

TWO TECHNOLOGY-ENTHUSIASTIC TEACHERS AND TWO VERY DIFFERENT LESSONS

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Valeria and Emilia are secondary-school teachers with more than 30 years of teaching experience. Both were enthusiastic about joining our project FlipMath (Andrà, Brunetto & Kontorovich, 2017), which explored the affordances of integrating instructional video clips in school mathematics instruction. The clips were developed as part of the Pre-Calculus Massive Open Online Course (MOOC)—the first Italian MOOC by the Politecnico di Milano [1] in 2014. A typical video clip lasts for two-three minutes, and shows a Presenter who explains the concepts that are central to a mathematical topic, reviews their key properties, and demonstrates characteristic procedures. Our interest in mathematical video clips draws on Niess and Walker (2010), who argue that watching appropriate videos can develop students' visualization skills, when analyzing the information that clips present provides opportunities for reasoning and communication. Recent pandemic events raise a new wave of interest in transformational spaces that digital technology can open in mathematics education, and specifically with respect to teachers 'borrowing' already existing online resources and using them in their teaching.

In our project, Valeria and Emilia decided to bring video clips to classes where they usually conduct traditional teacher-led lessons. It has been acknowledged that digital technology has the *potential* to transform traditional teaching to be more participatory, but not all teachers materialize this potential (see, e.g., Clark-Wilson, Robutti & Sinclair, 2014). Thus, we consider how differences in the changes in technology-infused teaching can be explained.

We acknowledge that the video clips constitute recordings of traditional teacher-led instruction. However, this does not mean that the clips cannot be used in a participatory manner, as a springboard for teachers to transform their practices. For instance, FlipMath designers envisioned a 'flipped' scenario, where the clips are assigned to be watched at home, and the classroom time is spent on interactive activities and discussions that build on the clip content. This is what happened in Emilia's classroom more or less. In contrast, Valeria turned the clip into the main focus of her regular teaching practice, while leaving minimal space for students' input. The goal of our research is to understand how such different lessons came about.

With Herbst and Chazan (2012), we consider "teaching as an activity system involving positions, roles and relationships, where individual choice is possible but not cost-

free" (p. 601). Accordingly, we argue that technology-infused transformations, or the lack thereof, can be explained with a mixture of individual orientations of the teacher and social influences, which we associate with students' reactions to a new technological resource (a video clip, in our case). Herbst and Chazan's view reconciles an individualistic approach to teaching that focuses on "teachers' personal beliefs or their knowledge of various kinds" (p. 601), and an approach that focuses on teaching as a cultural and socially-embedded activity. The power of practice (and students' reactions) to transform teaching has been conceptualized by many scholars (see e.g., Guskey, 1986; Desimone, 2009). Our approach focuses not on classroom practice (which plays a role of utmost importance, however), but on the space that a teacher is willing and able to make for a transformation to take place. Our data consist of interviews with teachers, lesson plans, classroom observations and comments on the given lessons. Specifically, we are curious about the common mismatch between the intentions as they emerge in the interviews and the lesson plans, and the actions that teachers take in the lessons. Let us start by taking a glimpse at Valeria's and Emilia's practices.

A short description of the two cases

Valeria teaches in a technologically well resourced school. When asked for details about her typical teaching practice, Valeria describes her usage of various technological devices (e.g., personal laptop for every student, a smart board) and online environments (e.g., GeoGebra, Wikispaces) as means to maximize the efficiency of her lessons (e.g., to "save time" and "better monitor students' progress", in Valeria's words). As part of our project, Valeria decided to introduce a video clip on power functions to a Grade 11 class that she described as "very slow", "needing a lot of guidance", and "not so open to innovations [...] that are different from a traditional lesson". As Valeria described, the FlipMath project provided an opportunity to promote her students' "readiness and autonomy" with respect to accessing and exploiting online mathematical video clips for exercise and understanding. In the clip that Valeria chose, the Presenter defines power functions, provides some examples of formulas and graphs of such functions, and recaps their main properties. Valeria planned to play a video clip twice on the classroom screen. After the first time, she planned to give students an individual task, asking them to

list familiar and new mathematical concepts. Then, Valeria planned to re-play selected segments from the clip and deliberate on its key concepts. However, in the lesson, she made several decisions that diverted from this plan: she displayed the video clips five times and she turned the initially envisaged individual task into a whole-class activity where she played a leading role.

Emilia teaches in a vocational school. She decided to integrate a video clip about mean and variance in a Grade 12 class. She described the class as “difficult”, because “one student [...] plays the role of a class leader, and she is keen to shine at any occasion. She is hostile to any one of her classmates who tries to contribute to the lesson”. Emilia explained that her lessons are mostly frontal with occasional interventions from that specific student. Emilia stressed that a change in this class dynamic was necessary. She planned to assign the video clip as homework, asking students to note unfamiliar concepts. Notably, this class usually did not submit homework and Emilia was skeptical that the students would do it this time. The clip that Emilia assigned used a formula for the variance that was different from the one she introduced in her previous lessons. Emilia highlighted that she wanted to mobilize a situation where two mathematical authorities, herself and the Presenter in the clip, propagate seemingly non-equivalent approaches to the same concept. To Emilia’s surprise, the entire class submitted their homework and she opened the lesson by expressing her appreciation of students’ efforts. Then, one student shared that he was confused by the variance formula and asked “which one [of the two formulae] is the most correct?” Another student exclaimed, “Yes! I got stuck on exactly the same thing! Which one is correct?” This was followed by another student suggesting, “I am convinced that they [the two formulae] are the same. There should be a way to prove it”. Then, Emilia proposed an unplanned example, to compute the variance of a small set of data, in order to address students’ comments. After the lesson, Emilia said that a spontaneous interaction where students related and built on each other’s comments “had never happened before!”

For different reasons, both Valeria and Emilia adhered to frontal teaching in their respective classes. Both Valeria and Emilia planned to assign a video clip for students to watch and then engage them in clip-related activities. Both were skeptical about their classes’ cooperation with their plans: Valeria’s students were impervious to novelties and Emilia’s were averse to homework. Nevertheless, Emilia’s students surprised her, and her clip-infused lesson turned out to be far more participatory than she envisaged. While Valeria planned her lesson as a first step in making her students “ready” and “autonomous”, she provided limited space for their activity at first. We ask, can the differences in the changes in technology-infused teaching between the two teachers be explained by the existing theoretical models in mathematics education? Or do we need to add more elements to characterize these differences? Speaking more generally, what makes it possible for a teacher to move, even within a single lesson, towards more participatory instruction?

Possible answers from the literature

Herbst and Chazan (2015) argue that in mathematics teaching “some precise technical knowledge is involved but is not sufficient; competence relies crucially on a relational know-how” (p. 272). This perspective invites us to challenge a common narrative, suggesting that realizing the potential of digital technology to transform teaching utterly depends on the teachers who embed, use, and maintain the use of digital technology in their classrooms. This narrative is consistent with research paying increased attention to teacher knowledge and affect that shape teachers’ relations with technology. Valeria and Emilia’s cases are meant to serve as illustrations for approaching the role of knowledge and beliefs in shaping a teacher’s change in practice. To recall, both teachers are experts with technology in mathematics teaching and they hold positive beliefs with respect to technology in general and instructional videos in particular. Hence, a focus on knowledge and beliefs may be not sufficient to understand the differences in Valeria’s and Emilia’s cases, with respect to the transformations that took place in their classes. It is important to consider also other aspects related to a teacher’s classroom practices, modes of engagement with students, and interacting with different resources.

We also suggest that a transformation can happen (or not) in relation to *any* new resource. Our focus is not on a specific resource, but is on a teacher opening the door to a transformation (or not), something that usually happens before the resource is actually used for the first time. The metaphor of a transformational space can help capture these ideas.

Transformational spaces

Herbst (2003) posits that teaching “involves interacting with students, partnering in activities and discussions wherein mathematical ideas may emerge and be developed [but also] involves designing and foreseeing those activities and discussions on behalf of mathematical ideas the students need to know” (p. 197). Accordingly, a teacher is expected to balance between a need for “responsiveness to what emerges from students’ work” and “accountability to an agenda for knowledge development” (p. 198). The latter summons planning and carrying out activities with certain goals, while utilizing available resources. These considerations invite us to consider the transformational potential of digital technology as it features in actual teaching and learning practices. To understand why a teacher takes one path to realize this potential and not the other, it may be useful to attend not only to teachers’ general attributes (*e.g.*, knowledge, beliefs) but also to what we term *transformational spaces*.

Transformational spaces are opportunities for a change in the established teaching and learning practices that a teacher recognizes and implements, often when a particular piece of technology becomes available to her, while responding to specific students’ work and relating to an overarching teaching agenda. A transformational space is sensitive to time and location. It relates to the way a teacher manages the tension between a predetermined knowledge agenda and the flexibility that responsiveness to students’ work requires. Accordingly, we suggest that an understanding of transformations of teacher practices is inseparable from accounting for the amount of space that the teacher makes for these

transformations to unfold in the first place. This space is influenced and shaped by the teacher's knowledge and beliefs, as well as by her ability to respond to students' work. But the construct primarily concerns the transformation as it has been desired, envisaged, and planned by the teacher as well as the whether and how it is carried out in a lesson. Metaphorically speaking, if a transformation were a material object, its size and shape would be substantially influenced by the physical space that has been allocated to it.

Drawing on these ideas, we propose that, more often than not, a teacher's usage of a digital resource in a classroom is a materialization of their decisions. These decisions are partially made in advance (*e.g.*, when planning a lesson), and partially in-the-moment (*i.e.*, during the lesson), on the basis of both a teacher's overarching goals, and her contingent need to respond to students' work, reactions, and activity. Correspondingly, we use the metaphor of 'making a transformational space' to refer to lesson-planning, while 'filling a transformational space' pertains to the in-the-moment decisions a teacher makes during the lesson.

Let us elaborate on the metaphor of 'making a transformational space'. Andrà, Rouleau, Liljedahl and Di Martino (2019) note that teachers often come to professional development with tensions, and they seek for a way to alleviate their tensions. At the same time, teachers are not 'prisoners' of their own tensions (Andrà *et al.*, 2019), but they enact transformations of their practices so as to resolve (or, better live with) their tensions. Tensions can be seen as a modality through which teachers make space for a transformation. In Emilia's case, we can identify a tension between the actual and desired class dynamic, while Valeria does not seem to have a tension to resolve. We might suggest that a change in Emilia's lesson is initiated by a new resource and a tension, but we also clarify that, notwithstanding the value of teacher's knowledge and beliefs, transformational spaces are sharpened towards *actions* that a teacher prepares to take in a specific instructional situation (*e.g.*, Herbst, 2003, p. 231), and a focus on actions entails also a focus on the *resources* employed to realize such actions, as well as on the *goals* the actions aim to address.

We are interested in understanding how teachers make space for a transformation of their lessons to become more participatory. This orientation emerges from their descriptions of their classes, their goals and their intended uses of resources. Hence, the first focus of our research is on the lesson plan. From the cases of Valeria and Emilia we argue that making space for a transformation is shaped by a teacher's goals, her description of the class and its dynamics, her tensions and needs, as well as the use of the resources she intends to make.

The second focus is on how teachers fill the transformational space during the lesson. This allows us to review the accounts of envisaged lessons that teachers make in relation to observable decisions they take. No matter how detailed the lesson plan is, deviations are hard to avoid. Drawing on Much and Shweder (1978), we claim that teachers' decisions can be directly observed in those moments when classroom rules are verbally "tested, employed, clarified and negotiated" (p. 20). Hence, decisions that teachers make and account for, during an actual lesson, are inter-

preted as "speech acts that correlate with behavioral episodes in which some breach of social expectation occurs" (p. 22). The function of these verbal accounts is to allow for the interpretation of an action or a decision so as to "make it more understandable according to the criteria for expectable, approvable behavior" (p. 22). In other words, deviations from the planned lesson, specifically those that are justified by the teacher, can help clarify the central needs and goals a teacher has.

Transformational spaces are not static entities that once constructed are incapable of changing. Instead, we see them as dynamic structures that mold as a response to significant encounters between a teacher and a resource. According to Herbst (2003), teachers design activities in and through which students learn ideas (and, thus, a teacher has goals to achieve). Hence, the resources available are crucial to the students' process of producing understanding and this, in turn, affects the teachers' attitude towards a resource.

To sum up, it seems reasonable to start tracing the evolution of a transformational space from a teacher's first encounter with a new resource. The planning of the resource-infused lesson is important since this is where the most fine-grain details of the resource's usage are engendered. This is the phase where a teacher can make space for a transformation to occur. The actual teaching with the resource is also important. When decisions need to be made 'here and now', it seems reasonable to expect the teacher to act in a way that constitutes an organic extension (also with respect to a teacher's knowledge and beliefs) of the transformational space with which she entered the classroom. We associated these contingency-driven actions with a teacher filling a transformational space.

The metaphors of making and filling a transformational space is, in what follows, concretised again in the re-analysis of the cases of Valeria and Emilia. We add further information and details from their records of practice, which are taken as "experiential grounds to explore an intellectual problem and discuss some theoretical ideas that might shed light on that problem" (Herbst, 2003, p. 208).

Valeria making and filling a limited transformational space

Before the lesson mentioned in the opening of this paper, we observed Valeria's class three times and we had five meetings with her. Our observations and conversations suggested that she usually employed the following teaching format in that class: Valeria started her lessons by projecting slides and/or using the blackboard to show procedures or to introduce mathematical concepts before assigning exercises for individual solution. The lessons typically concluded with her solving the exercises at the blackboard for students to check their work.

The emerging transformational space of Valeria on her first encounter with mathematical video clips is tied to the school context, which is rich in technological devices. The premises seem very promising, at a first glance. We notice that Valeria acknowledged the educational opportunities that the video clips can entail for students (*e.g.*, she described them as "a good way to foster collaborative learning"). But we also note that these acknowledgements were made at some general

level without grounding them in her own teaching. Taken together, these considerations bring us to comment that the orientations and decisions that Valeria bears at her first encounter with mathematical video clips may be read as markers of her positioning the new digital resource not as a vehicle for transformation but as another piece of digital technology to add to her existing repertoire. Moreover, Valeria chose to introduce the clip in a class that she described as “not open to innovations”, with her expecting that the students would not be able to cope with such a novelty. Recalling that Valeria tended to use technology as a pedagogical aid (*i.e.*, to “save time” and “better monitor students’ progresses”, in Valeria’s words), it seemed unlikely that this time was going to be any different, and that she would be making space to transform established practices. Furthermore, Valeria planned to use the clip in a lesson targeted at recalling the material that the class studied in the past, which turned the clip into a reservoir of the ‘same old’ mathematics. Accordingly, we suggest that Valeria made a limited transformational space in her initial lesson planning.

The class engagement with the clip was completely mediated by Valeria as she brought pre-selected concepts to the focus of the class attention and elaborated on them. Her decision to display the clip on the class screen is particularly noticeable since there were laptop stations available in the classroom and they could have been used for students to explore the clip at their own pace. Accordingly, we can conclude that Valeria fills a limited transformational space in the lesson.

Emilia making and filling a multidimensional transformational space

Emilia’s lesson plan contained multiple transformational opportunities, which we decompose in three dimensions. Firstly, she made space for transformation regarding homework assignments. Emilia planned to dedicate a considerable portion of the lesson to the apparent misalignment between the two formulae for the variance; thus, not engaging with the video clip at home would hinder students’ chances to make sense of what the discussion was all about. Emilia explained, “I want them to see that it is crucial to do the assigned homework in order to understand what is going on in class”. This could be viewed as an undeclared goal of making her students materialize the opportunities that homework assignments offer. Understandably, Emilia did not discard her past experience with assigning homework to the class (in fact, she was skeptical and prepared a ‘plan B’). Still, she was prepared to spend a part of the lesson discussing the students’ homework assuming that at least some of them would submit it.

The second dimension of Emilia’s making space for students’ participation pertains to the social dynamics in the class and to her goal of enabling more students to contribute to whole-class discussions. The assignment’s request to construct lists of confusing terms provided students with an opportunity to express their voices and share them with Emilia. Through addressing their lists in the lesson, she could create a secure environment for students’ voices to unfold, while lowering the chances for a problematic dynamic to emerge. Furthermore, voicing students’ lists

would celebrate their work, positioning it as a valuable contribution to the lesson.

The third transformational dimension concerns Emilia’s desire to have more than one student participate in this class, and the tension she feels with respect to actual dynamics and the desired ones. The first voice that Emilia is silencing is her own, leaving space both for a Presenter (in a video) to introduce a formula different from hers, and for students to reflect on the embedded mathematical tension.

The conducted lesson contained apparent deviations from what Emilia planned. The first envisaged part (*i.e.*, commenting on students’ homework) extended from a few minutes to twenty, the second part (addressing the difference between the two formulae) was led not by Emilia but by students, and in the third part Emilia spontaneously generated a task that accounted for students’ concerns. We consider these deviations through the lens of nontrivial in-the-moment decisions that Emilia made. Indeed, she could have ignored the concepts that the students sent her before the lesson and concentrated on the list that she prepared under the assumption that the class will not submit their homework. She could also have cut off a spontaneous student exchange and resolve the issue herself, but she let the communication unfold. True, these transformations in students’ and Emilia’s practices match the goals that Emilia set up for the class. Still, we do not take it for granted that she noticed, acknowledged, and supported these transformations ‘on the fly’ by creating more space for new transformations to come. In this way, we see the case of Emilia as corroborating our previous hypothesis that, in contingent situations, a teacher may act in a way that is consistent with the transformational space she made at earlier stages.

Some remarks on video clips

Since 1970, the Open University in the UK has used television programmes as part of the teaching material, in the same way as books. Nowadays, many universities leverage video-based distance lessons to reach a larger number of students than those who can physically be present on campus. With Hoyles (2018), we can say that engaging with short videos is a common contemporary way to obtain new knowledge outside the classroom. Combined with the abundance of high-quality and free video materials that are easily accessible on various online platforms, it seems shortsighted not to put them in use for the benefit of classroom learning (Niess & Walker, 2010). Moreover, in the recent Covid-19 pandemic a significant portion of students around the world resorted to video materials of this sort.

Notwithstanding the above, we located only a handful of studies that examined the use of videos in mathematics teaching and learning (*e.g.*, de Araujo, Otten & Birisci, 2017), including limited research on the 40-year-long experiences at Open University. We find this situation peculiar considering two aspects. First, mathematical videos are characteristic to ‘flipped classrooms’ and ‘blended learning’—contemporary pedagogies that draw increased research interest from the international mathematics education community (*e.g.*, Lo, Hew & Chen, 2017). Yet, these research areas are mostly concerned with the classroom activity that follows students’ engagement with the videos, and videos are construed simply as a replacement to in-class lectures. Second, videos are a

common object for analysis, discussion, and reflection in teacher preparation and professional development programs (e.g., Barton, Oates, Paterson & Thomas, 2015). Educational studies in other disciplines, such as health and medical education, often examine the use of instructional videos to support teaching and learning (e.g., Dong & Goh, 2015). There, the emergent conclusion goes along the lines of “the use of videos seems to be a promising, relevant and increasingly used instructional strategy” (Forbes *et al.*, 2016, p. 56). Furthermore, in his research review on the use of videos in (mostly higher) education, Kay (2012) points out that improving learning is the number one reason for watching videos that students mention. These encouraging findings from neighboring educational disciplines might motivate mathematics education to develop its own conceptual apparatus for better understanding how video clips can be used in mathematics education.

Concluding remarks

We have attempted to understand why bringing a video clip to a classroom entailed a transformation in Emilia’s lesson but not in Valeria’s. Valeria’s case provides us with an example of a new digital resource that had a limited transformational effect with respect to students’ participation, even though the teacher had strong knowledge of and positive attitude toward mathematics and technology, and the school was technologically well resourced. Emilia’s lesson illuminates some conditions under which a transformation can occur. We recognize the specificity of video clips as a digital resource, which can support students’ understanding and reflection (Niess & Walker, 2010). We also maintain that transformation toward more participatory lessons is shaped by students’ reactions to the new resource (Herbst & Chazan, 2012). Thus, we specify that transformation takes place not only for the teacher, but it involves the classroom, its norms and practices, as a whole. We are also aware that transformation, by itself, does not necessarily always have a positive connotation, as transformations can result in poorer experiences for students. This holds true especially for video clips, which might promote an even more passive interaction with the content of the lesson, if compared to in-presence frontal teaching.

Our main hypothesis, which we illustrated in the cases of Valeria and Emilia, is that in a lesson a teacher acts in a way that is consistent with the transformational space that she made at earlier stages. In other words, we argue that to understand why the introduction of new pieces of technology leads to a change for some teachers, researchers may want to trace ‘smaller’ decisions the teacher made before the particular lesson. In both Emilia’s words and in other studies (Andrà *et al.*, 2019), we notice that teachers can come from the place of a need for change, which turns their actions into potential vehicles for it. Emilia is a teacher who experienced a need for change in one of her classes and who actively searched for a different way of teaching. She was the one to recognize the potential in a new resource design for her lesson to mobilize this potential. Emilia was also the one to undertake in-the-moment decisions in a classroom to promote new classroom dynamics and practices. In her case, the three main conditions under which a transformational space was both made

and filled are: (i) the need for change towards more participative lessons, (ii) the usage of a new resource in a way that disrupted established dynamics, and (iii) contingent actions to advance the change further.

Valeria is a teacher who is experienced and open-minded so as to acknowledge the potential of a new resource. This can be viewed as a theorized change in terms of Clark-Wilson, Robutti & Sinclair (2014). However, Valeria embedded the resource in her routine instruction, without undertaking any substantial transformation.

The main finding of our study concerns not the transformation of a teacher’s knowledge and/or beliefs as drivers of change in practice, but that transformation is anticipated in a teacher’s needs and goals, it is planned to be brought in the lesson so as to disrupt the class dynamics and it is promoted through the contingent choices made during the lesson. A teacher’s practice, as well as students’ responses may change the beliefs and the knowledge with respect to a teachers’ teaching. This seems to hold true for Emilia, whose students’ positive reactions supported her change, but also for Valeria, as her unchanged transmissive practice tacitly perpetuates her existing knowledge and beliefs.

Interestingly, we can also notice that Emilia tries to invoke a mathematical tension in her students, as if she also tries to make transformational spaces for her students, in some sense. This is a point that needs further exploration, but we can say that a transformational space may not apply solely to changes in teaching, but also to changes in classroom dynamics and to student learning.

As a final remark, we observe that while we have used the terms ‘transformation’ and ‘change’ as synonymous, further reflection may lead us to make a finer distinction: the ‘transformational space’ can be thought of as the amalgam of cognitive, affective and contextual states that can (or cannot) promote a change, while the ‘change’ can be thought of as the result of the process of transformation, which can last in time but also have ups and downs.

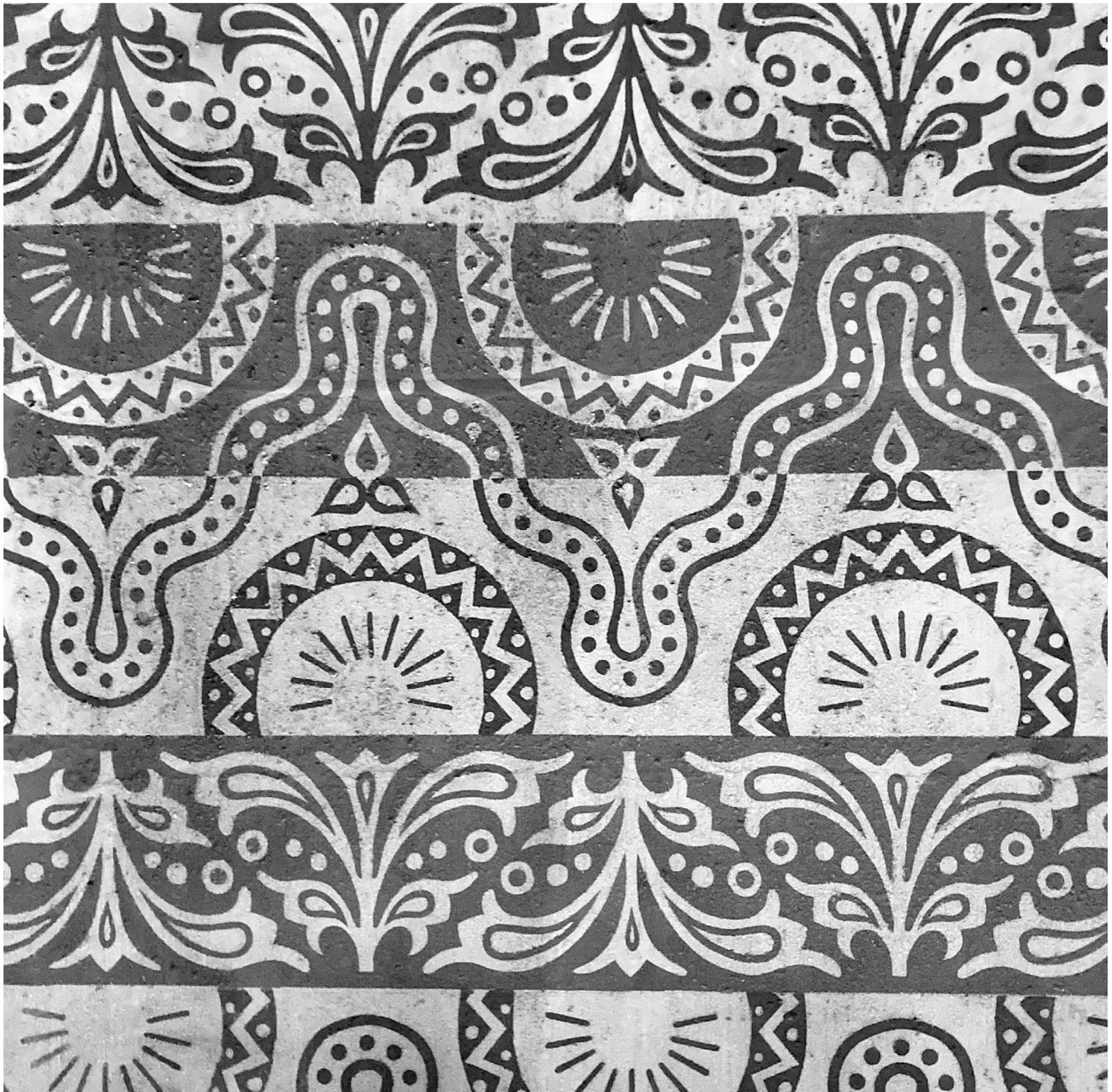
Note

[1] See <https://www.pok.polimi.it>

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Wall painting from the Matthias Church, Budapest.