The Questions Remain the Same: Only the Solutions Change

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This article was prompted by a request from the editor of this journal to help him in drawing up a list of mathematics education problems equivalent to Hilbert's famous list of mathematical problems. My immediate response (see FLM Vol. 4, No. 1) was to argue that there are considerable differences between a problem in mathematics and one in mathematics education, and more importantly in what constitutes a "solution" In particular, I argue that a "solution" of a problem in mathematics education was almost of necessity (with the exception perhaps of questions relating to genetical or neurological factors) limited in both (geographical) space and time Solutions are set within specific contexts, relating to social conditions, such as society's attitudes towards education (and mathematics education in particular), to current knowledge and interpretations of mathematics and psychology, and to the state of technological advancement As an afterthought I added that I believed the basic questions of mathematics education remained essentially constant; what differed was the appropriate "solutions" within a constantly changing setting. My offer to exemplify this using references from a recently-published book, *A history of mathematics education in England* [Howson, 1982], was accepted.

Before beginning to look at historical examples it is, I believe, of value to consider how I approached the problem. My first thoughts were to go through my book and see which questions were being asked and answered at varying periods. Soon, however, I began to see the difficulties attached to such an approach. The first is that one tends to define the list of key questions as those which one can identify in history—a procedure which guarantees that one's thesis will be triumphantly vindicated. The second is perhaps subtler but is encountered by every historian (and perhaps less consciously by most teachers) It is well illustrated by something which happened to me on a visit to a primary school some years ago when equivalence relations were much in fashion. I glanced round the class and saw one boy working away by himself in the corner of the room "Oh, he's learning how to partition a set into equivalence classes," said the teacher. I went over to see how this nine-year-old was dealing with equivalence relations and asked him what he was doing: "I'm tidying this drawer for teacher," he said Of course, every time we return cutlery to a divided cutlery drawer, we are "partitioning a set into equivalence classes." It would however, be rather rash to read more into general tidiness than is really there! In the same way it is easy to confuse general actions with considered responses to identified questions. Yet it must be realised that there was not the same incentive for educators in previous times to list problems as there is today (just as I do not know of any mathematician prior to Hilbert actually producing a list of key mathematical problems—although, as Hilbert points out in his Paris address, the problem (and, indeed, certain major problems which he describes) played a major role in the development of mathematics up to 1900). For this reason, in most cases it is a difficult task for the historian to judge to what extent a question had been clearly identified The reader is warned, therefore, of this problem and of the consequent subjective element to be found in this paper.

In order to meet possible objections to the first of the two difficulties described above, I decided that, rather than "infer" a list of questions from historical writings, I would begin with a present-day list, set out by a "neutral" authority, and attempt to find historical antecedents for the questions which it contained. The obvious list to which to turn was that prepared by Hans Freudenthal for the 1980 ICME held at Berkeley [Freudenthal, 1983]. Working in this way led me to two conclusions:

(i) that Freudenthal's list is deficient in particular respects, for example, in his avoidance of social issues (Here, let it be added, this is not intended to be seen as a criticism of Fruedenthal's Berkeley lecture, for he himself drew attentions to gaps. I also know with what misgivings he was persuaded to undertake this monumental (and, I would argue, slightly ridiculous assignment.);

(ii) that although I could perhaps produce "matches" for Freudenthal's key questions, it was in his explanatory asides concerning, for example, possible methodology, that one saw evidence of genuine progress within mathematics education.

I should like to put particular emphasis on this second point Mathematics educators would not in general appear to have either a great knowledge of, or even concern for, the history of their subject. This I believe to be unfortunate for a number of reasons. A real understanding of the position in which we find ourself today presupposes an understanding of how we arrived at this position. Moreover, this lack of a knowledge of the past can lead to a continual "reinvention of the wheel"—there is a need for a shared stock of knowledge which mathematics educators can take for granted and on which they can build. Yet a facile view of history can lead to the argument that "It has all been said before" and to despair. If we wish to gain help and encouragement from history it is essential that we probe more deeply and study in detail the gradual evolution and elaboration of responses to key problems (Thus, for example, there is a considerable difference between our present-day understanding of the difficulties of presenting mathematics *via* a text and that of Robert Recorde in the mid-sixteenth century. Nevertheless, Recorde succeeded in identifying some key issues concerned with the writing of texts which will always face the author—and the reader.) Also, it is in such studies that we shall identify that basic "foundation" knowledge For comparison we note how within the sphere of mathematics proper it is the good expository survey article which in tracing the historical development of a subject effectively defines what has now become the professional's basic knowledge within that particular area Mathematicians appear to have accepted this fact, and are beginning to recognise how highly the ability to write comprehensive and comprehensible survey articles should be ranked. Mathematics educators have yet to come to terms with the idea.

While stressing the need for such surveys (not only of contemporary thought but of how that knowledge has evolved), it must be clearly understood that this is not my aim here. In this paper I wish only to emphasize the historical roots of mathematics education—it is not a post-Sputnik discipline—and to hint at areas which merit deeper investigation. Let us then look at a particular key question and illustrate how opinions have been voiced upon it through the centuries. (Frequently there is a uniformity of utterance which can only perturb us It is a constant reminder that mere exhortation is insufficient; that mathematics education is determined to an enormous extent by external constraints and by the inertia of the institutions in which it is embedded and of its practitioners.)

What can mathematics contribute to a general education?

It is becoming increasingly necessary to stress the fact (if, indeed, it is a fact in which one believes) that schools are institutions for the *education* of children and adolescents, not factories whose output—measured in terms of correct responses to items on proportion, solution of equations, ... can be compared with, and with luck will surpass, those of other countries. What can the study of mathematics contribute to this education?

The questions has been posed on many occasions One answer was supplied by Mulcaster, a London schoolmaster of the 16th century:

> "such studies require concentration, and demand a type of mind that does not seek to make public display until after mature contemplation in solitude. The Mathematical Sciences show themselves in many professions and trades... whereby it is well seen that they are really profitable; they do not make outward show, but our daily life benefits greatly by them ... Mathematics are the first rudiments for young children, and the sure means of direction for all skilled workmen, who without such knowledge can only go by rote, but with it might reach genuine skill ... In the manner of their teaching [the sciences] also plant in the mind of the learner a habit of resisting the influence of bare probabilities, of refusing to believe in light conjectures, of being moved only with infallible demonstrations." [p. 31]
> (The page references refer to Howson [1982] in which full bibliographic details will be found.)

Mulcaster's list is in fact surprisingly comprehensive, particularly if one presents his aims in modern language:

1 the study of mathematics inculcates desirable personal traits,
2. mathematics has utilitarian value, both in life and in the pursuit of other disciplines,
3 a knowledge of mathematics can free one from dependence on remembered procedures and on other persons

That Mulcaster never appears to have included mathematics in the curriculum of the two schools of which he was head, is yet another reminder of the great divide which so often exists between theory and intention on the one hand, and practice on the other But who is to know the opposition which Mulcaster may have encountered or the constraints which he felt?

In succeeding centuries new claims were made for mathematics teaching. For one brief period, following Newton's revelation of a "system" of celestial mechanics which "could only proceed from the counsel and dominion of an intelligent and powerful Being ..[the] Lord over all" [p. 49], it was argued that the study of mathematics would lead students to God Thus, in the words of Isaac Watts, the eighteenth century religious leader:

> "If we pursue mathematical Speculations, they will inure us to attend closely to any Subject, to seek and gain clear Ideas, to distinguish Truth from Falsehood, to judge justly, and to argue strongly; and these Studies do more directly furnish us with all the various Rules of those useful Arts of Life, viz , Measuring, Building, Sailing, etc.
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> Even our very Enquiries and Disputations about .. Space and Atoms, about incommensurable Quantities ...which seem to be purely speculative, will shew us some good practical Lessons, will lead us to see the Weakness of our Nature, and should teach us Humility in arguing upon divine Subjects and Matters of sacred Revelation...
>
> .. there are many and great and sacred *Advantages* to be derived from this Sort of *Enlargement of the Mind*.
>
> It will lead us into more exalted Apprehensions of the *great God our Creator* than ever we had before." [p 50]

Alas, intentions are not always realised and Watts was later to change his mind:

> "To own a great but grievous truth, [mathematical studies] though they may quicken and sharpen the invention, strengthen and extend the imagination, improve and refine the reasoning faculty, and are of use both in the necessary and the luxurious refine-

ment of mechanical arts; yet having no tendency to rectify the will, to sweeten the temper, or mend the heart, they often leave a stiffness, a positiveness and sufficiency on weak minds, which is much more pernicious to society, and to the interests of the great end of our being, than all their advantages can recompense ... They are apt to beget a secret and refined pride, an over-weening and over-bearing vanity, the most opposite temper to the true spirit of the gospel. This tempts [mathematicians] to presume a kind of omniscience in respect to their fellow-creatures who have not risen to their elevation: nor are [mathematical studies] fit to be trusted in the hands of any but those who have acquired a humble heart, a lowly spirit, and a sober, and teachable temper ... " [pp. 51-52]

Would you encourage your daughter to marry a mathematician?

Yet it would be idle to shrug aside these objections. Does mathematics breed arrogance? Of course, it is easy to provide examples and counter-examples. What are more important questions are whether the usual methods used in teaching mathematics—emphasizing competition between individuals, and the continual ranking of the good, the average and the bad—are both socially injurious and unnecessary; whether society's use of mathematics as a filtering device (note, for example, how entry to the French élite in the early nineteenth century was based solely on a mathematics examination [see e.g. Dhombres, 1982], and how that influence has persisted in that country [see e.g. Revuz, 1978]) is not totally inconsonant with the educators' aims for teaching mathematics.

This last sentence serves to focus attention on two further issues:

(i) there are no universally held aims for teaching mathematics—or, to put it in a slightly different form, the weights which different sections of society, government, industry, parents, mathematicians, educators, teachers, ..., will assign to the various aims in Mulcaster's lists or any extended, modern equivalent, will vary greatly;

(ii) we have spoken so far only of the aims for *teaching* mathematics; there is no reason to suppose (although the supposition is almost always made) that these will translate automatically into aims for *learning* mathematics.

Both of these facts have enormous repercussions on mathematics education. As societies have developed the tensions induced by (i) have grown to such an extent that educators are now in danger of becoming completely demoralised by attempting to reconcile the irreconcilable. The curriculum developer cannot satisfy so many masters. The educational system, the school, or the individual teacher must evolve a *modus operandi* which best serves to satisfy contemporary pressures. This must be done in the knowledge that, as times change, pressures from certain directions will grow and those from others will temporarily diminish. The school is a control mechanism of society subject to many feedbacks. The lesson from history would appear to be that the nearer to the classroom that adjustments to the curriculum are made, the better. The teachers should be capable of carrying out finer tuning procedures than can the state.

As if to reassure us that everything in education has been said or done before, it must be remarked that in the past not a great deal of attention has been paid to the reasons which people might have for learning mathematics, and what place students see for it within their "general education" or their process of "growing up."

Robert Recorde, writing in 1551, clearly distinguished between two types of reader who might use his geometry text. There were those "who study principally for learning" and those who wished to acquire the knowledge, for some purpose or other, but who had "no time to travail [work] for exacter knowledge" [p. 19]. In a sense Recorde roughly distinguishes between those who, to use Mellin-Olsen's [1981] terms, wish to learn because they attach personal *significance* to what is being presented to them, and those who see such knowledge as merely *instrumental* in attaining other, possibly non-mathematical, goals. Recorde, however, was not writing primarily for the school pupil, but for the maturer student. It is highly unlikely that Shakespeare would have chanced upon his works at the school he attended in Stratford-upon-Avon. Yet he, too, had something to say on motivation—or rather the lack of it—when he described "the whining school-boy, with his satchel and shining morning face, creeping like a snail unwillingly to school."

For several centuries motivation was provided within the schools primarily by the use of the birch and rod. There was also the power of expulsion—Arnold, the famed nineteenth-century headmaster of Rugby School explained that "Till a man learns that the first, second and third duty of a schoolmaster is to get rid of unpromising subjects [i.e. pupils], a great public school will never be what it might be and what it ought to be" [Ballard, 1969, p. 29]. Today's teachers must smile ruefully at such advice—the "unpromising" cannot be removed from state schools quite so easily. The birch, which served Arnold well but which was never very successful in meeting its metaphorical ends—for often riots erupted in the schools and on occasions had to be quelled by the army—has in the past century been displaced as the prime motivator by the examination. Vast and complicated examination systems have been established, educational ladders have been erected, and meritocracies founded on systems where success has frequently been dependent on passing examinations, usually with a mathematical component. In expanding systems, with respect to both educational opportunities and subsequent occupational rewards, the examination has proved a very powerful motivator indeed. Now, however, that period of historical growth appears to be coming to an end—at least, so far as the West is concerned. A motivational vacuum is developing for many students within those educational systems which offer universal secondary education. For the first time it is becoming imperative to distinguish carefully between two questions. The first, asked by society, is *What can teaching mathematics contribute to a person's general education?*; the second, asked by the student, is

What can learning mathematics contribute to my personal growth?

In this paper I have hinted at how one particular question referring to mathematics has been stated and answered at various times and of how the way in which the question as traditionally posed has had to be refined. Clearly, very much more could be said on this matter. On the other hand there is not space for me to deal with other vital questions even in so cursory a manner. However, for the interested reader, I list below a few such key questions together with sample references drawn from my book which will indicate how such questions have been identified and answered in the past.

Granted there is a constantly changing body of mathematics on which to draw, how do we make the most fitting selection for particular types of student?
(Recorde, 19; De Morgan, 88-9; Godfrey, 158-9)

How should one best order the learning of mathematics and what is the interplay between technique and understanding?
(Recorde, 21; Godfrey, 158; Williams, 196; Ballard, 207. On the inadequacy of rote-learning see Recorde, 20; Newton, 38)

How does one give "meaning" to mathematics?
(Recorde, 22-23; Tate, 110-11; Godfrey, 150)

How shall we arrive at a science of education—are there no bad pupils, only bad teachers?
(Priestley, 57; Tate, 115-6)

How can one change traditional atttitudes and obtain acceptance for new ventures and initiatives?
(Pepys, 36; 201, 208)

What procedures and processes can be adopted to facilitate curriculum development?
(Wilson, 133ff; Godfrey, 154-5)

How does one demonstrate the indivisibility of mathematics—or is it divisible?
(Godfrey, 156)

How does one counter the bad effects of examinations?
(Wilson, 127; Godfrey, 152, 158; Williams, 187)

How does one cater for children of different attainment (ability, motivation)?
(Newton, 38; Williams, 197)

How does one help students to learn to learn?
(De Morgan, 92)

How does one make best use of the technological and other facilities available?
(Recorde, 19-20; Gregory, 42-3; Williams, 183, 198, 209)

What makes a good teacher and what are the best ways of training teachers?
(Pepys, 36; Tate, 105-13, 116-7; Williams, 179-181, 190-193, 200-202)

How does one involve the adult population in mathematical activity?
(Hutton, 70; De Morgan, 86; Tate, 98, 209)

The list of questions is by no means exhaustive and the examples I have quoted are restricted by being drawn from one particular country—certainly, the histories of mathematics education in other nations would reveal many similar examples. Nevertheless, I hope that this paper has served to indicate the rich heritage which we mathematics educators possess and the fact that it is essential that we are aware of that history if we are to view mathematics education in its true light—as a constantly evolving discipline which is continuously being called upon to react to changes in the society it serves and the mathematics it propagates.

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