On Mathematical Elements in the Tchokwe "Sona" Tradition

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1. The drawing tradition of the Tchokwe

The Tchokwe people (or Quiocos), with a population of about one million, predominantly inhabit the northeast of Angola, the Lunda region. Traditionally they are hunters, but since the middle of the 17th century they have dedicated themselves also to agriculture. The Tchokwe are well known for their beautiful decorative art, ranging from the ornamentation of plaited mats and baskets, iron work, ceramics, engraved calebash fruits and tattooings, to paintings on house walls and sand drawings.

When the Tchokwe meet at their central village places or at their hunting camps, they are used, sitting around a fire or in the shadow of leafy trees, to spend their time in conversations that are illustrated by drawings (*lusona*, pl. *sona*) on the ground. Most of these drawings belong to a long tradition. They refer to proverbs, fables, games, riddles, animals, etc. and play an important role in the transmission of knowledge and wisdom from one generation to the next. The designs have to be executed smoothly and continuously, as any hesitation or stopping on the part of the drawer is interpreted by the audience as an imperfection and lack of knowledge, and assented to with an ironic smile [Fontinha, 1983].

In order to facilitate the memorisation of their standardised picto- and ideograms, the "akwa kuta sona" — drawing experts — invented an interesting mnemonic device. After cleaning and smoothing the ground, they first set out with their fingertips an *orthogonal net of equidistant points*. The number of rows and columns depends on the motif to be represented. For example, in order to represent the marks left on the ground by a chicken when it is chased, one needs five rows of six points [See Figure 1; Santos, 1961, p. 48]. By applying their method — an example of an early use of a *coordinate system* [cf. Santos, 1960, p. 267] — the "akwa kuta sona" reduce the memorisation of a whole *lusona* to that of mostly two numbers and a *geometric algorithm*.

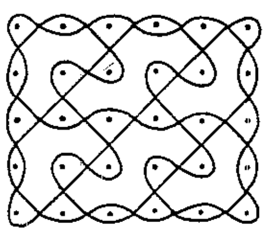


Figure 1

2. Analysis and reconstruction of mathematical elements

The *sona* tradition has been vanishing: "What we find today is probably only the remnant, becoming more and more obsolete, of a once amazingly rich and varied repertoire of symbols" [Kubik, 1987, p. 59]. For some years I studied those *sona* that have been reported in the ethnographical [especially Hamelberger, 1952; Santos, 1961; Fontinha, 1983; Kubik, 1987] and ethnomathematical literature [Zaslavsky, 1973; Ascher, 1988] and, on the basis of a systematic analysis of (implicit) cultural values, I succeeded in reconstructing important mathematical elements in the *sona* tradition. The first results are published in Gerdes [1989c].

2.1 Symmetries and monolinearity

Most *sona* display bilateral, double bilateral or rotational symmetries (see the examples in Figure 2). Many *sona* are *monolinear*, i.e. they are composed of only one closed, smooth line. In the aforementioned study I showed that some of the reported *sona* made out of two or more superimposed closed paths are in fact "degraded" versions of originally monolinear patterns. The probably original *sona* have been reconstructed (cf. my analysis of the Tamil drawing tradition in South India that is *technically* related to the Tchokwe tradition. Gerdes [1989a]). Figure 3 gives an example. This *lusona* represents a lioness with two cubs. The reconstructed original is monolinear (when one does not take into account the tails, which are drawn afterwards).

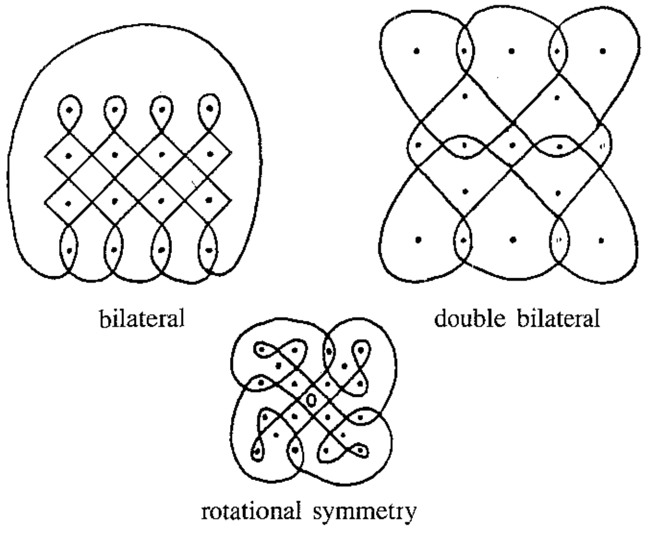


Figure 2

2.2 Classes and geometrical algorithms

The *sona* may be classified in accordance with the kind of geometrical algorithm that has been used to draw them. For example, all *sona* in Figure 4 belong to the same class as the reconstructed *lusona* in Figure 3b. The drawings in Figure 5 belong to the same class as the pictogram in Figure 1. These *sona* may be called *extensions* of the one in Figure 1. By varying the dimensions (but not arbitrarily, as not all dimensions satisfy!) of the point reference frames and by applying the same geometrical algorithm, the "akwa kuta sona" obtained many such extensions. Figure 6 displays the algorithm used in the case of the marks left on the ground by a chicken when it is chased.

2.3 Rules for the construction of monolinear sona

The "akwa kuta sona" knew a whole series of construction rules for monolinear patterns. Figure 7 displays such a rule in the case of the composition of a monolinear pattern out of two partially superimposed *sona* that belong to the class of Figure 4. For the representation of a leopard with five cubs (see Figure 8), this rule has been applied four times. Probably the "akwa kuta sona" who invented this and other construction rules [Gerdes, 1989c] knew *why* they were valid, i.e. they could prove in one way or another the *truth* of the theorems that these rules express.

3. Educational and mathematical potential

Initially I was mostly interested in the reconstruction of the mathematical knowledge that had been present at the invention of the *sona*. In order to guarantee that the drawings of the class of Figure 4 are monolinear, the number of rows and the number of columns of the point reference frame have to be relatively prime. This led me to the formulation of a didactical, geometrical model for the determination of the greatest common divisor of two natural numbers [see Gerdes, 1988a] and of a physical model for the determination of prime numbers [see Gerdes, 1989c].

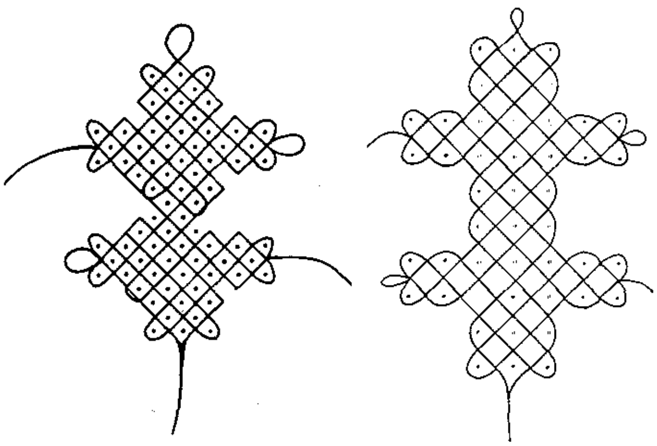


Figure 3

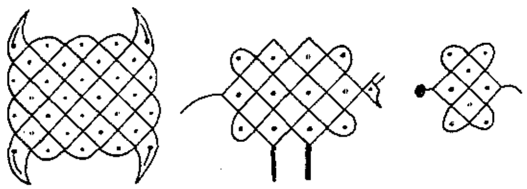


Figure 4

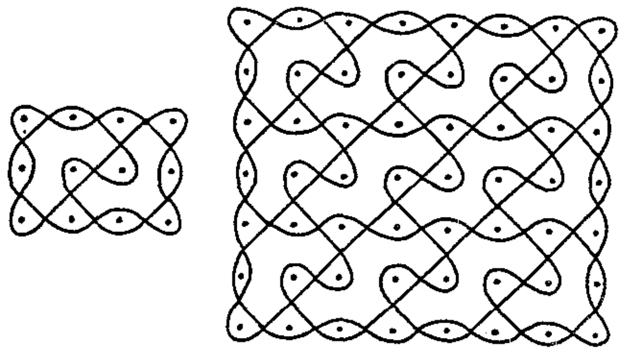


Figure 5

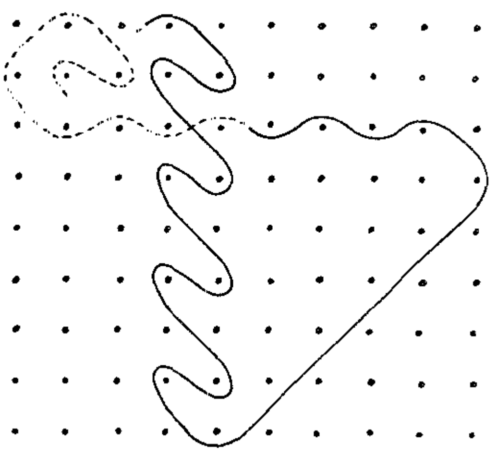


Figure 6

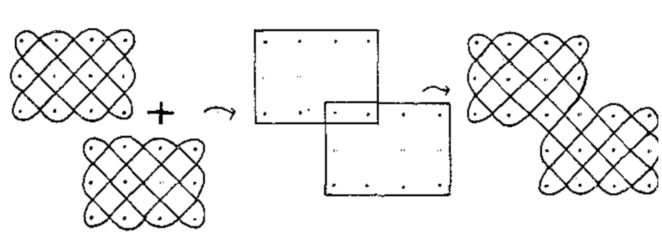


Figure 7

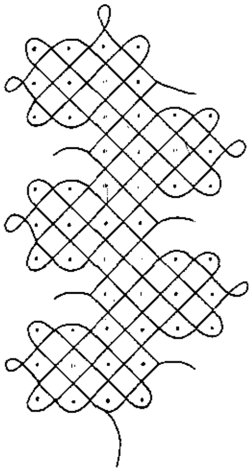


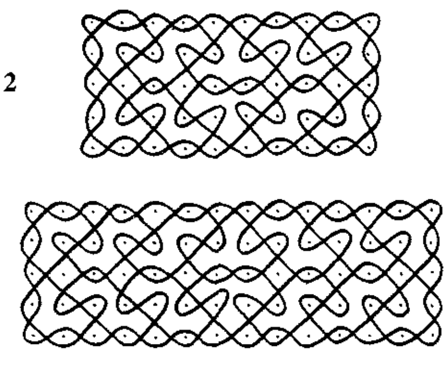
Figure 8

There are many ways to use the Tchokwe *sona* in mathematical education. In my paper, "On possible uses of traditional Angolan sand drawings in the mathematics classroom" [Gerdes, 1988a] examples are given that range from the study of arithmetical relationships, progressions, symmetry and similarity to so-called Euler graphs. As a variant on the wellknown theme of arithmetical problems of the type "Find the missing numbers", a series of geometric problems has been elaborated: "Find the missing figures" [Gerdes, 1988b and c]. Figure 9 gives an example. These problems have the objective to develop a sense for geometric algorithms, generalisation and symmetry. Other didactical uses of *sona* have been suggested in the booklet *Drawings from Africa* [in Portuguese; Gerdes, 1989d] and in the forthcoming book *Lusona: Geometrical recreations from Africa*

1?

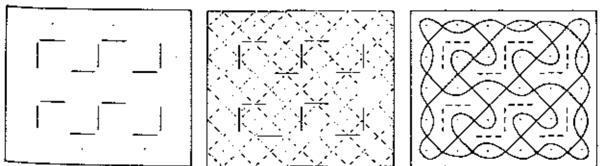
2

3



4?

Figure 9



A a

A b

c

Figure 10

Many reported and reconstructed (sub)classes of Tchokwe ideograms satisfy a common *construction principle*. The involved curves may be generated in the following way. Each of them is the smooth version of the polygonal path described by a light ray emitted from point A (see Figure 10 for an example) The ray is reflected in the sides of the "circumscribed" rectangle of the point reference frame. It encounters on its way through the point reference frame double-sided mirrors which are placed, at regular intervals, horizontally in the middle between two vertical-neighbour frame points and vertically in the middle between two horizontal-neighbour frame points. Once this common construction principle was formulated, it became possible to find a large class of single closed curves that satisfy the same principle. Figure 11 gives examples. The class of curves I found in this way is attractive and interesting for many reasons The curves are esthetically appealing. They may be used for instance in textile design. By filming them, starting the curve at one point, one sees a geometrical algorithm at work. Possibly they may be applied in the codification of information, in the development of laser memory circuits for optical computers, in the study of the topology of large scale integration chips, etc.

The study of the mathematical properties of these curves constitutes a new and attractive research field. A theorem with many consequences is illustrated in Figure 12: If one draws such a design on squared paper and enumerates the squares through which the curve successively passes, modulo 4, then one always obtains a scheme like the one in Figure 13

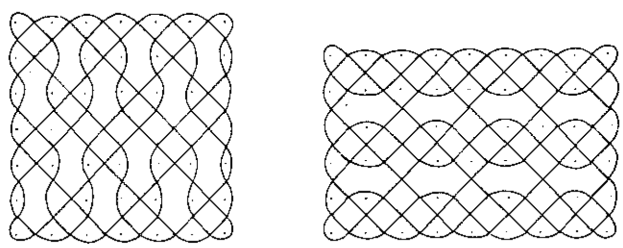
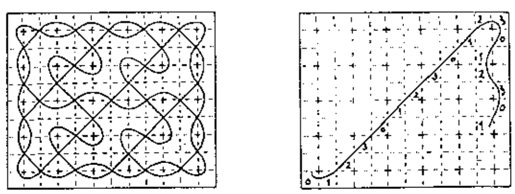


Figure 11



a b

Figure 12

3	2	2	3	3	2	2	3	3	2	2	3
0	1	1	0	0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0	0	1	1	0
3	2	2	3	3	2	2	3	3	2	2	3
3	2	2	3	3	2	2	3	3	2	2	3
0	1	1	0	0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0	0	1	1	0
3	2	2	3	3	2	2	3	3	2	2	3
3	2	2	3	3	2	2	3	3	2	2	3
0	1	1	0	0	1	1	0	0	1	1	0

Figure 13

4. Concluding remarks

The study of the Tchokwe drawing tradition, threatened with extinction during the colonial occupation, is not only interesting for historical reasons The incorporation of this *sona* tradition in the curriculum, both in Africa and in other parts of the world will contribute to the revival and valuing of the old practice of the "akwa kuta sona", will reinforce the apprehension of the value of the artistic and scientific heritage of our African continent, and it may contribute towards the development of a more productive and more creative mathematics education On the other hand, an analysis of Tchokwe *sona* stimulates the development of new mathematical research areas.

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Respecting the Idea of Time we may make several of the same remarks which we made concerning the idea of space, in order to show that it is not borrowed from experience but is a bond of connexion among the impressions of sense, derived from a peculiar activity of the mind, and forming a foundation both of our experience and of our speculative knowledge.

Time is not a notion obtained by experience Experience that is, the impressions of sense and our consciousness of our thoughts, gives us various perceptions; and different successive perceptions considered together exemplify the notion of change But this very connexion of different perceptions — this successiveness — presupposes that the perceptions exist *in time* That things happen either together, or one after the other, is intelligible only by assuming time as the condition under which they are presented to us.

Thus time is a necessary condition in the presentation of all occurrences to our minds. We cannot conceive this condition to be taken away We can conceive time to go on while nothing happens in it; but we cannot conceive anything to happen while time does not go on.

William Whewell