Visual Imagery and School Mathematics

KEN CLEMENTS

The conclusion of the article whose first part was printed in the previous issue.

5. Attempts to improve imagery skills

If we have not yet provided an adequate definition of imagery, and do not know how to measure imagery ability, then there would appear to be some virtue in confining attempts to improve imagery skills to fairly narrow contexts. However, if this is done there is a likelihood that transfer of new skills to apparently related tasks in other contexts will not occur. Be that as it may, there have been many attempts to improve imagery skills in specific areas and we shall mention just a few of these.

Atkinson [1975] used imagery mediation in mnemonic techniques to assist learning of foreign language vocabulary, and Ackerman [1975] developed an imagery technique for teaching sight-sound associations in reading. In the field of science education, Gropper [1966] found that a visual mode of presenting hydrostatics enabled learners to transfer acquired skills to rule application problems that were verbally presented, and that a verbal mode of presentation was less successful in promoting transfer to test items that were visually presented. Markle [note 5] presented a seven part programmed instruction sequence on crystallography to junior high students, with one treatment almost entirely verbal and the other diagrammatic (with words being used only for naming crystals and other important terms); it was found that subjects in the two treatments did equally well on the *Punched holes test,* and the *Cubes test.* Salomon's [1972, 1974] experiments, in which viewing of a movie film greatly assisted students' performances on the *Surface development test* suggested that audio-visual presentations can facilitate desirable use of imagery; and Richardson [1967] demonstrated that mental rehearsals and mental practice of athletic skills can lead to improved performance

While only a small number of the many studies in which attempts have been made to improve imagery skills have been reviewed here (additional studies in the field of mathematics education will be reviewed in a later section), we are a long way from knowing highly efficient teaching strategies which will achieve the desired end. The theory advanced by Gagné and White [1978], that the effectiveness of imagery-evoking stimuli, such as pictures, diagrams and demonstrations involving real objects, is likely to be improved when careful attention is paid to the descriptive words that link them to other memory structures, would appear to be especially worthy of investigation Gagné and White recommend that a direct effort to ensure the formation of images, and the linking of these to propositions and skills, be made to teachers. They comment that just as learners are required to paraphrase sentences to ensure that they have processed them as propositions, the teacher might require students to "paraphrase" diagrams That is, they could be asked to draw their own diagrams from memory, or possibly to render a given diagram from a different viewing angle. Kent and Hedger [1980] have been doing this sort of thing in school mathematics classrooms, apparently with success.

It is interesting to consult Soviet literature pertaining to the development of imagery ability. As Kilpatrick and Wirszup [1978] have stated, Soviet psychologists "assert that instruction broadens the potential of development, may accelerate it, and may exercise influence not only upon the sequence of stages of development of the child's thought but even upon the character of the stages" Thus, the Soviets do not accept the Genevan idea of a "natural" course of spatial development, and seek to study the development of spatial concepts and imagery under the influence of school instruction Chetverukhin [1978] who studied the development of imagery in children from grades 1 to 10, and first-year college students, has argued for example that difficulties in drawing stem from the overuse, in textbooks and instruction, of standard figures in standard positions, and from a tendency to use calculations rather than graphic constructions to solve problems. Vladimirski [1978] studied the effectiveness of a set of exercises for developing imagery. He contended that geometry teachers do not use visual aids effectively because they lack a properly sequenced system of appropriate exercises. In one experiment he studied the role of the diagram in developing imagery, and concluded that a diagram does *not* always aid reasoning and problem solving, that pupils have difficulty identifying concepts embedded in non-standard figures, and that knowledge of the rules for representing three-dimensional figures is a prerequisite for instruction in solid geometry. Yakimanskaya [1978] emphasised the role of imagery in geometry, drawing, geography, handcraft and manual labour, and concluded that there is an urgent need for more special exercises for developing imagery in elementary school children to be worked out, and that the task is one for teachers, psychologists and methodologists [Yakimanskaya, 1978, p 167].

Clearly, the Soviets have taken the question of how to develop imagery ability in children far more seriously than Western educators and psychologists. Unfortunately, we do not have readily available data which indicate whether their efforts have borne fruit. The assumption implicit in most of their work in this area is that although imagery ability can be developed by instruction, teachers and methodologists need to take care that children learn to use imagery in appropriate circumstances. As Twyman [1972], an English psychologist, has commented, the creation of an image can introduce difficulties associated with decoding the image. For example, images might possess irrelevant details which distract problem solvers from the main elements in the original problem stimulus, and make it more difficult for them to formulate necessary abstractions [see also McKellar, 1968; Hollenberg, 1970; Hasher *et al*, 1976; Corbett, 1977].

6. The verbalizer-visualizer hypothesis

Hadamard [1945], Poincaré [1963], Walter [1963], Menchinskaya [1969], Krutetskii [1976] and Richardson [1977b] are among those who have contended that individuals can be classified into three groups with respect to a visual-verbal dimension, and we shall summarize this conjecture by the expression "the verbalizer-visualizer hypothesis". The first group, consisting of "visualizers", contains individuals who habitually employ imagery or pictorial notations when attempting to solve problems; the second group, the "verbalizers", contains those who tend to use verbal codes rather than visual images or pictorial notations; the third group, the "mixers", consists of individuals who do not have a tendency one way or the other and, indeed, might like to use both methods when solving a problem. According to Walter [1963] most people belong to the last group, and in this regard it is interesting that Sternberg [1980] recently found that three-quarters of the subjects in a well-controlled experiment requiring linear syllogistic reasoning preferred to use a mixed linguistic-spatial strategy rather than a linguistic or a spatial strategy.

Sternberg's use of flowcharts for representing strategies which might be adopted under linguistic, spatial and mixed linguistic-spatial models for linear syllogistic reasoning provides some indication of the likely directions of future research. In the past no adequate instruments for classifying people reliably into groups have been available, and this has meant that individual researchers have not agreed on the processing modes individuals have used when attempting well defined tasks. Thus, A.R. Jensen [1971] demonstrated that although for over a decade many educational psychologists had been conducting research which was based on the assumption that "auditory" and "visual" learners could be identified, there was no ambiguous evidence that this was, in fact, the case. Subsequent research has failed to provide such evidence [see DeBoth and Dominowski, 1978].

Despite doubts concerning the verbalizer-visualizer hypothesis, Moses [1977 and note 6], Lean and Clements [1981, in press], and Suwarsono [in progress] have not only assumed the existence of a verbal-visual processing continuum but have also attempted to show how an individual's position on this continuum can be located. With respect to the Lean and Clements study, which involved 116 engineering students in Papua New Guinea, it is worth noting that the student who gained the highest scores on the spatial ability tests which were used showed no inclination at all to use visual imagery when solving mathematical problems.

The results of the studies by Moses, Lean and Clements, and Suwarsono are sufficiently promising to warrant further investigations of the verbalizer-visualizer hypothesis. The notion of a verbal-visual continuum, in particular, deserves attention.

7. Relationships between visual imagery and spatial ability

Although there is no widely accepted definition of the term "spatial ability", definitions of spatial factors (e.g. spatial orientation and spatial visualization) almost invariably make use of the notion of imagery. Indeed, one writer [Cook, note 7] has argued that mental imagery itself "is a type of spatial ability"; Cook described mental imagery "as the formation and retention of an image that involves no mental movement of the image once formed". Interestingly, Cook also stated that mental imagery is important in relation to both spatial orientation and spatial visualization.

Despite the fact that among psychologists there is almost universal agreement that visual imagery is necessarily involved in spatial tasks, the literatures on visual imagery and spatial ability have developed almost independently of each other [see Clements, note 1]. Shepard is the only writer who has made substantial contributions to both literatures. Undoubtedly there is a need for greater co-operation between workers in the two fields.

A recent paper by Liben [1981] has potential for unifying concepts involved in the visual imagery and spatial ability literatures. Liben distinguished three types of spatial *representations* — namely spatial products, spatial thought and spatial storage — and two *contents* of spatial representation, specific and abstract. She provided the following descriptions of these terms:

Spatial products refer to the external products that represent space in some way. Any kind of external representation of space, regardless of medium, is a spatial product (e.g. sketch maps, miniature models, verbal descriptions).

Spatial thought refers to thinking that concerns or makes use of space in some way; it is knowledge that individuals have access to, can reflect upon, or can manipulate (as in the manipulation of imagery in problem-solving). Clearly, spatial thought is needed to answer questions which appear on standard tests of spatial ability.

Interestingly, Liben [1981, p.12] specifically includes consciously evoked mental imagery within the ambit of spatial thought.

Spatial storage refers to any information about space which, although contained "in the head", is such that the individual is not cognizant of it. Once the individual becomes cognizant of it, or reflects upon it, it becomes spatial thought.

Although Liben does not specify how spatial storage information is, in fact, stored, she suggests that it might be in the form of mental pictures or truth propositions. Such information is implicit or tacit knowledge which guides an individual's spatial behaviour without the individual being aware of it. Liben illustrates her notion of spatial storage by referring to animals who move efficiently through real, complex environments even though there is no evidence they can formulate cognitive maps of these environments. Another analogy is with ordinary speech: why is it that individuals can say complex, grammatically correct, sentences without having planned exactly what they intend to say? Psycholinguists explain this phenomenon by referring to deep language structures within each language user, and users are not necessarily aware of these structures. In a similar way, spatial storage information influences an individual's behaviour without the individual being conscious of it.

Newcombe [1981], in discussing Liben's spatial storage concept, identified three controversial aspects of such information. First, there is the question of whether the representation of the information in long term memory is in an analogue or propositional format. Interestingly, Newcombe specifically referred to the visual imagery literature on this matter, and stated that Anderson [1978] has suggested that the question is unanswerable. This comment makes it clear that Newcombe has identified spatial storage information with visual imagery. With this identification in mind, the other two controversial aspects are well covered in the imagery literature. The second aspect is whether spatial storage information is consciously available; both Liben and Newcombe point out that it is difficult to determine if this is the case through empirical means. The third aspect is how an outsider might decide what information a person holds in spatial storage. Newcombe [1981, p.376] asks, in relation to this issue, "in what sense can we claim that one behaviour is a better guide to an unobservable mental entity than another?" This third aspect raises the question of externalization of imagery, a matter which was considered earlier in the present paper.

So far as the *contents* of spatial representation is concerned, Liben's [1981, p. 15] concept of a *specific* space corresponds to an individual's knowledge of, and ability to manoeuvre within, a particular environment, while that of *abstract* space corresponds to spatial abstractions which an individual may have developed (e.g. knowing that the left-to-right aspects of a configuration are "spatially reversed" for a viewer "on the other side" of the configuration).

Liben provides many more details of her proposed system of spatial representation than can be given here. For the present writer, the system has the potential to bring about a much needed unification of the imagery and spatial ability literatures. It would appear to be sufficiently well grounded in theory to serve as a foundation for serious moves in this direction. If Lohman [note 2, p. 188] can conclude, after his comprehensive review of the spatial ability literature, that "spatial ability may be defined as the ability to generate, retain, and manipulate abstract spatial images" then, surely, the suggested unification can be achieved.

8. Research into the role of visual image in mathematical learning

Gagné [1960] hypothesized that high verbal ability students would learn best from verbal presentations of information while high spatial ability students would learn best from more visual presentations. It would be wrong to suggest that mathematics educators have been greatly influenced by this hypothesis, but there have been a number of investigations during the 1960s and 1970s aimed at confirming or infirming it (or some variant of it) for certain topics and certain children. Typically, such investigations involved the same mathematical topic being presented in two modes, verbal and visual, to high and low performers on standard language and spatial tests, and the main interest in the results has been in whether significant aptitude-treatment interactions (ATI) occurred. The results of the studies have been difficult to interpret, but it is fair to say that, generally speaking, they have only rarely supported Gagné's hypothesis.

While support for the hypothesis was found in the study by Hancock [1975], the nature of the interactions in the sequence of studies begun by Carry [1968] and continued by Eastman [1972] and Salhab [1973], remained unclear [DuRapau and Carry, 1981]. The same is true of ATI studies by Behr [1970], and Peterson and Hancock [note 8]. According to Koran [1974] the failure of many such studies to find interactions could be traced to "constraints imposed by the nature of the subject matter and achievement measures used". Sometimes verbal subject matter cannot be easily translated into a visual message, and vice-versa; also critical aspects of a topic which lends itself to a visual treatment may be miscommunicated if a verbal version of that treatment is attempted. Despite difficulties of this kind, more recent ATI studies involving mathematics classes have produced especially interesting results [see McLeod, 1978; McLeod and Briggs, 1980; Threadgill-Sowder and Juilfs, 1980; DuRapau and Carry, 1981].

With respect to the present paper the results of the Threadgill-Sowder and Juilfs [1980] study are noteworthy. Results of the interaction analyses showed that achievement by grade 7 students in mathematics significantly interacted with an imagery-inducing manipulative treatment and a more abstract symbolic treatment. Students with low pre-treatment scores on mathematics tests received higher scores on the achievement posttest when instruction included manipulative materials, whereas students with high pre-treatment mathematics test scores found the symbolic treatment more beneficial. According to the researchers, these results exemplified Salomon's [1971] ATI compensatory model, by which high aptitude students "experience interference when given treatments which provide them with mediators they can provide on their own", whereas low-aptitude students "benefit when mediators they are lacking are provided overtly". Horwitz [note 9], in a study involving college students, obtained similar results; she concluded that "the visualizability of a problem affects its solvability by low performance subjects but not by high performance subjects".

Each of Moses [1977, note 6], Webb [1979], and Lean and Clements [1981, in press] investigated whether schoolchildren who prefer to use visual methods tend to outperform children who prefer more analytic methods on mathematical tasks. Webb and Moses reported in the affirmative, but Lean and Clements found that those who preferred more verbal, analytic methods did better. This apparent conflict could have been due to differences in the criterion mathematics tasks — both Webb and Moses used unfamiliar, and mainly difficult tasks in their studies, but Lean and Clements used mainly familiar tasks which were not unlike questions which the subjects had seen worked in class

In Marriott's [1978] study two comparable grade 6 classes were taught ideas about fractions Children in one of the classes made their own sets of circular cut-outs, and they then used these to embody fractional concepts throughout the time they studied fractions; the other class received a thorough, but traditional algorithmic treatment. While there was no statistically significant difference in the means of the pre-posttest gains of the two classes, a more qualitative analysis showed that the children who had used the cut-outs were much more likely to accompany their setting out with diagrams (almost always circular); furthermore, they tended to think about fraction questions by creating visual images which involved circles and sectors of circles, whereas the traditional group tended to think solely in terms of numerical algorithms. Not surprisingly, an analysis of errors made by children in the two groups revealed that the character of the errors varied considerably — in the "cut-outs" group a typical error might be $3/4 + 1/8 = 9/10$, whereas in the traditional group it might be

$$3/4 + 1/8 = \frac{3 \times 4 + 1 \times 8}{4 \times 8} = \frac{20}{32}.$$

In the former case, the answer $9/10$ would most likely be accompanied by a circular diagram

In the Marriott study the treatments clearly substantially affected the thinking of children about the topic under consideration This did not occur nearly to the same extent in a training study by Fary [1980], who found that when the topic of "operations on integers" was taught to comparable groups by three different methods, two of which emphasized the need for diagrams and visual thinking and the other more traditional algorithmic procedures, children in the visual groups tended to want to know algorithms for solving the problems — and if the teacher did not provide them with the algorithms they made up their own. When the time came for the criterion posttest, children in each group tended to use algorithmic procedures

From the preceding discussion, it is obvious that, considering the large expenditure of time and money on the research efforts which have been described, we know precious little about how and when use of imagery is likely to facilitate mathematical learning. This state of affairs should not be regarded as a signal for a lessening of research activity aimed at increasing our understanding of the role of visual imagery in mathematical learning One only has to read the article by Vinner and Hershkowitz [1980], on concept images in geometry, to be convinced that the different images which one associates with certain mathematical tasks can substantially affect both performances on, and understanding of, those tasks. It would be interesting to learn of the progress, or otherwise, of Soviet mathematics educators and psychologists, in their quest to understand the significance of visual imagery in mathematics learning They have been working persistently on the relevant questions for a much longer period of time than their Western counterparts and are probably much closer to reaching solutions.

9. Conclusions

The following conclusions would appear to be warranted from the reviews of the literatures which have been given.

1. Many highly original and significant creations of the human mind have been largely the result of nonverbal mental representations (mainly visual imagery).
2. Despite the controversy over the best way to describe visual imagery (various picture-in-the-mind versus propositional theories), there appears to be no strong reason at the present time for mathematics educators to discard traditional, but simple, picture-in-the-mind notions of imagery
3. Despite recent criticisms of research based on data gained from verbal reporting, introspections and especially retrospections can provide important and reliable data Mathematics educators need to develop better instruments for assessing the role of visual imagery in mathematical learning
4. At present the psychological dimensions of imagery have not been established, and this is hindering our understanding of imagery and its role in education
5. Although psychologists have talked about training to improve imagery ever since Galton [1883], there is little evidence of progress at either the theoretical or pedagogical level
6. Some recently developed instruments which enable individuals to be placed on a verbalizer-visualizer continuum would appear to hold much promise for research in mathematics education.
7. Although the literatures on imagery and spatial ability have, surprisingly, developed independently of each other, the recent model by Liben [1981] would appear to have potential for unifying imagery and spatial ability concepts.
8. Although previous research into the role of visual thinking in mathematics learning has not provided clear guidelines for classroom practice, there should not be a reduction in the amount of research which is aimed at achieving this end
9. Traditionally, psychologists and educators in the Soviet Union have been more inclined to investigate the role of imagery in mathematical learning than their Western counterparts.

Notes

5 Markle, N H *Increasing long term retention of knowledge: effects on learning and retention for students of different ability levels* U S O E final report, Project No. 7-48-7670-204, Palo Alto, California: American Institute for Research, 1968

6 Moses, B E *The relationship between visual thinking tasks and problem-solving performance* Paper presented at the annual meeting of the American Educational Research Association, Boston, 1980

7 Cook, N *Spatial ability: its impact on mathematics performance* Paper presented at the annual meeting of the American Educational Research Association, Los Angeles, April 1981

8 Peterson, J C , & Hancock, R R *Developing mathematical materials for aptitude-treatment interactions*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, April 1974

9 Horwitz, L *Visualization and arithmetic problem solving* Paper presented at the annual meeting of the American Educational Research Association, Los Angeles, April 1981

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