ON THE FUTURE OF DESIGN IN MATHEMATICS EDUCATION RESEARCH

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Mathematics education research is still a relatively young field, with a variety of different research topics, objectives and methods. As Mogens Niss critically discussed in FLM issue **39**(2), publications of empirical, theory-guided research are playing an ever-larger role, which does not reflect the true diversity of the field. He asserts that theoretical research is accepted into publication only to the extent it refers to and is corroborated by empirical studies.

We want to further Niss' discussion, focusing on an issue mentioned in his essay: the role of the *design of objects of study* in mathematics education research. Our case is that the design of such objects is not considered to be research [1]. We consider 'design' to encompass a wide variety of different activities and associated products. The products of design can be a mathematics textbook, a questionnaire for an empirical study, a collection of problems, a model of a learner trajectory through a certain topic, a software for mathematics learning, or even simply a picture visualizing a certain aspect of mathematics. As Bakker puts it, in his reply to Niss in issue **39**(3), Niss talks about how *form* is valued more than *content*; we join Niss in claiming that there is much content being disregarded.

Design and science

The relationship between design and science has been a lasting debate, as is the debate if design constitutes a science (Cross, 2001; Galle, 2014). The main arguments are that while science describes or explains, design, such as engineering, constructs: "the natural sciences are concerned with how things are. [...] Design, on the other hand, is concerned with how things ought to be" (Simon, 1996 [1969], p. 5). Another argument is on the fundamentally different object of study: the natural vs. the artificial. While this has been theoretically debated, in reality design has been a part of many sciences, and this 'scientific design' is "not a controversial concept, but merely a reflection of the reality of modern design practice" (Cross, 2001, p. 52).

Let us consider physics, for instance. There, artifacts are needed, for example complex measuring instruments for use in large-scale experiments, such as detectors for use in particle colliders. This requires the design of such instruments, which is accepted as research without 'scientific' experiments being done, nor reflection on the design choices being carried out. Nobel prizes have even been awarded for the design of a construction, as in the case of the scanning tunnel microscope (Binnig, 1982) [2].

This is an example of the incorporation of elements of an artificial *designing science,* in the sense of engineering as a science, in this otherwise empirical natural science. Interestingly enough, even though the design was considered research, the invention of the scanning tunnel microscope was neglected by the physics community until valuable insights were gained by its use (Mody, 2004).

The state of design in mathematics education research

Returning to mathematics education, we also construct objects to enhance our understanding of learning. In addition, mathematics education is not a natural science, so the phenomena we investigate do not, in most cases, arise naturally and without our intervention; design in the educational sciences is 'doubly artificial' (compare Cole & Packer, 2016) through artificial design of artificial settings. In most research models, some sort of artifact is placed at the center of the study, with research questions addressing the interaction of learners with it, either to understand the learning of mathematics, or to allow this learning in the first place. This artifact can be as simple as a problem, which learners will solve, or as complex as a curriculum implemented over years.

An example of such an artifact are dynamic geometry softwares. They have had an enormous impact on the mathematics teaching and the mathematics education research, engendering conferences and journals. Reports of research using dynamic geometry software have appeared in practically all high ranking journals in mathematics education. But would a presentation of the software itself, not framed by any empirical study or educational theory building, have found a way into one of these journals [3]?

This is the way that such artifacts are presented, as by-products of a theoretical or, more probably, an empirical study. Oftentimes, the artifact is constructed with the sole purpose of serving as a mediating object in the study, to exhibit some properties with which the learner will interact in a certain manner. The object may play a pivotal role; it is still presented only as a tool bringing insights on the learner, not having a purpose in itself. Even if the purpose of the study is the comparison of two or more such artifacts, the design of the artifacts themselves seems to play a subordinate role. An example could be the PISA studies (OECD, 2003), for which tasks were created that enable researchers to differentiate competency levels of students. This could have been a publication on its own. Instead, the tasks were constructed solely to serve the purpose of the study to measure student performance.

The prevalence of theory, methods and empiricism in such studies is not a problem *per se,* as their goal is to further understanding on the learning of mathematics by investigating learners, not the design of some object.

Approaches incorporating design in mathematics education research

There is an approach to mathematics education research which has, at first glance, design at its core. In the early 1990s, researchers argued for a refocusing of research, identifying the central role of design in mathematics education, portraying it as a 'design science' (Collins, 1992; Wittmann, 1995). This has then found its way into mathematics research through the 'design(-based) research' approach (van den Akker, 2013). This method consists of many design cycles, where an artifact and a theory are refined by empirical analysis. While the design research process includes the development of the artifact, the main purpose of this process is however not a refined designed object, but a *theory*, and the application of theory to practice (Gravemeijer, 2013). Researchers engaging in this type of research aim to further the application of their improved theory, it being field-tested in many cycles. This is a valuable aim, but again it does not focus on developing an artifact, but, as in the example of PISA, only to the extent that theory building requires it.

There is another variant of design-based research, named 'research-based design', in which the design of the object seems to be the main objective. But design in the sense of research-based design means a design to solve an empirical problem with its validation and optimization in several cycles, not design stemming from the perspective of mathematics education itself (Plomp, 2013). Even this type of design-based research sets the focus on and requires a cycle of empirical validation of an initial design. Returning to the example of the scanning tunnel microscope: Following the line of thought of research-based design, one would have started with the design of an earlier type of microscope such as an electron microscope. Then, to address a problem encountered in practice, in this case a resolution too low to picture single atoms, empirical refinement would be carried out. Would this process somehow end up with the scanning tunnel microscope? That is highly doubtful. The design of such a complex artifact requires mental leaps to be made and problems to be addressed which are not answered or even raised by empirical testing cycles. The ideas for its design were not gained through iterative improvement of an existing design, but by implementing a fundamentally new concept, using ideas derived from other fields of physics. Of course, every object is refined and empirical evidence can help in this process, but regarding its nascence, not every complex object is the result of a gradually improving design of a simple starting construction.

There is a strand of mathematics education focusing on design, named 'educational design' as opposed to 'educational research'. The community is small in comparison with 'educational research', but nevertheless they are organized in a society, the International Society for Design and Development in Education, and publish an online journal, the *Educational Designer.* It is devoted to the research into the design process of educational tools, with the hope of identifying best practices and in this way helping researchers to improve the design process, as well as "increasing the impact of professional design on educational practice" [4]. In Cross's (2001) characterization of design and science, this would correspond to the 'science of design'.

Allowing researchers to demonstrate their design process and in this way further the 'theory of design' will certainly help improving the design of future educational tools, and is a unique and important undertaking. But even here, the design, as in the construction of an artifact itself or 'scientific design' (Cross, 2001), is not considered to be research and cannot be published in mathematics education journals.

In any case, summing up the discussion of design-based research, research-based design, and educational design, despite the recognition of its central role in mathematics education research, the design of objects for the learning of mathematics has not been recognized as research in itself. Designers can only publish their designed object by attaching it to an empirical study, embedding it in a design-based research process or by reflecting and documenting the design process.

As the obligatory exception to the rule, some designs and constructions can be published in practitioner journals; these are however not considered to be reporting on mathematics education research. These journals are intended for teachers and provide ideas which relate to classroom practice, bridging the gap between research and school.

'Stoffdidaktik'

There is of course a school of research devoted to the analysis and construction of certain objects, called in German *Stoffdidaktik*, or subject-matter didactics (Jahnke, 2019). Its focus, however, is on purely mathematical analysis and construction. The design of a syllabus based on content analysis for teaching differentiation would be a prime example. If the design of a series of questions and tasks on a certain topic would be considered research in this sense could be debated. Finally, the design of an app engaging students in activities around fractions would certainly not fall into this category.

Subject-matter didactics, the dominant type of research in the early 20th century, faced much criticism for not including empirical evidence. Curricula were designed at the drawing board, without considering how students actually reacted to them in this process. This led to the inclusion of empirical studies in subject-matter didactics, to the point where Schubring (2015) states, "In contrast to the traditional subject-matter didactics free of empiricism, nowadays an empirical component should be self-evident in every teaching proposal" (p. 36). This quote also showcases the shift of German-speaking didactics from content analysis to teaching proposals and their evaluation; traditional subject-matter didactics has faded from the journal-published research landscape (Jahnke, 2010). There is still research being carried out in subject-matter didactics, but its decline certainly falls into the category of "conceptually or theoretically oriented reflective research without an empirical component" which Niss (2019, p. 6) identifies as the kind of research suffering the most from the trend to empiricism in modern mathematics education.

In the case of empirical studies of the kind Niss describes, the view of the object as a tool limits the amount of work put into it. By its very nature, this kind of 'small-scale research' also produces small, strictly bounded objects; every step beyond these bounds would not only mean unnecessary work, but also a blurring of the exactly defined research parameters. Again, the PISA studies are a prototypical example. While this may be useful to showcase behavior or performance of learners, it is the opposite of a rich learning environment. It is designed to understand and sometimes measure learning, but not to promote it or to extend our understanding of it.

The role of technology

Technological advances amplify this problem by expanding the field of learning software design. In the past, mathematics was learned only in classrooms with textbooks, a context within the reach of mathematics education research. Nowadays, there are thousands of mathematics learning apps and programs on the market, which are used at home more than in the controlled and supervised school environment. They stem from companies, with only a small percentage having any relationship with mathematics education research. The 'edtech' market is growing, resulting in disrupting technology such as apps that can solve textbook equations instantly, and also investments in education by some of the world's largest companies, seeking to shape education through their own policies.

The development of artifacts, especially larger projects, is happening in the education industry and through independent developers outside of academic education research, leaving mathematics education in the passive role of an observer (see Abrahamson, 2015). To manage a large project in mathematics education would mean to have to divide the projects into several parts, such as artifact design, theory, and empirical validation. The latter two roles count as mathematics education research, but design in itself does not, and thus is done by researchers as a side-line. This is a limiting factor in the project size and scope, restricting research to projects in which the design part is small enough to be handled by researchers not using their full resources for it. Technology influences this trend by enhancing our ability to collect and generate data as well as providing a larger variety of comprehensive learning environments. This leads to the number of large projects growing, aggravating the need for designers, which need their place in the mathematics education community.

Furthermore, dismissing the constructions would also mean the *inability* to construct such artifacts, and with it not having knowledge on their functioning. It would not only make mathematics education research passive, but blind, forced to treat artifacts as black boxes, only capable of measuring outcomes. It cannot be in the interests of the research community to exclude design knowledge from mathematics education.

Examples and future directions

An example of research, which should be considered as such in mathematics education, is a well-thought out curriculum proposal together with the motivation and reasons why the author might consider it especially well suited for implementation. Another kind could be a set of interesting problems, which are capable of enlightening aspects of a certain theory, for example able to provoke moments of intuitive reasoning.

The foundation of these examples is a collection of mathematics, which has undergone didactical exposition of some kind. Given this vital role of mathematics exposition, this line of research in the tradition of German subject-matter didactics should certainly find its place in modern mathematics education. Jahnke (2010) already pointed out this "gradual disappearance of the subject" (p. 22) from mathematics education research, relying on publication records of the *Journal für Mathematik-Didaktik* [5]. While this kind of research is strictly speaking not design, the two are strongly tied, and it represents another aspect of not well-represented mathematics education research.

Of course, not every design constitutes research. As Niss (2019) puts it,

> as most important designs and constructions are required to have certain properties and meet certain specifications before the resulting constructions are installed, design disciplines are scientific only to the extent they can provide well-founded evidence and reasons to believe that their designs possess certain such properties to a satisfactory degree. (p. 10)

In the case of physics, the test which a construction has to pass is subject to engineering and physical standards, requiring for example a detector to achieve a certain fidelity with regard to measurements. In mathematics education, such criteria would have to be established in a dialectic process. A starting point could be given through the criteria for intervention in design-based research given in Plomp (2013): relevance, consistency, practicality, and effectiveness. As an example, consistency might describe the degree to which a learning software demonstrates its fidelity to mathematics, by explaining the mathematics behind the software and its representation for the user, together with the manipulation choices of the user on this software. Another criterion could be practicality, *i.e.* if the design has a potential use in mathematics education, justified by a comprehensible argumentation. These criteria do have to be applied carefully, as not to confine designs into strict boundaries, thereby restricting creativity and innovation.

Our aim is not to diminish and not even to criticize the ways researchers work or do design, but quite the opposite. We believe that mathematics education research encompasses a variety of different approaches that all make valuable contributions, and that designing artifacts for mathematics education is part of these valuable approaches.

Mathematics education as a scientific discipline has over the time acquired autonomy from mathematics, as its importance was acknowledged as being too great to be just research being done as side projects of mathematicians such as Pólya, Hadamard, or Klein [6]. Given the importance of design in mathematics education research, it is reasonable to give it space in the communication channels of our community. Designs also play a crucial role in the link of mathematics education research and practice, a link which would be strengthened by legitimizing design as research.

In the same way that physicists chose to consider the construction of experimental tools to be a part of physics and not to outsource it into engineering, to accept design as research in mathematics education is a *choice* that the field can make. This would not change the nature of research in mathematics education in any way. It represents a division of labor in the current type of research. There is no need for every researcher to do every part of a research project; some researchers may design an artifact, others collect data using it [7], and again others build theories explaining observed phenomena. The acceptance of this would be a choice that can benefit both sides, designers and empirical researchers, and avoid the pitfalls portrayed by Niss of research which lacks clarity and purpose. This would of course require journals and conferences to accept contributions presenting artifacts of interest to mathematics education, following certain standards.

With this in mind, it is pleasing to hear the editor-in-chief of one of the most influential journals in mathematics education emphasizes the importance of "non-empirical articles with important messages" (Bakker, 2019, p. 44). This message cannot only be a theoretical perspective on a topic as asked for by Niss, but also the presentation of a certain way of seeing, doing or interacting with mathematics through a well-thought-out design.

Notes

[1] This is disregarding some exceptions that prove this rule. See, for example Hewitt (2016) or Jankvist and Niss (2015).

[2] The Nobel prize was in fact "for their design of the scanning tunneling microscope"; see https://www.nobelprize.org/prizes/physics/1986/binnig/facts/

[3] Again, there are rare exceptions (*e.g.,* ZDM issue 43(3)), but their rarity supports our point.

[4] From the inaugural editorial in *Educational Designer,* by Burkhardt, McKenney & Pead; see https://www.educationaldesigner.org/ed/volume1/issue1/article0/

[5] Publishing design in mathematics seems to follow a similar decline, noting the publication of Papert's LOGO and the Labordes' Cabri-Géomètre in the sixties and eighties together with the absence of similar, more recent publications.

[6] Of course, mathematics education has its roots not only in mathematics but also in other fields such as psychology, sociology, and in the profession of mathematics teaching which should not be forgotten.

[7] There even exist journals for publishing data, such as 'Scientific Data', https://www.nature.com/sdata/.

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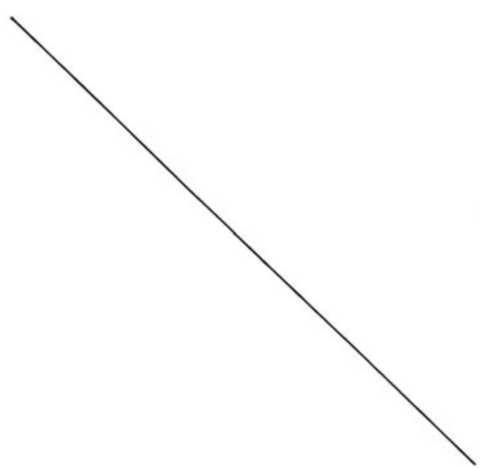

Drawn by Arthur, age 15, on being asked to "Draw something mathematical".