al.,* 2011).

Modelling mathematics and modelling *in* mathematics

Models and representations are central to mathematical activity. Physical materials, diagrams, charts, symbolic representations, exemplary problems of a type, *etc*., dominate mathematics teaching and learning. Much of our pedagogy rushes to representations. We want pupils to become very good at using them to model concepts and relationships. This desire, indeed, seems to be at the heart of mathematics as a living, growing discipline: pupils who can move from one model to another are usually taken as understanding the mathematics. It seems that the very act of working *with* representations as *models* might actually characterize mathematics.

We often assume we know what we mean by representations and models, yet there are multiple ways to conceive and apply them as researchers, as developers of curriculum materials, as teachers of mathematics, and as mathematicians. In particular, there is a widespread assumption that models are used to create a picture of a real-life situation to facilitate problem-solving, or that a representation within one mathematical area might be employed as a model of relationships in a previously disconnected branch of mathematics. If we were to work with one or both of these assumptions, we could summarily dismiss the ignorant schoolmaster as ignorant of these otherwise routine mathematical practices. However, as ignorant schoolmasters ourselves, we do not jump to these assumptions. To presume that models are tools of problem-solving, for example, contains the further expectation that we would explain how to use models to someone who does not yet know how to do this. What if the people with whom we interact are using models for other purposes? If we assume their ignorance, then we are reifying our own ignorance of the ways they are modelling and using models.

The sculptor Josiah McElheny (2007) [2] analyzes the purposes of models and the act of modelling, categorizing practices as typical of a scientist, architect, or conceptual artist. The point is not to caricature professions, but to use the dominant stereotypes to generate types of practices. Each practice applies metaphors that, in turn, represent assumptions, values, fears, desires, structures of discourse, and so on. We might imagine teachers or pupils of mathematics similarly employing models and the representations within them with different aims at different times and, in this sense, acting in ways analogous to different iconic conceptions of representation in general. For example, a teacher or pupil might work as an architect-mathematician, scientist-mathematician, or artist-mathematician, at different times during a school day, in order to achieve various goals, just as they probably do in their everyday life when the situation calls for these orientations. This is not to say that such supposedly non-mathematical approaches would somehow rescue mathematics, nor that students might find working like an architect, scientist or artist more relevant or motivating than working as a mathematician. We are using these professions metaphorically to describe the experience of social interaction and implicit epistemological conceptions of representation within a cultural discourse.

One might employ blocks, pictures or equations as part of an argument to convince others of a conclusion or the reasonableness of a result, metaphorically acting as an architect convincing clients with a blueprint or scale model of a building. Or, one might invent a way of representing a system or set of relationships, and then proceed to analyze the representation for what it further implies regarding these relationships, much as a scientist uses models to better comprehend the relationships among natural phenomena. Architect-mathematicians might employ a construction by straightedge and compass to convince another pupil that the perpendicular bisectors of the sides of a triangle will intersect in one unique point; we might create an animation using dynamic geometry software, or create a series of logical statements that include details about these parts of a triangle in algebraic symbols

If learners graph parabolic data and use the graph to study parabolic functions (noting various characteristics of the types of relationships that are demonstrated among the variables that are depicted by their graph), they could be metaphorically labelled as scientist-mathematicians; similarly, drawing a picture of a person and their shadow from a lamp-post as they walk, marking items that could be measured at discrete distances from the lamp-post along the ground, a scientist-mathematician would then use the picture to generate a chart or graph from which they could further clarify particular aspects of the relationships involved, *i.e.,* proportionality between heights of the light source and the person walking, length of shadow, distance from lamp-post, and so on.

Sculpture of utopia

Artist-mathematicians in McElheny's (2007) sense seem rarer in school. Yet, I believe that this way of using models is more central to the work of mathematicians than architect- or scientist-mathematicians, at least, more at the heart of what the mathematical experience is all about. Conceptual artists do not use models to convince others or to solve problems. McElheny's artists use models to create new worlds, imaginary spaces of learning outside of time and space. The models become "proposals"—invitations to come and play and explore ideas. The other kinds of models drag us down into realms of accuracy, correctness, and explanation: "Is it a 'good model'?" is too often taken to mean, "Is this model a true replica of the real world?" Such models are tossed aside as soon as they fail to live up to the demand that they precisely mirror nature. McElheny refers to Isamu Noguchi's playground designs. Noguchi's model for a UN Playground in New York was never built, but the model became a well-known work of art in museum exhibits. Used in *this* way, the model provokes us to imagine what playgrounds could be, to think about our assumptions about what a playground should or should not be, and to question the decisions that we generally make about what might take place in a playground. The artist's model differs in value from other models, it seems; perhaps it raises the potential for all models, regardless of their surface intent, to carry this provocation. Not tied to true mirroring, the pleasures and utility of an artist's model are in itself, and in the way it questions the very act of modelling. Conceptual art leads a person interacting with it to reflect on the process of making the art and the concepts that are invoked with the art. It is in this sense that I believe McElheny's artist is like our mathematicians, contributing to a living, growing discipline.

Noguchi's playground is, simultaneously, a potential playground in the world and a playground of the mind, generating all sorts of experiments and questions in time and space distinct from our direct experience of our world, yet maybe influencing that world through the ways we might act in the future, even more than a playground might affect the lives of whoever had played in it. It is in this sense that the model becomes a "sculpture of utopia"—an ideal conception that does not exist in the world. A mathematician, artist or practitioner of any kind who establishes this sort of standpoint on his or her use of models evokes what Rotman (1993) once called a meta-subject—someone compelled to consider the relationship to the act of idea generation or artistic invention itself; the work provokes reflection on the meaning of the work with the very action of constructing the model. For mathematics education, model creation is critical, because it enables the teacher and student to talk about the specific purpose of mathematical ideas, as well as to reflect on the processes of idea-development: it instantiates those aspects of learning and concept creation most difficult to address in ordinary classroom practices. Both teacher and pupil examine and discuss what is typically left to chance: processes of creating, representing, and modifying ideas.

How is this provocation different from the political theatre earlier in this article? How does it avoid a pedagogic stance? For practitioners of the pedagogic stance, that is, enlightenment revolutionary playwrights or traditional mathematics teachers, inequality inadvertently establishes passive spectator observers instead of social change agents or inventive mathematicians. Conceptual art sculpts utopia as an affirmation of the revolutionary capacities of everyone involved in the co-creation of the model, calling attention at once to the model and the act of modelling. Mathematics needs to work for us and our pupils in this way, offering provocations to construct new conceptions of relationships and systems and make visible the model invention processes. Models in the more traditional sense of applications of mathematics to the real world do not challenge presumptions; they provide algorithms for obtaining solutions. Such models do not help people focus on the mathematics, but instead on the algorithms and recipes for answers. We are left dissatisfied that pupils are merely memorizing lists of steps toward a formulaic solution, rather than genuinely understanding the mathematics or authentically understanding with mathematics.

McElheny's models of utopia provoke questions and conversation, confusion and fascination, contemplation, new philosophic inquiries, fantasies, repulsions, and more. [3] Noguchi's model provokes questions: What *is* a playground? What *could be* a playground? Why do our playgrounds all look the same? Analogously, base-ten blocks might provoke questions: Why do we work in base-ten? How does a base-ten way of organizing numbers of things influence the ways that we think? How does thinking about numbers in terms of three-dimensional volume lead us to different questions and conclusions when compared with the types of questions that emerge when working with 100s-charts or number lines? When we think with base 10 numeral systems of functional relationships, that is, as we think *about* base 10 models of relationships, this use of models generates opportunities to interrogate the meanings within the mathematics while simultaneously inventing our own algorithms, both of which make it possible to easily approach standard procedural knowledge critically and meaningfully, to appreciate its power as well as its limitations. A comparison of algebraic representations, graphs, and tables for the same functional relationship should likewise provoke: we consider, in response, relationships *among* relationships and the representations for these meta-relationships of relationships.

Mathematics as sculpture of utopia by emancipated spectators

Representations can be thought of as the actual material and content of mathematics. I mean here simple things like numerals to represent numbers of things; drawings of shapes to represent ideal geometric relationships; fractions to represent parts of wholes, proportions, and ratios; equations to represent functional relationships; letters to represent variables that may take on different values; base-ten blocks for arithmetic operations; drawings of rectangles or circles for fractions and ratios; graphs that visually represent algebraic equations, modelling mathematical concepts and relationships. One interesting point to then consider is that art historians and critics sometimes suggest representation is not always the aim of art; in fact, for some, representation violates art. What might learners as conceptual mathematical artists do, then, if they are not primarily practicing forms of mathematical representation? What would it mean for youth who are learning "stuff that many adults already know" to be artists—creators and producers—when we seem to want them to consume and use mathematics instead?

The critical issue turns out to be the way we make sense of the "art." Susan Sontag (1966) wrote of the "highly dubious theory" (p. 10) that art contains content. When we take art as containing content, we are led to assume that art represents and interprets the world, and that these acts of representation and interpretation are the essence of art itself. Similarly, we often imagine the mathematics curriculum as content and move quickly to the assumption that this curriculum represents and interprets. This makes art and curriculum into articles of use, for arrangement into a mental scheme of categories. What else could art or curriculum do? Sontag (1966) suggests several things: "To avoid interpretation, art may become parody. Or it may become abstract. Or it may become ('merely') decorative. Or it may become non-art" (p. 10). Parody, abstraction, decoration, and/or non-art are three types of tactics for art and curriculum.

Mathematical work is not always done within the framework of McElheny's architect or scientist. More importantly, there is a way in which even the work of an architect or scientist relies on the kind of practices that McElheny calls an "artist", or eventually leads to this artist-like way of working. The artist-aspects of such work are prior to, anterior to, or independent of the work of the architect or scientist, performed, in Rancière's terms, by emancipated spectators.

Given the extensive, wide-ranging research on modelling and models in mathematics education, I suspect my argument is challenging. Most of us already have a working understanding of models and representations, and most of us employ a number of assumptions about them in our own work. I am asking us to reconceptualize some aspects of mathematics that are very fundamental to what we do and, in the process, opening myself to the criticism that I am doing what I ask us *not* to do: explaining to the ignoramus what he or she needs to know. It may take a great deal of effort to explore this together. I have found recent conversations collapsing back into a discourse that presumed, within a pedagogic stance, a scientist-mathematician framework that could not allow the comparisons that this article makes. If a teacher presupposes that most activity in a classroom should have the purpose of solving a problem or of practicing methods of solving types of problems, then that teacher is going to think of models only in the way of a scientist-mathematician, since the main purpose of models in such a classroom is to accurately present a mathematized analogy for the situation occurring in the problem to be solved. This means that such a teacher will have to reorient themselves in order to take advantage of what I am saying. He or she might, for example, begin to introduce new types of activities in their classroom, during which pupils are not solving problems or practicing methods of solving problems; such activities would involve pupils comparing and contrasting models, with no intention of using the models to solve a problem; or taking a model for one mathematical concept and absurdly using it to demonstrate a seemingly unrelated mathematical set of relationships.

A teacher who values problem-solving for its clear selling points on the usefulness of mathematics will also find this article challenging. In some ways, the artist-mathematician seems to be less concerned with the real world or with the application of their ideas. This distancing from reality is not really the case, but the issue deserves some attention. Zbiek and Connor (2006) offer modelling as an activity that replaces motivation, providing a context for learning the mathematics itself; this, too, is different from the artist-mathematician, who might, in fact, care more about the potential of a model or representation than the mathematics for which it was initially brought into use. The history of mathematics is filled with those, such as G. H. Hardy (1940), who have extolled the virtues of pure mathematics, whose efforts seem to exist independent of the practicalities and necessities of an architect, or whose work has no intended scientific application. Even a pure mathematician, however, employs the habits and skills of an architect when convincing others of their conclusions or proofs, and those of a scientist in elaborating and evolving models of systems of relationships.

At one point, I tried to write this article with numerous examples of classroom practices. However, as I finally understood, it is not the lesson plans or curriculum materials that are the critical element. What matters is precisely what is not in those plans or materials: the orientation to mathematics as *sculpture of utopia,* in which the relationship with the models and representations is analogous to conceptual artists working with found objects. Rather than refining how well a model represents reality, teachers could employ a series of exercises adapted from Boal's (1995) *Rainbow of Desire.* Pupils would *critique* models as if they are works of mathematical art. Many lessons would look the same on the surface, yet the students would routinely juxtapose, appraise, analyze and create models; place models in historical context; and use models to provoke emotions, such as joy, nostalgia, outrage, or constancy. Each representation—every picture, diagram, chart, graph, equation, *etc.*—would, in such classrooms, always be taken as a potential model of many concepts and relationships. And this notion that a particular representation is always potentially many different models all at once is important, because such different uses of the representation as a model can be discussed, explored, applied, critiqued, modified, *etc*. Seemingly unrelated concepts and relationships modelled by a common representation are drawn together into new worlds of similarity and difference within classroom conversations and pupil investigations. This all occurs outside of the time and space of the literal modelling processes that make up the usual or traditional focus—in other words, within a *utopian* place, both inside and outside of time and space.

In one educational environment outside of the usual time and space of school mathematics, an after-school "math circle" with secondary school students in a high-poverty, high-crime, low-school-performance neighborhood of Philadelphia, youth used statistics to design a basketball "dream team". A traditional approach would have led the students through an explanation of existing sports statistics and the practice of using them within a well-known model of basketball skills to deduce what many sports analysts already know about the perfect team of players. Instead, university pre-service teachers facilitated the investigation, which was designed by the high school students, helping them pursue a series of experiments in a gym at the local church to determine particular skills and statistics upon which their dream-team would be based. Multiple models and the dependent importance of one statistic or another and how they might apply to the dream-team design were proposed, tested, and compared. In their discussion of which models to employ, students needed to consider how to represent their model for themselves and for their colleagues.

In a primary school math circle, repeated addition on a calculator led students through a series of efforts to model the phenomena that arose, using a number line, a 100s-chart, a collection of tesslation tiles, and a circular representation of modular arithmetic. In many classrooms, students would have been shown how to use the models as part of being guided to deduce the patterns in the units digits of such repeated addition, and the lengths of those patterns, in order to learn particular ways to seed repeated addition with a starting number and a constant adding number for specific outcomes. The university students facilitating this math circle encouraged a small group of children who happened upon repeated addition on their calculator to share what they had noticed with the rest of the group. Discussions emphasized differences among the types of models for what they revealed and obscured. Similarly, attention to sketches of an African village's architecture, shadows cast by a sculptor's collections of human trash, or a graph of recent income distribution would help students understand the kinds of relationships and questions that can be examined and posed as much as such attention would help them obtain answers to particular mathematical problems.

Conclusion

It may seem grandiose or inappropriate to evoke theories of social change for school mathematics. However, entrenched ideologies thrive in both revolutionary theatre and the teaching and learning of mathematics. Indeed, the shared, deep-rooted, pedagogical stance is mutually reinforcing. Epistemological transformation in school mathematics could be the spark that ignites social change, and work for greater participation in democracy in all facets of everyday life and politics. To contribute to society in this way requires that we work against the presumption of ignorance through the stance of the ignorant schoolmaster: beginning with a blindness to the ignorances that are called forth by our pedagogical stance is, in a strange twist, a direct confrontation with ignorance itself. Every pedagogical act conjures up an epistemology of ignorance in two senses. First, it constructs a set of ignorances to be remediated; differences between the one who knows and those who do not yet know establish indicators of inequality. Remediation may occur through minimizing behaviours construed as misguided, through imitating others who know, or by identifying with a teacher who serves as a model of one who knows. Second, a pedagogical act constructs a set of ignorances regarding the learner; any form of teaching and learning makes many characteristics of the learner unknowable for the teacher. Becoming a mathematician of some kind, a mathematically literate person, does not, in this conception, mean that someone is learning mathematics that they did not know. Becoming a mathematician can be observed by the teacher in the movement from one form of modelling practice to another and, especially, in the sculpting of utopia. In this sense, becoming a mathematician is to change the world by changing ourselves.

Notes

[1] Applying Rancière's term and McElheny's potentially gendered professions raises many questions for future research about gendered implications.

[2] McElheny's presentation can be heard at www.moma.org/explore/mutimedia/audios/91/180.

[3] I employ utopia referring to conceptions not found in this world. The more contemporary interpretation of utopia as paradise might be relevant in the sense of crafting a fantasy of wonderful mathematical classrooms, but is not directly intended. The critical mathematics education focus on dystopia—a society in a repressive or controlled state, often under the guise of being utopian—is similarly appropriate, in the sense of working to change with mathematics the most repressive practices of mathematics education and its social context.

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I'm thankful that no one has ever put me up before a blackboard and asked me to explain what I mean when I use the word *explain*. I'm also puzzled that this tends not to happen to me, or to anyone else: we are courteous, and adapt ourselves to an impressionistic sense of each other's general usage of extremely important words—such as explain—happy enough that formal and semiformal words such as *prove, demonstrate,* and *show* seem, at least, to have a clearer significance. (p. 193)

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